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THE HOLLOW UNIVERSE

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FOREWORD

The Whidden Lectures were established in 1954 by E. C. Fox, B.A., LL.D., of Toronto, the senior member of the Board of Governors, to honour the memory of the late Reverend Howard P. Whidden, D.D., LL.D., D.C.L., F.R.S.C., 1871–1952, Chancellor of McMaster University from 1923 to 1941. The purpose of the lectures is to bring to the University scholars who will help students to cross the barriers existing among the academic departments of a modern University. The lectures are not restricted as to general theme.

Dr. Whidden was a member of a family resident in Antigonish, N.S., since 1761, after earlier settlement in New England in 1700. Born in Nova Scotia, he was educated at Acadia University, McMaster University, and the University of Chicago, and subsequently served as the minister of Baptist churches in Ontario, Manitoba and Ohio. From 1913 to 1923 he was President of Brandon College in Manitoba, then affiliated with McMaster University, and served in the House of Commons in Ottawa from 1917 to 1921 as the Union Government member for Brandon. Assuming responsibility as chief executive officer at McMaster in 1923, he was responsible for negotiations and actions leading to the transfer of the institution to Hamilton in 1930, from Toronto where it had been established in 1887. He is remembered as a man of striking appearance, unusual dignity, effective leadership, ready tolerance, deep Christian conviction, and broad educational outlook. Chiefly to him the University owes what amounted to its second founding.

Charles De Koninck, D.Ph. (Louvain), F.R.C.S., who delivered the fourth series in January 1959, is Professor of the Philosophy of Nature at Laval University in Quebec. Born and educated in Belgium, he has taught in Canada since 1934. In the course of these twenty-five years as a member of the Laval teaching staff, he served as Dean of the Faculty of Philosophy from 1939 to 1956, as a lecturer in the Faculty of Theology since 1937, and as a visiting lecturer in many places in Canada, the United States, Europe and Central and South America. He has been a Professor Extraordinary at the National Autonomous University of Mexico, and a visiting Professor at the University of Notre Dame, Indiana. He is a Fellow of the Royal Society of Canada, President of the Canadian Academy of St. Thomas, Knight Commander of the Order of St. Gregory and a member of the Roman Pontifical Academy of St. Thomas Aquinas. Among his recent publications have been *Un Paradoxe du devenir par contradiction*, 1955, and *Random Reflections on Science and Calculation*, 1956. Forthcoming soon is *An Introduction to the Study of Nature.*

His theme, *The Hollow Universe,* is a challenge to the view that, since science has been spectacularly successful in the investigation and control of nature, scientific knowledge is of greater value than knowledge obtained in any other way. He chose to examine this from the points of view of mathematics, physics and biology. So challenging a thesis met with immediate student attention. The lectures were largely attended, and

provoked the desirable prolonged discussion that is a sign of undergraduate interest and sprightly difference of opinion.

G. P. GILMOUR

PRESIDENT'S OFFICE
MCMASTER UNIVERSITY
July 1959

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PREFACE

THESE lectures are not intended as a brief course in the philosophy of science. They are concerned only with views of thought and nature suggested by certain advanced interpreters of the scientific outlook. The final opinion of these men is by no means accepted by all scientists, and indeed is not even the common view amongst them. Yet it is the one which, in the popular mind, is thought the most truly scientific. We have all heard of computer-men who would lead us to believe that they are on the way to constructing machines as truly capable of mathematical thinking as any man has ever been, with the implication that we shall have to change our mind about the nature of man as characterized by reason. As another sample of the advanced views, we have Bertrand Russell's pronouncement that physics proves man to be a mere collection of occurrences, that 'Mr. Smith' is in fact a collective name for a bundle of events. Again, there is the assertion of certain outstanding biologists that 'what life is' has become a meaningless question; that if the behaviour of organisms is to be explained at all, this will have to be done in terms of physics and chemistry.

We do not question the competence of those who hold such views: all of them have made signal contributions in the field of science and its philosophy. But it may be opportune to determine the import of their opinions by comparing them with doctrines which they are believed to replace. Every conception which seems the consequence of genuine progress in science, no matter how nihilistic and disturbing, must be honestly faced, and this is all that I mean to attempt. It should be clear, then, that my intention is not to attack the new positions, except where they are intended as total and final. Rather, my aim is to show that unless we examine them in the light of earlier views, we shall fail to appreciate what has actually been achieved; I also hope to make plain that if we generalize this new scientific outlook as it is commonly understood, if we accept it as the one true way of thinking about nature, the only wonder left to us will be wonder at the hollowness of the universe, both of nature and of thought. Even Einstein's conception of the material world as the manifestation of a wondrous intelligence would be out of date. A friend tells me that his little daughter, having seen the title of these lectures, begged him to ask me: 'Is the hollow in the universe or in our heads?'

I wish to apologize in advance, if I may, to those who will find my English a strain. I am not sure that I should not have to voice a similar warning were I to address a French or a Flemish audience in a language more natural to me; I suspect that even my close friends share my misgivings.

Finally, I must meet a debt of gratitude to the Chancellor and to the President of McMaster University, as well as to the members of its Faculty, who showed me such cordial hospitality.

C. De K.

I

The World of Symbolic Construction, or Two is One Twice Over

One thing cannot be made out of two nor two out of one.

DEMOCRITUS

What each number is is what it is once, e.g. what six is is not what it is twice or thrice, but what it is once; for six is once six.

ARISTOTLE

If you feel dismay at the subject of this first lecture let me reassure you with the promise that we shall not go far beyond the equation $2 = 1 + 1$. Indeed, I can offer you at once even better relief, since our main concern will be, not primarily with 'two equals one plus one', which could become difficult enough, but with the simpler equation $1 + 1 = 1 + 1$. (We shall not even approach the vastly more intricate $2 + 3 = 3 + 2$, let alone the general law $x + y = y + x$, which is a more convenient way of putting down 'If a second number be added to any given number the result is the same as if the first given number had been added to the second number'.)

The problem of whether there is any real difference between the expressions $1 + 1 = 1 + 1$ and $1 + 1 = 2$ has divided philosophers from the beginning, so that I cannot promise that our reflections upon these equations will be as simple as they themselves appear. In fact we shall soon find ourselves with so much infinity on hand that we shall hardly know what to do with it.

Nor should we expect obvious and trivial findings when the best minds, from Thales to Wittgenstein, have been at loggerheads over the force of these simple expressions. How casually we may sidestep the difficulty of apparently easy questions did not go unnoticed by Wittgenstein. He had in mind the now generally accepted position that time is defined by the way in which we measure it, and it may be remarked in passing that there is indeed a context in which this attitude is fair and true. But he faces an objection which anyone might raise at this point: If someone asks what *time* is, you ask in return, 'How do we measure time?' But time and the measurement of time are two different things. It is as if someone asked 'What is a book?' and you replied, 'How does one obtain a book?' We shall see that number and thought are also treated in this evasive way nowadays. Meantime, your attention may be quickened by the promise that in our reflections on such elementary points we shall have to inquire in what exact sense it may be allowed that our great computing machines can think.

You may have recently noticed in the newspapers that groups of psychologists, neurophysiologists and linguists gathered, late in November, at Britain's National Physical Laboratory for an international conference on The Mechanization of Thought Processes. The scientific correspondent of the *Manchester Guardian* reported on this occasion

> that it is rash to rule out the likelihood that one day it may be possible to design machines which can for themselves

> (given suitable experience of practical problems) form the kind of judgment which characterizes the higher flights of creative intellectual activity. It will be a long time, of course, before a machine will do for modern physics what Einstein did for it at the turn of the century. But it is important that at this stage there appears to be no obvious reason why this should not turn out to be possible.

I am not shocked at this bold anticipation. The challenge of the machines is a fine thing for us all if it drives us to investigate more clearly what thought is, and, if need be, to disengage the 'higher flights of creative intellectual activity' from the machinery which attends them in the human brain. Meanwhile, in the sense in which logic and mathematics are understood nowadays, the new electronic hardware certainly proves that machines can be splendid logicians and mathematicians.

The foundation of our ultra-modern logic and mathematics was already discerned, and very clearly, by a Greek philosopher, Democritus, some 2,400 years ago. The position which Aristotle attributes to this predecessor of his is again so disarmingly simple that I have some misgivings in offering it as the basis of the vast edifice which in the last few centuries has been reared upon it. But that was the way with Greek philosophers: in pondering the simplest things and searching in them for the basis of whatever needs to be explained, they showed themselves to be possessors of true wisdom. Edgar Allan Poe makes an interesting remark somewhat to this effect in *The Purloined Letter*:

The principle of the *vis inertiae* . . . seems to be identical in physics and metaphysics. It is not more true in the former, that a large body is with more difficulty set in motion than a smaller one, and that its subsequent *momentum* is commensurate with this difficulty, than it is, in the latter, that intellects of the vaster capacity, while more forcible, more constant, and more eventful in their movements than those of inferior grade, are yet the less readily moved, and more embarrassed and full of hesitation in the first few steps of their progress.

I waive what Poe calls ' metaphysics ' and come to Democritus's opinion as reported by Aristotle: ' One thing cannot be made out of two nor two out of one.' Democritus meant something which may appear quite trivial, viz., that the number two is exactly the same as one plus one, so that to say $1 + 1 = 2$ is just another way of saying $1 + 1 = 1 + 1$. Two, then, is nothing new over and above one and one. Though Democritus is the first to put it so clearly, the same idea was already held by Thales, who believed that numbers were actually just heaps, or, as one would say today, mere classes or bundles.[1]

[1] ' . . . Things that are thus two in act [i.e. each having a complete and distinct actuality of its own, such as Socrates and Plato], are never one in act, whereas if they are two only in potency, they can be one [in act], like things that are double, as two halves potentially; for the complete actualization of the halves divides them one from the other; therefore if the substance is one, it will not consist of substances present in it and present in this way, which Democritus describes rightly; he says *one thing cannot be made out of two nor two out of one*; for he identifies substance with his indivisible magnitudes. It is clear therefore that the same will hold good of number, if number is no more than an aggregate of units, as is said by some [e.g. Thales, who is said to have described numbers as heaps or bundles of units]; for two is either not one, or the unit is not present in it in act.' *Metaph.* VII. 13. 1039*a* 1–15.

The statement of Democritus may seem trivial, but I hasten to urge that it is not. Aristotle was fully aware of its grave implications and would have recognized at once how it illustrates the most basic principle of what our age intends by ' logic ' and ' mathematics ', as distinguished from some earlier meanings of these names. It contains, and not merely in embryo, the modern conception of number as a mere collection or aggregate, as a logical fiction and symbolic construction. Nearly half a century ago Bertrand Russell, in his *Introduction to Mathematical Philosophy*, explaining to the English public Frege's definition of number, observed quite pointedly ' that number is a way of bringing together certain collections . . . '. However, the implication of this conception of number was eventually more fully realized by that early pupil of his, to whom we have already referred, Ludwig Wittgenstein. In *My Mental Development* Lord Russell says that he himself ' had thought of mathematics with reverence, and suffered when Wittgenstein led me to regard it as nothing but tautologies '. Russell's disappointment must not obscure the fact that Democritus's way of treating number provides the most exact of the sciences with an indispensable tool, one that had been available of course from the earliest days of arithmetic as the art of calculation, but the vast potentialities of which had passed unnoticed. Plato called the technique ' logistikè '; Aristotle's term was ' logismos '. But I must beg you to notice that both distinguished this technique of calculation from the science which it served. Science was the effect of reasoning by way of syllogistic demonstration, as exhibited later and more completely in the *Elements* of Euclid.[1] Demonstration, in this sense, was not the same as calculation—although in mathematics one could not demonstrate without computing. But the computation itself was no proof.[2] We shall return to this point later. Meantime, let it suffice that mathematics nowadays is almost wholly identified with and confined to the art of calculation. This is what is implied by the conception of number as 'a way of bringing together certain collections'. Numbers are defined by the operations that can be performed upon them. As Hermann Weyl put it: 'their being exhausts itself in the functional role which they play and their relations of more or less.' As a result, 0, 1, -1, $\frac{1}{2}$, $\sqrt{2}$, &c. are in this sense just as much numbers as 2 or 3.

That mathematics is now largely equated with its technique, or rather that all mathematical entities are defined by the technique for manipulating them, is an observation based upon what mathematicians now

[1] By syllogistic demonstration I do not mean the syllogistic rules of Aristotle's *Analytics*, which are there demonstrated but which are not themselves demonstrations.

[2] Unless proof be taken in a very broad sense. For example, if you tell someone that Mr. Smith is in the next room, and he asks you to prove it, you could do so by opening the door to show Mr. Smith really there. Such a proof is called 'demonstration to sense'. There is no reason why 'proof' should not be used apropos of computation, or even 'demonstration', so long as we recognize that each of these terms has several distinct meanings.

attend to, especially those who reflect upon what they are doing. This assimilation of mathematics with the mushrooming art of calculation was explicity asserted by the late John von Neumann: The new calculus, discovered by Newton and Leibniz, or rather all of analysis, which sprang from it, must be held 'the first achievement of modern mathematics, and it is difficult to overestimate its importance. I think it defines more unequivocally than anything else the inception of modern mathematics, and the system of logical analysis, which is its logical development, still constitutes the greatest technical advance in exact thinking.'[1]

The art of calculation enjoys a curious freedom, which is exhibited by the fact that, when we carry it on, we are never concerned with what those things are which we calculate about, but only with how we can operate upon them. These things we define in terms of what we can do with them—much like the physicist who does not care what time is apart from

[1] Whether von Neumann saw the new calculus, or all of analysis, as a mere method, like elementary algebra, or as a distinct branch of mathematical science, in the manner of plane geometry or number theory, I cannot say. He did write, however, in the recently published notes on *The Computer and the Brain*, that he regarded the human brain as 'a computing machine, in the proper sense'. I do not mean that the modern mathematician does not demonstrate (even though some philosophers of mathematics profess to reject all of Aristotle's *Posteriora Analytica*); the point is that the demonstrative form goes unnoticed. After all, one may reason correctly without expressly attending to the second intentions of logic. The way the problem 'Whether there is a last prime number' is still discussed nowadays, reveals the definition of prime number being used as a middle term; but no one will tell you so, and some may insist that the definition is a creative one and therefore not a middle term in Aristotle's sense.

his measure of it. My statement about the indifference of calculation remains nonetheless ambiguous. I must therefore explain at least two ways in which it may be understood. We are all familiar with Bertrand Russell's remark that 'Mathematics may be defined as the subject in which we never know what we are talking about, nor whether what we are saying is true.' A French mathematician, Jacques-Hadamard, gives an illustration of what this means, although I do not believe that he intends all that Russell has in mind.[1] Here is the example:

Having bought 6 metres of cloth at 12 francs a metre, how much does one have to pay? In raising this problem, are we really talking about cloth? Not at all. Instead of

[1] What Lord Russell intended in *Mathematics and the Metaphysicians* is that pure mathematics starts from certain rules of inference, by which we can infer that *if* one proposition is true, then so is some other proposition. If we adopt this so-called axiomatic method and consider mathematics as no more than a hypothetico-deductive system, the conclusions arrived at may be 'good' as to consequence, but their truth remains undecided. There is a world of difference between '*If* the exterior angle of the plane triangle is equal to the two opposite interior angles . . .' and 'The exterior angle of the plane triangle *is* equal to . . .', even though the conclusion will in either case be materially the same: 'The three angles are equal to two right angles.' Both consequences are good, but only the latter would be true. (By 'truth' we mean the conformity of the mind with what is—*is*, in this instance, referring to the existence of the triangle in abstraction, not to physical existence.) Now, it seems to me that once we have decided that hypotheses are all that is needed in mathematics and that truth is irrelevant to this subject, we have ruled out the problem of foundations. What, then, can be meant by the assertion that nowadays the foundations of mathematics are less certain than ever before if they were never intended to be certain? A valid consequence does not depend upon the truth of its principles. If the moon is made of green cheese, we may some day enjoy a slice of it.

asking the price of 6 metres of cloth at 12 francs a metre we could just as well have asked the price of 6 pounds of meat at 12 francs a pound. We might have replaced the meat by copra, and the pupil could have provided the answer without even asking the teacher what copra is. Hence, in raising this problem one does not know what one is talking about; or, to put it otherwise, there is no need to know it. Here, then, in a first, simple instance, we have the notion of mathematical abstraction.

This is one way of understanding mathematical abstraction, but there are other ways, too. (There was Plato's, and then the widely different one which Aristotle taught—not to mention M. Hadamard's own in *The Psychology of Invention in the Mathematical Field.* None of these however has anything to do with the case in point.)

Here is a further interpretation which we should find helpful. M. Hadamard points out that in computing we forget all about cloth, meat, copra, and francs; but we do not forget about numbers. Yet number itself is a very ambiguous term. It could be a name, but also a mere symbol. Permit me to dwell upon this as a possibly relevant distinction. 'Man' is a name which stands for a certain type of animal, and 'Socrates' is the name for an individual man. But 'the freckled, vagrant, accordion-playing, unshaven barber, at the corner of Blank Street last night' is not exactly a name, yet we can pin down this agglomerate by a single symbol, (let it be Y), and use it in the calculus of propositions. The reason we cannot cover with a single name all the incidentals of our barber—whose name is actually Oscar—is that they are incidentals; they do not have the kind of unity which naming requires. What we have symbolized by Y is an accidental whole, a mass of things which need not belong together: for a man can be freckled without being a vagrant, vagrant without playing the accordion, a barber without being unshaven, and play the accordion on some other street-corner. All this taken together is what we call an accidental whole—the difference between a heap and a man. And though to name the aggregate is impossible, to assign a symbol to it is the simplest thing in the world. For what is not in itself *per se* one, in the way a man or a circle is, can nevertheless be gathered together by the mind, set apart, and assigned a symbol that is one. We are not of course thinking of *heap* or *aggregate* as such, of which a definite notion is easily possible, but of a particular heap which, by itself, cannot have a name. We can identify one as 'the heap in Mr. Smith's back yard', but this is not to name it. But, though the heap be made up of old tires, rusty pipe, broken milk-bottles, a wilted pumpkin, &c., nothing can stop us from saying: let Y stand for this heap. Our queer jumble may be the only one of its kind, but it is designated clearly enough by being in Mr. Smith's back yard, and there is no ambiguity about the symbol, if clearly referred to such a heap.

In other words, symbols, unlike names, abstract from what is one *per se*. Aquinas pointed out that 'Symbolum collectionem quamdam importat', as the

sign of what is 'thrown together', meaning that, as distinguished from the sign that is a name, the symbol refers to what is no more than a collection, an aggregate, a heap, an accidental whole.[1] Now, if the number two be taken as a collection, prescinding from any unity the collection may have, two becomes not really a name, but a symbol—though it surely looks like a name. This assumption has the advantage of dispelling ambiguity from calculation. For it is plain that, while computing with 2, we need not trouble ourselves as to whether 2 is in fact one two, a *per se* whole; or two ones, a class or collection. Hence, from this point of view, $1 + 1 = 2$ is exactly the same as $1 + 1 = 1 + 1$; 2 is $1 + 1$ for short, which is how Democritus wanted it. For, if 2 be taken as something more than $1 + 1$, if two enjoys its own irreducible unity, so that by removing one of its units you would destroy the new thing which is two, this number would not be just

[1] The ancient distinction between what is *per se*—either in being, in unity, occurrence, &c.—and the abysmal *per accidens*, the subject of Plato's *Sophist*, is largely ignored nowadays, even by representatives of traditional philosophy, as one can see in their writings on existence and contingency. But all the main principles of this philosophy stand or fall with the old distinction. Failure to make it must lead to the identification of name and symbol; of logic and mathematics with the mechanics of computation. We must agree nonetheless that there are domains in which things can be both studied and handled efficiently, where the distinction is indeed of no account. The trucking business ignores it: a heap can be hauled away as well as an elephant. Moreover, the following sentences are grammatically correct: 'All three-cornered spheres are yellow'; 'nothing exists, including me'; 'Mr. Smith never was nor ever shall be more than a mere bundle of events'. We can say ever so much more than we can possibly think, and say it well.

$1 + 1$. Now, notice that this aspect of number, whatever be thought of its validity, will have to be disregarded by the computer, human or mechanical. For the computer must be as indifferent to what numbers are, apart from their being collections, as the highway policeman who checks the weight of trucks is indifferent to whether the loads are of potatoes, cement, horses, or men. From the computer's point of view, the only thing found in 2, which is not found in $1 + 1$ is the symbol, and who is responsible for that?

Traditional philosophy could now make itself very troublesome, though. If numbers are only accidental collections, why should they reveal distinctive properties that follow analytically from the kind of number they are? The number two, for instance, is of a different kind than the number three: the former is even, the latter odd, and each, as such, has certain demonstrable properties; again, both are prime numbers, i.e., numbers that cannot be broken up into factors; 2 is the first of them, and 3 is the next. To what do these properties belong? How does symbolic construction account for them? But this problem—if it is a problem—does not concern the computer: characteristics like these arise apparently from the operations he performs upon certain collections, and in terms of these operations he will cheerfully define both numbers and properties. Thus, 2, 3, 5, 7, &c. will be prime numbers because they are divisible only by the unit; 'divisible' refers to his operation, not to what prime numbers may be apart from it. And the same

holds for geometrical entities. As Hermann Weyl put it: 'For the mathematician it is irrelevant what circles are. It is of importance only in what manner a circle may be given . . .'. It is as if we said that it is irrelevant for us what man is; it is of importance only to know in what manner we can meet one. In terms of traditional philosophy, this means that the mathematician is not at all concerned with definable natures, nor with properties demonstrable from the definition of 'what they are' absolutely, taken as the middle term; which is another way of saying that the mathematician abstracts from science in Aristotle's sense—a position which Aristotle and Euclid would never accept, although both would readily grant that while computing the mathematician must so abstract.

If computation, then, can be such a rigorous technique, it is precisely because, in it, number is taken as no more than a collection, so that $1 + 1 = 1 + 1$ is exactly the same as $1 + 1 = 2$. Bertrand Russell states this very plainly when he says that

> there is no doubt about the class of couples: it is indubitable and not difficult to define, whereas the number 2, in any other sense, is a metaphysical entity about which we can never feel sure that it exists or that we have tracked it down. It is therefore more prudent to content ourselves with the class of couples, which we are sure of, than to hunt for a problematical number two which must always remain elusive. Accordingly we set up the following definition:
> *The number of a class is the class of all those classes that are similar to it.*

Thus the number of a couple will be the class of all couples. In fact, the class of all couples will *be* the number 2, according to our definition.

In other words, the calculator neither does nor should commit himself to what numbers are in any other sense. And we must grant that it is thanks to the extrusion of the 'metaphysical' from what Russell calls logic and mathematics that their technique could become an amazingly powerful tool, even for the investigation of nature. But if this be what is meant by logic and mathematics,[1] Russell had no reason for grief when led to regard them as nothing but tautologies. An equation is resolved when the part on the right turns out to be exactly the same as the part on the left. Computation is the most noncommittal operation possible. For example, when we divide two

[1] The kind of subject-matter which the names 'logic' and 'mathematics' now stand for differs almost beyond recognition from what these words originally referred to, while their new meanings can be understood without any knowledge of the earlier ones. To believe that Aristotle's treatment of the syllogism as to form in the *Prior Analytics* is formal logic in the modern sense of the term is like holding that Euclid's geometry was intended to be natural. On the other hand, as John von Neumann said, 'The prime reason, why, of all Euclid's postulates, the fifth was questioned, was clearly the unempirical character of the concept of the entire infinite plane which intervenes there, and there only'. Even the meaning of 'contradiction' has thoroughly changed, when we are told, with relation to parabolic, elliptic, and hyperbolic triangles, that to have their three angles equal to two right angles, to have them greater, or to have them less than two 'are contradictory statements'. Why should 'biped mammal' and 'quadruped mammal' be contradictory unless referred to the same thing in the same respect? Does anyone believe Euclid would demand that the angles of a wilting triangle be equal to two right angles or that the shortest distance between two points on the surface of a sphere be a straight line like that on a flat surface?

into one and one, our one and one are not something new—they were never even 'parts', to begin with; we just thought them up as parts of a symbolic 2. In Democritus's view, we readily divide two, not because two is divisible, but because two is already divided. To be quite literal, we should say 'two *are* two', and divisibility actually refers to our operation of dividing, not to what two *is*. If we were to ask him about it, the computer would maintain that in the statement 'two is two', the 'is' is nothing better than a linguistic obfuscation of the fact that 2 is merely a calculator's fiction, a symbolic construction: what the symbol 2 stands for is exactly the same as what $1 + 1$ stand for. The symbol 2 is $1 + 1$ for short.

This basic idea was clearly perceived by the poet Goethe, in a passage that echoes Democritus, and which is quoted with approval by modern logicians and philosophers of mathematics:

> Mathematics has the completely false reputation of yielding infallible conclusions. Its infallibility is nothing but identity. Two times two is not four, but it is just two times two, and that is what we call four for short. But four is nothing new at all. And thus it goes on and on in its conclusions, except that in the higher formulas the identity fades out of sight.

Analytical philosophers tell us expressly that 4 is just $1 + 1 + 1 + 1$, *abbreviated.* The symbol 4 is an arbitrary sign, bringing $1 + 1 + 1 + 1$ together by a fiction which, in computing, soon becomes indispensable.

Symbolization, even of this primitive type, did not come about as easily as we might think. The birth of 0, 'the most modest of all symbols', is a fascinating illustration of our point. Mathematicians of the ancient world had no effective symbol for zero. Yet once established it was no more difficult to use than counting with the fingers of the hand. And so, in the mechanics of the computer, 0 has become a number just as valid as 2; and -1 as valid as any positive integer; which could hardly be the case if, as Aristotle did, we defined number as 'a plurality measured by the unit'.

Before moving on to more basic questions concerning these simple elements, we must see what is meant by rigour or exactness in the operation of computing. Again we need not look beyond the implications of Democritus's dictum which, as he applied it to nature, provided the basis for his physical atomism. For if 2 is exactly the same as $1 + 1$, then, to put down $1 + 1 = 2$ is to be about as noncommittally exact as can be. In fact, it does not have to be said: it is there for everyone to see, whether anyone actually sees it or not. Is anything gained by knowing or saying that '$1 + 1 = 2$,' when 2 is neither more nor less than $1 + 1$? Mirrors can plurify the stuff of our numbers and reflect these as well as eyes or brain. A speedometer will grind out exact numbers in this sense until the whole machine breaks down. If it is a good meter, it guarantees that the number of miles covered is

(roughly) equal to the number last turned up. Nor is this number without its own kind of abstractness, for the very same number could be registered by the speedometer and by the kind of photo-electric eye that counts people entering a stadium. Whether anyone paid attention to the equality of the two numbers or not, there it would be nonetheless. If there were as many miles as there were people, the number would be indubitably the same, and one's knowing or saying it would not alter the case. So you see, once the symbols have been agreed upon, the operations thenceforth involved are obviously mechanical; the machines are in some measure more reliable than a human computer. And if you object that the machines can't read their own numbers, you will have to admit at least that they can store them away, and eventually recover them. For it is a simple matter to construct a machine which, without knowing in the least what it is doing, 'tells' us what it has accomplished, what it has recorded, and what it will next perform. And the basic condition for all this is simply that $1 + 1 = 1 + 1$, or 2 for short. If 2 were something radically new and distinct from $1 + 1$, even the most refined electronic contrivance would fail us. Perhaps we should add that, to call for a definition here, is the same as to ask for an interpretation of the symbol. Where John Stuart Mill said that 'All definitions are of names, and of names only', we could say, in this context, that all definitions are of symbols. But if the game is to work, there can be no definition of what the symbol stands for. For, if there were a definition of 2 that was more than an interpretation of the symbol, viz. $1 + 1$, it would mean that 2 is something over and above the mere collection which is the business of mechanical computation. The machine, if obliged to take this kind of two in earnest, would face something like a nervous breakdown.[1]

[1] The reader should be warned that nothing obliges us to assume that our 'one', in 'one plus one', has the same meaning as the 'one' in the statement: 'two is something that is one', where the word 'one' can again be understood in two ways. In short, the word 'one', like the word 'being', is highly ambiguous. Although always related, the various meanings of 'one' remain simply many. One meaning is seen, for example, in the expression 'a man is one'; another in 'an arm is one', still another in 'one man', 'one arm', and so on. Further types of unity are indicated by 'a house is one', 'a man's size is one'; and a peculiar sort by the phrase 'time is one'. It is not surprising that some people would prefer, if they could, to do away with language altogether, and never to use anything but unambiguous marks or symbols. Nor do I see why we should hesitate to admit that this denial of analogical terms, as a means of intelligible expression, means the impossibility of all philosophy. For if, in naming things, we follow the progress of intellectual knowledge and, if this knowledge proceeds from the more known to the less known with dependence upon the former, it is only natural that we should transfer names of things more known to things less known. Thus the word 'distance' has been transferred from things that are apart locally, to distance in time, distance between simple and complex systems, between ideas, and philosophies. Extended meanings of what is morphologically the same word indicate progress in knowledge. But if the meaning of a word be either unique, or sheer metaphor, so that once the word has been used to refer to something in the order of external sensation or of making, it may never again be used to mean anything else in any proper sense, it must follow that philosophy will have nothing to name, for the excellent reason that there will be nothing else we can come to know so as to need to name it. The very sentence just pronounced, containing such names as 'reason', 'know', 'name', must now stand as a series of mere scratches or noises of which it would be meaningless to ask whether it is likely, unlikely, true or false.

Now we know what is meant by a 'logical fiction', such as the symbol 2. It is not logical in the sense used by Aristotle, but should rather be related to what he called 'logismos'. We also know what 'this magic of symbolic construction' amounts to, at least as regards the number 2. Yet there remains a still more basic and peculiarly teasing question: where do we get the raw materials for the construction? How are the members of classes given? How do we achieve the members of classes, and the members of the classes that are classes of classes? Some philosophers of mathematics (Weyl, for example) believe that in the end this problem reduces to 'Where do the natural numbers or integers by which we count objects, where do *they* come from?' The German mathematician Kronecker declared that 'God made the integers, all else is the work of man.' This statement is intolerably ambiguous. I can believe it no more than I can the statement: God made 'what it is to be a man', 'what it is to be a house', or even 'what it is to be a heap', although there be no such thing as a man, a house, or a heap, that was not somehow produced. Still, Kronecker had a point: for we do construct all other numbers, such as zero, or the square root of minus one, in a way that is apparently different from that in which the integers are obtained. Now the question is: Are the integers themselves a construct of our own mind? Hermann Weyl, following Brouwer, was emphatic that they were. Reflecting upon the present status of the foundations of mathematics, he added:

However, it is surprising that a construct created by mind itself, the sequence of integers, the simplest and most diaphanous thing for the constructive mind, assumes a similar aspect of obscurity and deficiency when viewed from the axiomatic angle. But such is the fact; which casts an uncertain light upon the relationship of evidence and mathematics. In spite, or because, of our deepened insight we are today less sure than at any previous time of the ultimate foundations on which mathematics rests.

But let us pass over this statement for the moment. How *do* we construct a sequence of whole numbers?

Let us put down a series of strokes one after another. They are strokes, but they can function as symbols; dots would do as well, or apples, horses, or even as many nothings, each of a different kind. If we put down / / / / /, we might match these with the fingers of one hand, with the same number of apples; or with a finger, and an apple, and a star, and a thought, and a witch's broomstick. The number of strokes will be the same as the number of fingers or of apples, or of the five incongruous entities, if there is one apple for each stroke, and one finger for each stroke, or one member of our wildly heterogeneous group for each stroke. The strokes, then, may be taken as the articulated symbol of all classes that have the same number, no matter what members these classes may contain: they are a class of classes. But the more convenient Arabic symbol 5, the cipher, is employed rather than a row of five strokes; and we can now forget all about strokes, dots, horses, apples, and so on; all of

these will be classes having the same number, namely 5.[1] What then *is* the cardinal number five? Professor Kasner answers this question in no uncertain terms:

> The cardinal number of the class C is thus seen to be the *symbol* representing the set of all classes that can be put into one-to-one correspondence with C. For example, the number five is simply the name, or symbol, attached to the set of all classes, each of which can be put into one-to-one correspondence with the fingers of one hand.

If you doubt the validity of this observation, just watch a cash-register and see what it does with numbers. Now why should we want to do anything different?

We now seem to have constructed an integer. How do we construct the series of integers? We began with 5. (We did so, not only to reach Professor Kasner's statement, but also because the fingers of the hand were once the most useful standard for counting, until some one thought of *calculi* or pebbles, possibly a shepherd to keep track of his sheep.) Now let us compare one series of strokes with another. We might use our five strokes in this way:

(a) / / or 2
(b) / / / or 3

In this process, which is again called 'constructive', we can decide which of the groups is the larger, by

[1] Notice that, as the members of classes become more numerous, the need for symbolization becomes more urgent. How could we distinguish at a glance between the class 21 and the class 22 if we did not have these handy symbols, but were obliged laboriously to compare large groups of strokes? Symbolically, nothing could be more simple than 10^{12} for a thousand billions.

checking one against the other, stroke by stroke. Now we have different numbers, different symbolic constructions, and there seems nothing to prevent us from 'constructing' as many more as we choose.

But what have we done? What does this work of construction amount to? /// is // + /, or 3 for short. As we add one more stroke, we obtain a different collection. Can we keep it up? If it were simply a matter of time, then, if time went on forever, and if we were *there* for as long—we, or machines to take over the drudgery—there might be no end to our sequence, which is a tautology (in the modern sense of this term), for those who know what is meant by 'forever'. Yet, without actually carrying it out, how can we be sure that the sequence can go on and on, in such a way that whatever number n is given, it is always possible to pass to the next, or that, if n is possible, so is $n + 1$? (By 'possible' we here mean that, a thing being posited, nothing impossible follows.) How do we know that nothing impossible follows from $n + 1$? If, as Weyl says, 'the open countable infinity is basic for all mathematics', that is, if the process by which the integers are created, and in which functions of an argument ranging over all integers n are to be defined, is the ground of all mathematics, does this process, as so many have believed, constitute the truly basic problem? For it seems a little odd to inquire whether $n + 1$ is possible, and to think of this question as lying at the heart of the mystery, when $n + 1$ lies before us; not merely possible, but realized and actual, since that

is what it is, namely, the symbolic expression $n + 1$. All the same, some modern philosophers of mathematics insist upon finding the root of this infinite process and this in an age when nearly all standards of verification have long since been jettisoned.[1]

Seeking the root of the infinity in question, Hermann Weyl fell back on our intuition of an 'ever one more', opening the mind 'on to a manifold of possibilities which unfolds by iteration and is open into infinity'. This may recall what Poincaré laid down as the basis of mathematical induction.

Pourquoi donc ce jugement s'impose-t-il à nous avec une irrésistible évidence? C'est qu'il n'est que l'affirmation de la puissance de l'esprit qui se sait capable de concevoir la répétition indéfinie d'un même acte dès que cet acte est une fois possible. L'esprit a de cette puissance une intuition directe et l'expérience ne peut être pour lui qu'une occasion de s'en servir et par là d'en prendre conscience.

This assertion by the great French mathematician caused quite a stir at the time. It might amount to no more than, 'We just know it to be so!', and so stand as a kind of ultimate vindication of the still more famous maxim: 'Le coeur a ses raisons que la raison

[1] Allow me a marginal note before facing our so-called problem. The idea that time as the measure of physical motion is related to the sequence of integers is a rather commonplace misunderstanding. Of course, I consume time as I add a unit to a given number, and still another unit to the new number, and so on. But such time is of no consequence to an understanding of the expression $n + 1$. It tells me only that I could spend as much time on the process as there will ever be, so long as I am there to waste it, but tells me nothing whatever about the process itself.

ne connaît point.' But it might also suppose the most penetrating kind of thinking. Aristotle pointed out long ago that

> belief in the existence of the infinite comes . . . most of all from a reason which is peculiarly appropriate and presents the difficulty that is felt by everyone—not only number but also mathematical magnitude and what is outside the universe are supposed to be infinite *because the intellect never gives out.*[1]

Whether any infinite exists, and in which of its many senses 'existence' would here have to be taken, is not the question, of course. The infinity which interests us just now is an infinity related to the nature of our intellect. Poincaré and Aristotle appear to be saying pretty much the same thing. As Aquinas explains in his commentary on the passage from the *Physics*:

> the intellect never fails but can add something over and above any given finite thing. . . . Number appears to be infinite because the intellect, by adding a unit to any given number, makes another species [*facit aliam speciem*]. And for the same reason it seems that mathematical magnitudes, which exist in the imagination, are infinite, because no matter what magnitude is given, we are able to imagine a greater. And for the same reason people are inclined to believe that space beyond the heavens is infinite, because we are able to imagine dimensions beyond the heavens to infinity.

The intellect does not give out, so that, by adding a unit to any given number, the intellect 'makes' another

[1] *Ph. III.* 4.203b20.

species of number. But how do we know that intellect 'does not give out'? And what are these numbers that intellect 'makes'? Do we make numbers in the sense that we make houses? Well, there is at least one obvious sense in which we can make numbers. I *made* this 2 by adding 1 + 1. But did I, really? And if so, did I also make 'what it is to be one' and 'what it is to be two'? Is it by my doing that 1 + 1 = 2? Suppose I put down 1 + 1 = 3? There it is. But what does it mean if we have agreed that 3 stands for 1 + 1 + 1? Is it my doing that 1 + 1 is not the same as 1 + 1 + 1? Again, dare I assert that the number two is *this* number two that I have set down here, just as one might state Man to be Socrates? For, if Man is Socrates, what is Plato? If two is *this* number two, what is *that* number two? And what can be a number two which is neither this one nor that?

But this is traditional philosophy making a nuisance of itself again, and even threatening to drag us on to the old battleground of the argument over universals. To avoid all such tedious quarrels we have only to allow ourselves to be confined to numbers as sheer symbolic constructions: within the area of Democritus's 2, none of these problems can ever bother us. A number two other than 1 + 1 is now out of the question, for we do not pretend that the unity of our symbolic 2 is more than a convenient fiction. Of course the problems are still there, to be raised, debated, or rejected, nor may we allow ourselves to think that by confining ourselves to symbolic construction we have solved them. Nevertheless, such a restriction at least teaches us what questions *not* to ask in the present context. Once we have agreed to take numbers as mere collections, as we simply must do in calculation, why raise difficulties which assume that they are anything more?

Let me suggest an example of the sort of question which should be rejected as unfair by all organized computers. Once the decision has been taken that ' man ' stands for mankind in the sense of a collection, will it be proper to ask whether man is a member of this class? A change of example will show how absurdly the question may now present itself. If by mathematician is meant the class of all mathematicians, it is manifestly inconceivable that this class be identified with one of the mathematicians who are members of it. So it must be admitted that such a class is not a member of itself. Yet it seems that there do exist classes that are members of themselves: an instance would be ' the class of all classes,' which is at once a class and the class of *all* classes. Notice that words are being used here, but it is not clear whether they are being used as words or as substitutes for symbols. Meanwhile, granted that the class of all classes is a class, will that class be both a class and a class-member in exactly the sense in which the other members of that class are classes and members? If so, we have a paradox of our own devising, which is unfair to our computers, who cannot acknowledge this kind of question. But if it is words we are using, as words, we may disturb people by reviving the ancient problem of self-reflection. By

self-reflection I mean that I can not only consider an object, such as a man or a circle, but can also consider the act whereby I know that object, so that the act becomes an object in its turn: in knowing, I know that I know. Similarly, one can love something, and love one's own love for that something. But how mind can thus double-back on itself, and what the activity supposes, are problems which have nothing to do with the way things are being viewed nowadays.[1] The old problems represent the kind of thinking that gives rise to endless debate, and are neither to be raised nor solved by computation. (Some will see proof in this that they are senseless problems.) To put it briefly: if the class of all classes is taken as a symbolic construction, why sneak in questions that arise only because we are making needless trouble for ourselves by using words? If we are in trouble it is only because we are clinging unconsciously to certain elements of traditional logic[2] which we ought to have divorced completely

[1] There exists none the less an analogy between Goedel's theorem and the nature of reflection in the order of sensation. For a sense-power, being organic—that is, intrinsically dependent upon a corporeal tool—cannot reflect upon itself, though one sense-power can reflect upon another. Hence, if all knowing powers were such that none of them could perceive its own knowing, but always required still another sense, then so long as there was to be reflection of this kind, these powers would have to be plurified in the manner of a series converging towards a limit at infinity, or there would have to be some ultimate power unaware of its knowing.

[2] The paradoxes of traditional logic were of a different kind, as in the cases of ' name ', ' definition ', and ' genus '. For instance, ' horse ' is a name; but ' name ' too is a name. The definition of definition makes definition both definition and definitum. If genus is definable,

from logistic, whereas we go on using names and symbols as if they were interchangeable. After all, 'the mathematical game is played in silence, without words . . .'. 'Class' as a word, and C as a symbol, whether of a class, of a class of classes, or of the class of all classes, are in different worlds.

It must be agreed, then, by those who confine themselves to symbolic constructs, that such problems are properly ignored. To pass judgment on $1 + 1 = 1 + 1$ is no part of their business after all; they do not even have to *know* these elements. The symbols and functions are ultimate data and will eventually appear 'out there' as actual mechanical entities. Whether their meaning be grasped or not does not invalidate the computation which may be ground out by a machine. Computation, let it be repeated, does not require understanding of what is being computed. Even when performed by man, a computation can be purely mechanical and, in fact, ought to be. But I am sure you will agree that a curious compliment is being paid to the human mind by the assertion that its supreme degree of precision is achieved when nothing is either apprehended or stated, and when reflection is more of a hindrance than a help—as demonstrated of

and the definitum a species, genus is a species, from which it differs by definition. Such paradoxes were resolved by the distinction between essential predications (e.g. 'man is an animal') and denominative ones (e.g. 'man is a name' or 'man is a predicable species'); and also by the now forgotten *suppositio nominis* of Aristotle's *Peri Hermeneias*, where he points out that, while it is true to say 'Homer is a poet', even though dead, it is false to say 'Homer is'.

course by the machines. All of which calls to mind something written by Whitehead half a century ago. It was not an original thought: we find it in Raymond Lull, Leibniz, and in the English logicians Boole, de Morgan, and Jevons. In the example $x + y = y + x$, anyone can see for himself that, by the aid of symbolism, he can make 'transitions in reasoning almost mechanically by the eye, which otherwise would call into play the higher faculties of the brain. It is a profoundly erroneous truism, repeated by all copy-books and by eminent people when they are making speeches, that we should cultivate the habit of thinking of what we are doing. The precise opposite is the case. Civilization advances by extending the number of important operations which we can perform without thinking about them.' A modern *Introduction to Logic* (Copi) adds: 'From this point of view, paradoxically enough, logic is not concerned with our powers of developing thought but with developing techniques that enable us to get along without thinking.' This, of course, is where the machines are taking over. And they can take over quite a lot that we used to believe demanded thought. Poincaré says so very plainly in the following passage of *Science et Méthode*:

What strikes us first of all in the new mathematics is its purely formal character. 'Imagine,' says Hilbert, 'three kinds of *things*, which we will call points, straight lines, and planes; let us agree that a straight line shall be determined by two points, and that, instead of saying that this straight line is determined by these two points, we may say that it passes through these two points, or that these two points are situated on the straight line.' What these *things* are, not only do we not know, but we must not seek to know. It is unnecessary, and any one who had never seen either a point or a straight line or a plane could do geometry just as well as we can. In order that the words *pass through* or the words *be situated on* should not call up any image in our minds, the former is merely regarded as the synonym of *be determined*, and the latter of *determine*.

Thus it will be readily understood that, in order to demonstrate a theorem, it is not necessary or even useful to know what it means. We might replace geometry by the *reasoning piano* imagined by Stanley Jevons; or, if we prefer, we might imagine a machine where we would put in axioms at one end and take out theorems at the other, like that legendary machine in Chicago where pigs go in alive and come out transformed into hams and sausages. It is no more necessary for the mathematician than it is for these machines to know what he is doing.

Our symbols possess many aspects. They are conventional signs, artifacts. Various types of notation exist and, to this day, there is debate about which is the more convenient, that is, mechanically the more useful. (My teacher of mathematics used to say that to define a fraction it is enough to point out two marks on the blackboard, separated by a line—which of course need not be straight.) The rules of operation are like artifacts too, built into the machine; so that operations upon symbols according to fixed rules may henceforth be carried on without thinking, if thought be conceived as something distinct from a mechanical operation. If your machine put down $1 + 1 = 3$, you would

know that something had gone wrong, because you have chosen this symbol 2, not this one, 3, to stand for $1 + 1$.

But we must leave off playing the fascinating game sooner or later, of course, and when we push aside our complex toys and begin to chat with one another about the results, we cannot help noticing that our words, unless forced into the role of mere symbols, are not the indifferent, objective counters that our symbols are. The reason is that, by the very fact that they are words, they must bear one or several meanings, which suppose a knower and relate them to a knower. What words stand for is something which we have got to know before we can use them as words. But the operation upon symbols does not require knowledge of what they stand for, and this is why mathematics can be called a game with ' meaningless symbols ', played according to arbitrary rules. But take this statement itself, the statement just made, what can be done with it if the words are meaningless? How can it be used? To what can it be made to lead? The words which compose it could be copied, of course, *ad infinitum*, and most faithfully by a typist or machine ignorant of their meaning. That is all. Hermann Weyl's classic statement cannot be repeated too often: ' The mathematical game is played in silence, without words, like a game of chess. Only the rules have to be explained and communicated in words, and of course any arguing about the possibilities of the game, for instance about its consistency, goes on in the medium of words and appeals to evidence.' Weyl is justifying, fairly and fully, all the claims of the computers; but, in allowing this function to words, he also, perhaps without being aware of it, grants the traditional philosopher his time-honoured right to have the last word.

After this digression into symbols and mechanics, let us return to Weyl's 'intuition', Poincaré's 'esprit', and Aristotle's 'intellect that never gives out'. As I have already suggested, modern analysts should reduce this way of interpreting infinity to a pseudo-problem: $n + 1$ is valid because it means just that. They ought to suggest that if you insist on a proof, you may go about it in two ways: (a) you may define a machine, or a brain, able to go on and on constructing new integers, without end and, if you know what you mean, you should see that your question contains its answer; (b) but if $n + 1$ is to be true in the experimental sense, as it is true that some automobiles actually attain speeds of over 100 miles per hour, then you will have to construct such a machine. There's the analytic rub. Imagine what *that* would require! Any other way of viewing $n + 1$ as a problem, as one analytical philosopher puts it, will take us back to the Stone Age.

But let us consider a bit more closely this intellect that 'never gives out'. Mine gives out on me each time I fall asleep—as far as I'm aware. Of course, that is not what Aristotle means. He observes, elsewhere, that 'identity is a unity in being either of more than one

thing or of one thing when it is used as many, as when we say that a thing is identical with itself, for we then *use one and the same thing as two*.'[1] This is what we do when constructing the proposition of identity 'Socrates is Socrates'. One and the same Socrates is doubled into a subject and a predicate, the terms of a relation of identity. The two extremes involved are of course not distinct in reality, for in this case we would be denying exactly what we intend to express: the multiplicity, as well as the relation, is wrought by reason. If it were real, Socrates could never be himself. Now notice that the very relation, which we conceive to express the identity of Socrates, can in its turn be conceived and expressed as identical with itself. Let it be a, so that a *is* a, where the first a is subject, the latter predicate, viz. a_s and a_p. Thus we have a second relation of identity, and we could establish another and still another, to infinity, without bringing in anything new. We started with Socrates, but we could have started just as well with nothing, for 'nothing is nothing'; or even with whatever impossibility you like, since there is no denying that 'what is impossible is impossible'. Hence there is no being or non-being about which we cannot form and conceive a relation of identity: the *terminus a quo* for an infinite series of well-defined relations is completely indifferent—indifferent to real or rational, possible or impossible. Whichever may be chosen, and there is nothing to limit a choice, each or any can yield an infinite series of

[1] *Metaph.* V. 9. 1018a5.

relations. Now the root of this infinity, we are told, can lie only in the intellect, in its ability to reflect upon itself. For in knowing anything you please, or in knowing that I do not know anything you please, I know that I know it, or know that I do not know it; or know that I wonder whether I know it or not; further, I know that I know that I know it, or that I do not know it, or that I wonder about it; and I know that I know that I know this, to infinity.

This remarkable ability of the intellect to watch itself at work has more than one aspect. One of them is utterly boring. For who wants to sit down and wind himself up into an actual series of 'I know this, and I know that I know this, and I know that I know that I know this, &c.'? But there is also another aspect, one which might be of crucial importance to us just now. Suppose I put down a pencil-stroke for each of these acts of self-reflection, or, to avoid all commitment on the nature of this act, a stroke for each relation: the result is a clear multiplicity tied down by these strokes. Please attend very closely to what is happening. I have said that we are achieving a clear multiplicity represented by our strokes. The pencil strokes have nothing to say about the nature of the initial relation of identity and, if we choose we can now ignore it too; we may in fact ignore whatever it was that prompted us to put down the first stroke. From now on it will be the strokes that are important, not as strokes, but as symbols. (We could use dots, letters, crosses, or anything you please.) Thus, from material much less

dense than the stuff of dreams, we have spun an utterly endless symbolic construction. The kind of relations with which we are really concerned in such construction will now turn up automatically, so to speak, as we proceed: for example, a (a); $a = a$; $a < 2a$, or $2a > a$; $n > -$ n, . . . In this fashion we can obtain those numbers whose being exhausts itself in the functional role which they play and in their relations of more or less. ('They certainly are not concepts', Weyl adds parenthetically, 'in the sense of Aristotle's theory of abstraction'. And of course we agree.)

Now, regarding the first object of investigation, namely, symbolic construction as exhibited in 'the simplest, and in a certain sense most profound, example: the natural numbers or *integers* by which we count objects', is there any difference between Weyl's 'intuition of the "ever one more," of the open countable infinity, which is basic for all mathematics', and Aristotle's intellect which 'never gives out' and knows that it knows? Or between both of these and Poincaré's 'esprit qui se sait capable de concevoir la répétition indéfinie d'un même acte dès que cet acte est une fois possible'? But we shall not allow ourselves to be puzzled over the nature of this intuition, or self-reflection, or 'esprit'. It will be enough to consider the implications of the symbolic construction itself, without regard to what it issues from; to consider not what intellect is, but what it makes possible. And what it makes possible, first of all, is symbolic construction. By a symbolic construction we mean, in our simplest example, a thing like Professor Kasner's cardinal number five, which is a symbol, out there, on the blackboard, on paper, or wherever you want to put it down. For it was designed as something that can and should be engaged in a mechanical process, and so might take our place in such a process, by its very nature relieving us from drudgery. Hence, if you ask, 'Where are these numbers?', I will reply by simply pointing a finger at them. I can hardly designate to you the images I have in my imagination. And that the mental images are irrelevant is proved by the fact that machines can handle numbers and store them away without imagination, and also without memory, simply because they are holding them all the time in a demonstrably physical way. If 'memory' is the faculty of retaining what is no longer present, and of recognizing it as no longer present, the machines plainly have no need of such a power to do their work. To have a memory in this sense of the word, it is not enough to retain traces of the past. Recorded traces of the past, and the ability to exploit them, are not all there is to memory. (In my brain there are presumably traces of my whole past, and I can only feel grateful that most of it never turns up; or that, when it does, I am able to recognize it as no longer present.)

It also follows that the infinity, like that of which the series of integers is an instance, can be considered in different ways. It may be taken as a potential infinite, defined by Aristotle in the *Physics*, namely

'that beyond which there is always more that can be given'. This holds true of any single integer: we can have as many particular instances of two as we wish, to correspond with a series of cardinal numbers as great as you choose, and as many particular series of these as well. However, as long as we consider the infinite as that beyond which there is always more that can be given, it will forever retain the nature of part, as distinguished from a complete whole, namely 'that of which nothing is outside', such as any 'whole number'.

But this is only a beginning. If, as Aristotle does in his *De Caelo*, we choose to consider an infinite (e.g. the series of integers) as a *whole* to which nothing can be added, we can then speak meaningfully of 'all the integers' and symbolize them as an infinite set. Operationally speaking, nothing prevents us from calling this infinite set a number, nor from laying down a symbol for it (let it be Cantor's aleph-null, or $\aleph_0$), to be defined as the first instance of a transfinite cardinal number. And there is again not a reason in the world why we cannot have an infinite set of each finite cardinal number; and an infinite set of that first instance of a transfinite cardinal number; and, not only an infinite set of which this number is a class, but even an infinity of infinite sets each of which is greater than the other, indeed infinitely so, and so on, as Anaxagoras implied. In other words, whether or not you choose to relate it to an intellect that never gives out, to an 'intuition of ever one more', or to an 'esprit qui se sait capable . . .' there is plenty of infinity available. As Aristotle said, anticipating Cantor as it were, the mathematician may use as much infinity as he needs. The real problem is not so much how we get started, but where we ought to stop, or how much infinity we can put to use. (In the context of symbolic construction it would appear that the infinite set of all infinite sets will be forever a matter of choice, here and now, since there is no reason why this set could not be taken over and over again to infinity. Unless of course 'all' signified a mode of universality going beyond *ut nunc*, as of now; which would take us into a world far more confined than that of symbolic constructs.)

Notice again that, so far as symbolic construction is concerned, it makes no difference whether the infinity in question is one that has the nature of quantity, or whether it is just a limitless heap. If properly symbolized, an order can be made to arise within any heap, however disorderly: and not only order, but widely different types of order, in spite of the indifference of the stuff out of which these grow. So long as the intellect does not give out, or, to be quite non-committal, so long as we know what to put down and how to put it down, order will declare and maintain itself more firmly than the calculator's nose.

Poincaré suggested that all of mathematics might in a sense be 'no more than a roundabout way of saying that A is A'. But this formula actually holds more than is needed for Jevons's logical piano. I mean that,

once 'l'esprit' has provided 'A is A', the 'is' can, and should be, dropped, if the computer is to be trusted. The analytical philosopher, like his machine, can do nicely without a trace of intellect, intuition, or *esprit*. And, once it can reproduce itself, the mechanical computer will presumably dispense with them in a fashion even more thorough, and will eventually bring forth machines more efficient than those that gave it birth.[1]

[1] Let me repeat that I do not mean to belittle the so-called analytical philosophers. Besides showing so convincingly the sense in which Democritus was right (and they might do the same for what he said about nature and chance), they have helped to clear away much of the rubble that made philosophy an inane exercise in the use of words with unknown meanings. For it is entirely possible, as Aristotle warned the instructors of youth, to teach them the vocabulary of primary philosophy (later called metaphysics) yet leave them with no understanding of it, though perhaps with a passionate conviction that they do understand. Philosophy is all too frequently an attempt to explain what we know in terms of what we do not know. But I am obliged to say that I feel quite helpless in the face of the analytical philosophers who held, at least until recently, that the meaning and verification of a proposition are obviously one and the same; that the notion of class is the clearest thing in the world; that 'man' is always and never can be more than a collective name meaning the class of all men, or that 'triangle' stands for the collection of all triangles. They make the person who sees that these things are by no means evident feel like the man named Nuñez in H. G. Wells' *Country of the Blind*. It is a sad fact that the most striking innovation of the analytical school is its assumption that so many obscure, unanalysed and undemonstrated things are self-evident, with self-evidence itself disposed of as one of these. One feels more helpless still to hear them emphasize the supposed evidentness of these difficult and unproved opinions with the emotional observation that the man who does not share them is mentally back in the Stone Age. Most of our statements, it may be granted, show an emotional overtone—a tone which, in lectures on unemotional subjects, may at least serve the purpose of keeping the listener awake. But it seems to me that, in all history, no philosophers have indulged more freely in passionate outbursts, or have shown greater need for a contemptible adversary against whom to hurl their charges, than some representatives

If we take the Aristotelian view that all such symbolic construction is sufficiently explained once and for all—if explanation be required—by the mind's ability to reflect upon itself (and I do not see how this differs from Poincaré's *esprit*) we may wonder how so much can be done with so little. There is even more cause for astonishment at the fact that these fictions and constructs have such relevance to nature that Professor Einstein, in his Herbert Spencer Lecture, could say:

It is my conviction that pure mathematical construction enables us to discover the concepts and the laws connecting them which give us the key to the understanding of the phenomena of Nature. Experience can of course guide us in our choice of serviceable mathematical concepts; it cannot possibly be the source from which they are derived; experience of course remains the sole criterion of the serviceability of mathematical construction for physics, but the truly creative principle resides in mathematics.

And how right he was has since been proved even more fully by the use of Group Theory in quantum physics. It all makes one wonder. What can our constructs, mental or physical, have to do with the things of Nature? Can it be that both our works of head or hand, and Nature's works, somehow have Reason back of them?

of the analytical group. Surely they should emulate the calmness of the machine which, as some of them predict, may one day take their place. (But I must be careful, for I have read that computers, too, get their nervous upsets.) And I think this is the reason why disciples of the school are so oddly disconcerted by any serious objection (even of the kind made by Bertrand Russell) to the doctrines of those whom, for the time being, they acknowledge as masters. They betray a greater dependence on authority than was ever felt by the lowliest peripatetic.

But is mathematics really no more than what the computers can do for us? Has the machine entirely swallowed up the study which was at one time held to be the most exact of human disciplines? Is this model science to be reduced to the status of mere tool? A strange tool indeed, when it is without agency or purpose. We will return to this subject in the last lecture of the present series. Meantime, allow me to assure you of my hearty agreement with E. T. Bell that that part of mathematics which is not mechanical drudgery remains, that the computers and all that they imply leave this mighty science enhanced and free. The machines are here. They work, bless them! And there is no more reason to get bewildered at their feats than there is at the performance of cars that travel faster than we can run. For these reasons, I hold our case to be one where we can have our cake and eat it too. We can burden computers with the mechanics that can serve the science, and still rejoice in a mathematics that no machine or beast will ever possess.

Hegel, who could be so very wrong on basic points (as in that confusion of his of contradiction with contrariety, which some logicians are beginning to discover), was not so wide of the mark when he observed, in his *Science of Logic*, that:

Number is a non-sensuous object, and an occupation with it and its combinations a non-sensuous business: thus the mind is urged to reflect in itself and to do inner and abstract work; which is of great though one-sided importance. For, on the other hand, insofar as number is based only on external differentiations in which mind has no share, this occupation becomes thoughtless and mechanical. The effort mainly consists in seizing units void of concept and combining them without the use of concepts. The content is formed by the empty unit; that rich content of moral and spiritual life, and its individual growth, on which, as its noblest nourishment, education should rear the young mind, would here be ousted by the empty unit; and, when such exercises form the main matter and the main occupation, the effect can be no other than to hollow and blunt the spirit in form and concept. Calculation being so extremely external and therefore mechanical a matter, it has been possible to construct machines which execute the operations of calculation in the most perfect manner. If this one circumstance were known about the nature of calculation, we would see what it means to believe that calculation should be the chief instrument for educating the mind, and realize that we are trying to perfect the mind by converting it into a machine.

II

Mental Construction and the Test of Experience

> Nothing that is has a nature, but only mixing and parting of the mixed, and nature is but a name given them by man.
>
> EMPEDOCLES

> Nature likes to hide.
>
> HERACLITUS

The word ' mental ' in the title of this lecture may be disturbing to those of my hearers who know that advanced philosophers of science are now inclined to reject any distinction between mental and physical, mind and matter, reason and nature. It is more than likely that these thinkers are justified in this rejection, at least as regards their own work, for the distinction has little relevance to the context in which they usually place it. Anyhow, just at this moment, I am not disposed to argue the point, because, for the purposes of the present discussion, the distinction between mind and matter does not concern me either, and it would be a mistake to pay particular heed to the implication of it in my title. What I mean is that I have not the smallest objection to being considered mindless, so long as I am allowed to go on doing certain things which in earlier times were related to what was called mind, reason, or intellect. Right now it is my harmless intention merely to take advantage of the distinction, which I readily concede may be only verbal and of no account, and to use my head (if I dare not call it my intellect) in an attempt to learn what the word mind can have been intended to signify.[1]

Take those symbolic constructions we laid down in our last lecture. Are they mental, or are they physical? The question at once gives us something to quarrel over. Because the apparently neat alternative, 'a thing is either mental or physical', I emphatically reject; just as I do, 'a thing is either black or white', 'hot or cold'. Even the far more determinate 'white or not-white' could be ambiguous, as you can see by comparing the following cases: 'the wall is not white', 'the square root of minus one is not white' (unless

[1] If called upon to justify the distinction between mind and nature, or between rational and real, I would point to the difference between contraries as in our knowing, and as in fact. In fact a man cannot at the same time see and be blind; but in knowing blindness, he must simultaneously grasp what sight is. For sight is implicit in the very notion of blindness, just as any positive term is essential to its negation, and the perception of one term as contrary is dependent upon the representation of its opposite. This supposes a radical difference between the corresponding real subjects of any contrariety. So, if the differences between contraries are held to be finally one and the same, the real, as distinguished from the rational, will involve contradiction (which is the way some people want it): just as one cannot conceive blindness without simultaneously conceiving sight, nor think death without thinking life, so one could not actually see without being actually blind, nor be alive without being also dead. This impossibility cannot be escaped by anyone who refuses to allow a significant distinction between mind and nature. All the same, there are instances of simultaneous contrariety outside the mind—providing ample room for confusion. A plant, for instance, grows in contrary directions; and a thing becoming white is neither determinately white nor not-white. But these cases differ widely from that of the mind: the first involves parts that are quantitatively external to one another, while becoming remains this side of full actuality.

you refer to the chalk symbol on the blackboard, which I take to be irrelevant.) Of the wall we truly understand that, if it is not white, it must bear some other colour; but the square root of minus one we do not expect to have any colour at all. In other words, 'not white' could be a relative negation, viz. holding good only in the genus colour; but it could also be an absolute one, as in the case of $\sqrt{-1}$, for 'not white' could be said just as truly of 'square circle', or of whatever is impossible, or not impossible. To return to our first alternative, 'either mental or physical', you should now be able to see that it might bear witness to a mind utterly confused. One could try to cope with the muddle by using an example such as 'I thought it was going to rain, but it did not'; in which 'thought' would be something mental, opposed to the physical rain you expected; and the rain, had it rained, would then have been something 'physical'. But it will not do to be too confident, for what reply shall we make to the man who reminds us that the barometer, too, expected a rain, which did not fall, and inquires where our distinction is now? To this sort of fellow I would make no reply.

All this is enough to make anybody feel a bit out of his mind; but there is no need to lose heart. Let us see what can be accomplished by a series of sure and cautious steps. Suppose we take a fresh look at what we so unthinkingly called '*mental* construction'. Whatever others may think *mind* stands for, an actual engine of some sort, or parts of one, or their function, suppose you and I attach the ancient word to something relatively clear. You remember how we put down $1 + 1 = 2$? Following Democritus, we added that 2 is no different from $1 + 1$; that it is nothing new except as to symbol. Now, let us fix our attention on that insignificant novelty, the symbol. Where did it come from? It did not grow from $1 + 1$. No, it was our doing. But how did we do it? Where did we find our symbol? Of course, the question may be dismissed with the remark that it cannot matter, so long as we have such symbols. But if we are stubbornly inquisitive, and resolved that symbols must have some source, and a special source; and if we agree further that we may name it as we choose, then suppose we call it ' mind '. Of course, we do not mean to commit ourselves to ' what ' mind, or mental, is, apart from our name. Mind now is simply a name for the source of our symbols. We must keep an open mind (a hard word to do without, isn't it?) on the subject, for mind may turn out to be only the function of a certain congeries of minimal electrical charges, which is conveniently not too clear; and what I know of physics makes me confident that I would not be here at all but for my electric charges, nor if their jumps got a bit mixed up, as by my touching a live wire, much less that I would be able to think without them. (There are those who will hold that to call them ' my own charges ' is no more than a linguistic convenience, and that the same is true for ' I ' and ' me '. This is indeed a fascinating position, and one far too much

taken for granted, since ' linguistic conveniences ' have now been invested with power to dissolve nearly all philosophical problems.) I insist, all the same, that these charges are vital to me, since at the very least they allow me to ask what kind of a swarm of them is I. A particular kind of swarm, no doubt, but perhaps not so very particular, for we humans must not unduly exalt ourselves and lay claim to a dignity which one day we may have to share with machines.

But let us get on with our hollow universe. We have seen what is meant by the hollowness of the world of mathematics. We have shown what this emptiness means in connection with number, which is sufficiently clear in the elementary example of 2 conceived as exactly the same as $1 + 1$. We might have added an example from geometry, such as the ' creative ' definition of a circle, which defines by showing how a circle may be given, as distinguished from the circle defined as a special kind of *nature* (taken in the extended sense of ' what a thing is '), viz. a plane surface bounded by a single line which at every point is equidistant from the point within called its centre. We have shown, further, that when we disregard certain mathematical entities as definable natures in the sense just described, we can proceed to arithmetize the continuum, and to geometrize numbers by filling in the gaps between them with other kinds of numbers to the point of density. And no one can hold that this cannot be done. Nevertheless, the process does suppose mind in the sense just established, for it requires the particular kind of symbolic abstraction so often referred to. What I choose to call the hollowness of such constructions was again made plain by the complete indifference of their raw materials. There is a sense indeed in which these scaffolds of symbols grow out of nothing, as it were, but thanks once more to that activity which we have tentatively called mental.

I now mean to disclose a similar state of things in the physical world, that is, in the world presented to us by mathematical physics. This problem has two aspects. The first is that revealed by Sir Arthur Eddington when he showed how the most rarefied mental constructions, namely those of Group Theory, can penetrate some of the deeper secrets of nature. Group theory, begun in 1830 by the French mathematician Evariste Galois, who died at twenty, was until recently considered to be the most useless, and presumably the most abstract, branch of pure mathematics. (The simplest illustration of a group would be the class of whole numbers taken along with the familiar rule of addition, in accordance with which any member of the class can be combined either with itself or with any other member of the class.) Manipulating groups is a game, which eventually becomes so vacuous that it has been stamped as suitable only for the most withdrawn lunatics. To appreciate just how withdrawn, we may consider a few lines from Eddington's famous Messenger Lecture on *The Theory of Groups:*

It does not trouble the mathematician that he has to deal with unknown things. At the outset in algebra he handles unknown quantities x and y. His quantities are unknown, but he subjects them to known operations—addition, multiplication, etc. Recalling Bertrand Russell's famous definition, the mathematician never knows what he is talking about, nor whether what he is saying is true; but, we are tempted to add, at least he does know what he is doing. The last limitation would almost seem to disqualify him for treating a universe which is the theatre of unknowable actions and operations. We need a super-mathematics in which the operations are as unknown as the quantities they operate on, and a super-mathematician who does not know what he is doing when he performs these operations. Such a super-mathematics is the Theory of Groups.

Various mathematical tools had been tried for digging down to the basis of the physical world, and the last quarter-century has shown that group theory is a most suitable tool. To show just how mathematics first gets a grip on basal entities whose nature and activities are essentially unknowable, I can only refer to that lecture of Eddington's from which we have just quoted. For the present we shall accept his authority.

The application of group theory in quantum physics actually gives more weight to the observation by Einstein, made a few years earlier, which we cited in our first lecture:

It is my conviction that pure mathematical construction enables us to discover the concepts and laws connecting them which give us the key to the understanding of the phenomena of Nature. Experience can of course guide us in our choice of serviceable mathematical concepts; it cannot possibly be the source from which they are derived; experience of course remains the sole criterion of the serviceability of mathematical construction for physics, but the truly creative principle resides in mathematics.

We must be careful, however, not to confuse this mathematics, and its use, with the mathematics of traditional philosophy and its application to nature, as in Aristotle's ingenious theory of the rainbow. As already suggested, ' mathematics ' in Aristotle's day meant a very different type of knowledge, whose demonstrations were derived from definitions of what the subjects were apart from the way in which they were given. Even the ' operational demonstration ' (the expression was used by Aquinas), in which we construct an equilateral triangle to show that it exists (that is, as an abstract subject entailing definite properties, of which we assert neither that it exists in reality, as Plato thought, nor that it does not), is not concerned with the triangle as a construct: construction and construct in an operational demonstration are only means of getting at what an equilateral triangle is. For the old philosophers, mathematical abstraction was of a very special kind, differing from that of all the other sciences, and one little understood by Aristotelian scholars even to this day. In our time, however, the construct is the subject *qua* construct, and it is only this operational aspect of mathematical entities that is applied to the investigation of nature.

Now, sheer mathematical constructs, endowed with that hollowness which renders them so efficient as tools, must make over into their own likeness the natural world to which they are applied. The science which uses these symbolic devices, mathematical physics, thus finds itself able to reveal the world of nature only to the extent in which it can be caught in what has been termed the selective screen of symbolic construction. As Eddington remarked: ' We have found a strange foot-print on the shores of the unknown. We have devised profound theories, one after another, to account for its origin. At last, we have succeeded in reconstructing the creature that made the foot-print. And lo! it is our own.' Nor must we forget Einstein's warning, that pure mathematical construction ' can give us no knowledge whatsoever of the world of experience; all knowledge about reality begins with experience and terminates in it. Conclusions obtained by rational processes are, so far as Reality is concerned, entirely empty.' If our rational constructs and processes are not entirely empty, then, it is thanks to experience alone. This is what we mean by the test of experience.

There is a further sense, one even more generally agreed upon, in which the world of mathematical physics is hollow, or, as Eddington called it, ' a shadowy world of symbols '. This aspect of the matter is implied in the basic definitions of the science, and it was Einstein who did most to reveal it. I am referring to the methodological principle that lies at the very foundation of Relativity Theory, namely, that the mathematical physicist must define his terms by describing the way in which he reaches them, that is, by a process of mensuration which results in a measure-number; a symbol whose meaning is defined by reference to a certain operation and the incidentals that attend it. For instance, length could be defined as 'what is extended in one dimension'; and such a definition is intended to establish 'what length is,' no matter what length it may be. Whatever may be intended by such definitions, as distinguished from mere interpretations of names or symbols, they are of universals only (it does not matter for the present what universals are, whether they are, or in what sense they would be if they were). The point is that the mathematical physicist, if held to such a definition, even though not obliged to reject it, can make no headway. What *he* means by a length is 'when we take a reasonably fair copy of a certain platinum-iridium bar kept in Paris at a controlled temperature of O° centigrade, &c., and apply it, once or more, successively or by division, to know a given distance between A and B, the result of the operation will be expressed by *xm*'. But in this mode of defining, the standard of length can of course have no length, when there is no prior standard. Length, then, only appears when the measurement is actually made. It is not an abstract operation upon something abstract: it is performed *here and now*, or *then and there;* and the time-factor may be essential; nor, if the metre be his standard, may the physicist

forget the reference to Paris. He cannot afford abstraction of universal from particular. Now, to this whole process, means and result, to this complex operational unity, he cannot give a true name, but he can embrace all of it by the particular value of *m*, let this be $\frac{1}{4}$ or 1000. Such length will turn up on the graduated scales of weighing machines and clocks, by which weight and time are defined in their turn, and even on the scale of the thermometer which entered into the definition of length. As I have noted elsewhere, in this type of definition, the crucial term is *when.* If the physicist said 'length is . . .' instead of 'length is when . . .' he would revert to a mode of definition that tells, or pretends to tell, 'what' a thing is absolutely. However, having defined length in the only way profitable to him, the physicist may assert that 'this *is* length'—much as Russell handled the number 2: 'In fact, the class of all couples will *be* the number 2, according to our definition.' But the physicist who so takes length can only mean that this understanding of it is the only one with which he will concern himself. Allow him this sort of definition and he will astonish you with what he can do with it. Let us notice, however, that if this type of definition were the only valid one at all levels of science, the definition of man would have to be something like this: 'when I bump into something and it produces a series of sounds like "Where do you think you're going?", this *is* man'—which would be a possible enough interpretation of the name.

That remark of Wittgenstein, which we quoted in the previous lecture, suggests, at least to some, that this mode of defining, however indispensable, does leave something out: 'If someone asks what *time* is, you ask in return, "How do we measure time?" But time and the measurement of time are two different things. It is as if someone asked "What is a book?" and you replied, "How does one obtain a book?"' By proceeding in this fashion we not only leave out 'what time is', but oblige ourselves to deal with a particular span of time in such a way that we can only express it by means of a symbol. Time here becomes a symbolic construction, not quite like the one we discussed in the previous lecture, because this one has reference to experience, but which is still far from telling us what time is. We are left with the grin: the Cheshire cat has departed from the scene. But this grin has a structure, and that is something. It is the hollow of the hollow world of physics.

In this analysis we have taken an important step, and we ought to make ourselves very much aware of it lest we extinguish a basic kind of wonder about nature. It is particularly gratifying to observe that very eminent physicists (e.g., Heisenberg, de Broglie, Schroedinger, Jordan, Oppenheimer, Born, von Weiszäcker, to mention only a few) show a growing interest in the golden age of Greek philosophy, and hence a genuine concern for a philosophy of nature, and for questions prior to those which can be disposed of by mere

measuring. This concern supposes that wonder about nature which I have called basic. For example, I can wonder what time is; or I can wonder what the time is that it will take me to get to the station. Fully to answer the latter question is not to erase the first. Yet there is now a whole school of thought ready to call the first kind of wonder a disease of the one animal that obviously wonders what kind of animal he is, a state of mind, we are told, which was normal to him in the Stone Age.

Let us indulge for a moment the mentality of that primitive epoch. The Greeks not only asked what they were using the word 'nature' to mean, they also asked what nature itself is. In his book on the intentions of names (*Metaphysics* V) Aristotle lists seven meanings of the term, while he himself employed even more. In relation to the physical world 'nature' was for him 'a principle and cause of movement and rest in that in which it resides, primarily, per se, and not per accidens'. This definition implies that nature, which grows eyes, differs from art, which makes eyeglasses and radar screens. He thereby started a discussion that lasted for centuries, and is still going on. A modern philosopher like Whitehead will offer a quite different definition: 'Nature is that which we observe in perception through the senses.' Others declare nature to be just sense-data, including chairs as well as trees. Volumes have been written on the concept of nature without any attempt to define or describe what 'concept' means, or what 'nature' signifies. We will not press the matter here, but will go on to some other apparently simple questions and difficult answers.

What is movement? Some early philosophers thought that it was the same as ' inequality '; others, the same as ' otherness '; and finally Aristotle argued to and settled upon ' the act of what is in potency as in potency '. This statement provided Descartes with a hilarious time in his *Regulae ad directionem ingenii*. How is it possible, he remarked, to render utterly obscure a thing so obvious and simple! Yet Aristotle, having defined it, observed that movement, however obvious and certain as a fact, remains nonetheless difficult to conceive, for the simple reason that it is something not clear in itself. We might recall that several predecessors of his found movement so difficult to account for that they simply denied its very existence, labelling it no more than an appearance. The modern physicist's point of view, on the other hand, approaches the conception of movement found in Aristotle's own mathematical physics,[1] where he does not conceive of movement as a ' becoming,' but rather as a quantum, or a ' state '—or, as it would now be phrased, as ' a succession of immobile states '. But here again, to the question ' What is movement?' the answer offered is ' How do we measure it?' and wonder as to what movement itself is disappears.

[1] The best example is that theory of the rainbow to which reference has already been made. In *Mathematics in Aristotle*, Sir Thomas Heath pays tribute to Aristotle's brilliant attempt to explain this natural phenomenon.

Place and time can be made to vanish by the same sort of prestidigitation. If place is ' the first, immediate and immobile inner surface of the surrounding body ',[1] what about changing places? and in what place is the whole universe? If time is ' the number of movement according to its before and after ', where is this number to be found? Now that the outer sphere of Aristotle's cosy universe has been erased, what is the cosmic constant of movement which, according to his definition, would, by reason of its regularity, be the measure of all other movements? In other words, what is the cosmic chronometer? The earlier definition said nothing

[1] While ' to be in a place ' is the first and most obvious case of ' being in something ', and the name *place* is readily enough identified by an example like ' the water is in the jug ', exactly what place is remains a matter of considerable difficulty, although of no concern to mathematical physics. Of the relevant terms essential to the definition we have quoted, that of ' immobile ' now becomes the most intriguing. ' Immobile ' supposes place, and a sort of place which is quite different from Newton's ' absolute ' space. My proper place (proper as opposed to common) is something which I seem to carry with me as I walk about. But, if this be so, how can I ever get to a different proper place? All things are forever changing place, as when the water in the sailor's bucket is dashed along the deck of a ship sailing up-stream, counter-clockwise to the rotation of a planet simultaneously being swept around the sun, and so forth. Yet change so described can be no more than material, while place, in the sense of ' proper ' stands formally unchanged; just as the light in this room remains the same while its material is forever renewed, or as Socrates remains the same person while his physical stuff is constantly being altered. It is only when we identify place with the material of the container, or with its shape, that we feel compelled to imagine some kind of background against, or through which, things move, like the space of classical physics, which is of course a pure fiction. If there is to be movement according to place, then place must be something relatively immobile, which, like the passage of time, relates to the whole universe, although the universe itself be nowhere and its component parts be forever elsewhere.

about where this standard might be found, no more than the definition of measure provides us with the metre. Our household clocks are secondary standards which we adjust to what, for practical purposes, has been selected as the primary standard, namely the sidereal day or observed rotation of the stars, which our time-pieces imitate. But is this primary standard the right one? is it really primary? Or is it merely one cosmic clock among many, and itself in need of correction? The question amounts to asking 'what is nature's constant of velocity?' And it is not a question of some ideally high and constant velocity conceivable in the mind, but of a physical one, of an utterly regular movement which we must seek to verify in experience. It is this that the mathematical physicist demands.

It is easy to see that the first type of question (for example, 'What is time?') is, and indeed must be, put aside by mathematical physics, which confronts a very different type of question without laying claim to a definitive answer. Hermann Weyl observed that 'the first step in explaining relativity theory must always consist in shattering the dogmatic belief in the temporal terms, past, present and future. You cannot apply mathematics as long as words still becloud reality.' And diagrams we may draw to represent the space-time causal structure of the universe—which metrically speaking is obscure as long as we use words like past, present, future to convey time—must be replaced by its 'construction in terms of sheer symbols . . . the intuitive picture must be exchanged for a

symbolic construction'. In this connection, Weyl quoted Andreas Speiser:

> By its geometric and later by its purely symbolic construction, mathematics shook off the fetters of language, and one who knows the enormous work put into this process and its ever recurrent surprising successes cannot help feeling that mathematics today is more efficient in its sphere of the intellectual world, than the modern languages in their deplorable state or even music are on their respective fronts.

Now, we may not have to take into account all that these gentlemen are telling us, but we ought to notice at least this much, that what they are telling us is being told in words, not in symbols, and that we have a right to wonder why. Can it be so certain that the use of words, and the framing of questions in words, does carry us back to the Stone Age? When our physicists have scored some great triumph in their shadowy world of symbols, can philosophers of science find no more effective vehicle with which to record it than 'modern languages in their deplorable state'? It is hard to see how a living language, that is, a constantly changing one, can escape the state which they deplore. These men, though sometimes fine writers, might be more helpful if they told us why they use words when they do, and why they cannot confine themselves always to symbols that, as they themselves say, are not words. They continue to use words, in spite of the fact that these, whether of a dead or a living language, bear those shifting values which these writers sometimes exploit so well. But the problem of what a word is would take us back to the *Peri Hermeneias*, and it seems we can no longer afford the turn of mind which characterizes that work. It is frightening to think that we may have to agree with the late F. M. Cornford, who observed, with something like despair, that we seem to have passed the stage of asking simple questions, the answers to which turn out to be so difficult and, so long as men will think and talk, forever debatable. The principle now reigning is that anything debatable is not worth debating. We are still brewing potions for any possible Socrates, though in amiable ignorance of our own intentions.

And it may be noted as well that scientists are most often bewildering because they use words where there is no need for them, when they should stick to their symbols; and where words in fact turn out to be as meaningless as some of them say they are. Allow me to illustrate such a case. Bertrand Russell's famous Mr. Smith is, he assures us, really a mere bundle of occurrences, a collection of events; and his name is hence no more than a collective one for such a swarm. This, according to Russell, does away with 'substance', reducing it to a mere linguistic convenience. On his understanding of 'substance', this opinion is unassailable. The meaning he reads into substance is a very strange one, one that I am sure never occurred to Aristotle, and which in fact makes no sense. Substance, for Russell, is something that supports accidents in the way in which a floor supports a table: if you

could scratch off or peel away the accidents, he thinks, you would be left with something reassuringly solid, like a stone for an unscientific imagination, chock full of stone through and through, with no hollow left. Swiss cheese would therefore be rather insubstantial; and a sponge, or a flurry of snow, far less substantial than Sir Arthur Eddington's familiar table. Now, it may well be that some people associate such an irrelevant image with the notion of substance, but nowhere shall we find support for it in Aristotle.

Here is how the Philosopher reached his conception of substance. He first observed our way of speaking of things. When speaking, we always have something in mind, yet it may be no more than the sounds or shape of the words, which may nevertheless be used with an air of erudition. (This could be true of philosophers *ut in pluribus*.) But he goes on to show that we do say things like 'Socrates is pale', or that 'he is five feet eight inches tall', or that 'he weighs two hundred pounds, knows Greek', and so on. All of this we may say of Socrates, but we never say *him* of anything else. We do not think of saying that 'five feet eight inches tall is Socrates' (except in Latin syntactically, or in poetry!) And, though in a proposition of identity, we can say Socrates of Socrates, we can never say him of his wife, Xanthippe.

Now, has this way of speaking, and indeed of thinking, some foundation in reality? Is Socrates Socrates? Does he have colour, size, weight, and grammar? Is he the husband of Xanthippe? Is he a corruptor of youth (for showing them that their thoughts about virtue were somewhat confused)? Can this way of speaking be true? When I *say* that he is seated, and he *is* in fact sitting down, is it not true as I say it? What is there about Socrates that would make it false to say that he is his wife? Or to say that he is grammar? or that the grammar he knows is Socrates? Not even Russell, surely, would allow this manner of speaking—at least not in the cases just mentioned.

As we see and say that Socrates is pale, our mind forms a certain relation expressed by this way of speaking. This immanent product of our mind, a work that remains within and could not possibly exist outside the mind, is called a second intention or understanding: a something with which our own mind invests the subject, as known to us and in our mind (whatever 'mind' may stand for). Now it is invitingly easy to define realities by means of these second intentions, but the fatal shortcoming of this sort of definition is that it tells only how the reality is to be thought or spoken of, not how it is. To define substance in this way, by its mode of predication, is to define substance *logically* (in the ancient sense of this term). Substance, logically, is a mere intention of reason, something on a plane with linguistic conveniences. Yet if it is true to say that Socrates knows his grammar, and false to say that the grammar Socrates knows is Socrates, there must be some grounds for this: our way of speaking is dictated by something

in this pale, literate Socrates himself. A lengthy dialectical pursuit will bring us to what is called a natural or real definition of substance: 'that which is in itself and not in another.' And whatever we may profess about this general definition of substance, as found in man, horse, or tree, our normal language supposes it, and we certainly treat our neighbour as if he were such a thing. Of some objects, like stones or snowflakes, we may not feel so sure—since the physicists have upset us by showing that, in a sense, these are bundles of non-stones or non-snowflakes—but even these we consider to be substantial at least in the way that a crowd is, though we cannot identify the kind of unity they have. One thing is plain despite the difficulty of certain examples: we never feel that substance needs the reinforcement of solidity in order to be substantial, that it is like Parmenides' geometrical sphere, or that it demands a resistant material in order to exist. To be a substance, I no more require what Russell calls by that name than the earth requires an elephant to rest upon.

That I am a swarm of sparsely scattered electric charges, rushing headlong in nearly empty space, the combined bulk of the swarm less than a billionth of myself, does not in the least distress me as a substance. Neither they, in their mad rush, nor I, have any doubts as to whom they belong. Though I keep on losing and acquiring them in billions, I sit here quite unperturbed by the frightful turmoil, nor do I feel any more divided up by their multiplicity than by having two eyes, two legs, and a heart, all outside of my feet. I am a loose enough assemblage of limbs, but does that prevent me from being myself? Indeed, how could I be myself without these odds and ends? But if I were truly described by these multiplicities as such, if I deserved nothing more than Russell's collective name, then, speaking literally, I should always use the royal ' we ': ' We ' the bundle of events.

In plain English, what are we asserting? That the physicist need not know what a man is any more than the shipping agent weighing trunks need be aware of what is in them. Far less, in fact. To the physicist, Mr. Smith is part and parcel of the shadowy world of symbols; and how else can a physicist treat of Mr. Smith? Indeed, it would mean a much closer approach to what Mr. Smith is to state that he can be turned into a good soap—which is demonstrable fact. So all we ask is that the mathematical physicist realize what his science supposes: that if he wishes to deal with men he cannot permit himself the use of words. It is precisely the improper use of words which makes Russell's statement concerning Mr. Smith so delightfully comic.[1]

And now I am going to surprise you by asserting that, in Russell's understanding of substance, there is

[1] In justice to Russell, we should keep in mind what he said in *Human Knowledge* (1948): ' All nominal definitions, if pushed back far enough, must lead ultimately to terms having only ostensive definitions, and in the case of an empirical science the empirical terms must depend upon terms of which the ostensive definition is given in perception. The Astronomer's sun, for instance, is very different from what we see, but it must have a definition derived from the ostensive definition of the

one respect in which that bundle of occurrences of his is far more substantial than an Aristotelian can allow. Bundle and occurrences are both far too spread out in respect of time. We simply can't tolerate so much of Mr. Smith at once; he is too ample in point of duration. Poor Mr. Smith is actually far more tenuous and eerie than a bundle of events. Nay, even a single specimen of his events is intolerably bulky. For an event implies time, and time for an Aristotelian is composed of non-existents, namely, of a past, which is no longer, and of a future, which is not yet. As to the present, strictly speaking, it is not time, any more than a point is a line; or a moment (the indivisible of motion), a motion. For whatever is called present is so called by reason of the indivisible of time, the instant.

Let me explain myself. Russell is inclined to dismiss Aristotle's treatment of time as a curious discussion in which time is said to be composed of non-existents. But let us see how Aristotle

word " sun " which we learnt in childhood. Thus an empirical interpretation of a set of axioms, when complete, must always involve the use of terms which have an ostensive definition derived from sensible experience. It will not, of course, contain *only* such terms, for there will always be logical terms; but it is the presence of terms derived from experience that makes an interpretation empirical.

The question of interpretation has been unduly neglected. So long as we remain in the region of mathematical formulae, everything appears precise, but when we seek to interpret them it turns out that the precision is partly illusory. Until this matter has been cleared up, we cannot tell with any exactitude what any given science is asserting.' I find myself more often in hearty agreement with Russell than my allusions to him in these lectures might lead one to think.

proceeds, if only as an illustration of what can result from a direct attack on the question of what time is. We shall find that the Philosopher baffles Russell simply because he talks about time as time is (or is not), rather than about how one might measure a length of it.

The present day may be divided into hours, the hours into minutes, these into seconds, and so on. But neither the first half of the day, any more than the first half of its half, is ever at the same time as the other half. (Do not forget that it is time I mean, not the symbolic construction of time.)[1] And so it goes for each of the parts into which time may be divided. Now if we say, of the day, or of the hour, that it is present, we say this never by reason of one or the other of its parts, each of which, no matter how long or how short, is divided into past and future by the present. It is therefore by reason of something belonging to the day, but which is not the day, that the day itself will be called present, and so on for each part of time. The present is precisely that which divides all time, long or short, into past and future; and it is because of such a division that any time may be called present. No doubt, a time past may be divided into past times, as yesterday and the day before, but it is thanks to a

[1] We may recall what Eddington said in his Romanes Lecture (1922) on this subject: 'Those who suspect that Einstein's theory is playing unjustifiable tricks with time should realize that it leaves entirely untouched that time-succession of which we have intuitive knowledge, and confines itself to overhauling the artificial scheme of time which Romer (in 1675) first introduced into physics.'

past present that yesterday was preceded by the day before, and today by yesterday.

Now, this present, which is so vitally important to time, is not itself part of time, as can be easily shown. For time past, the present is the extreme limit beyond which there is nothing of the past, and this side of which there is nothing of the future. For time to come, the present is the extreme limit this side of which there is nothing of the future, and beyond which in the other direction there is nothing of the past. Now, since all time that is to come follows after the time that is past, and the two are strictly consecutive, nothing can lie between them that is one in kind with them, and therefore no time. Hence, if the present, inasmuch as it divides time into past and future, were itself a time, the future would not be consecutive, or next to, the past.

Besides, if the present, the in-between of past and future, is the final extremity of time past, and the initial extremity of future time, since the extremity belongs to that of which it is the extremity, it follows that the same present, the *Now*, is both the end of the past and the beginning of the future; but if it be indeed this in-between which is the present, and if this present be also time, as time itself is inevitably divided into past and future, so it will follow that something of the past is still to come, and therefore is not yet; and that something of what is not yet is already present in what is no longer present.

Furthermore, if the present, the in-between of times past and future, be itself a time, it will be continuous, and divisible into parts, that is, into a past and a future, each of which would be a time. It must follow that this present cannot be wholly present, but present according to something of it which is neither its past nor its future but which divides its own past and future one from the other. But nothing divisible is also the division which divides it. Therefore, lest past and future be confused, lest that part of the present yet to come encroach upon the part which is no longer, and lest the part which is past encroach upon that which is to come, still another in-between will be required to distinguish them. And since this new in-between, in its turn, must be a time, to prevent the same overlapping of one part of it upon another, yet another in-between will be demanded, and so on to infinity. But in this case no present can ever be present, no past time ever be past, nor can there be any time to come—there will simply be no time at all. Hence, the present, the in-between, which divides past and future, cannot be divided and is therefore not a time, nor even a part of time, in the sense in which a part of a line is a line, for a part of time is always a time.

Now, that which divides time but is not time is called the instant. And it is at this instant, and only at this instant, that Mr. Smith actually exists. His time is composed of past and future, that is, of non-existents, while he himself clings to the indivisible of time which is forever passing away. In other words, in that element of time at which Mr. Smith actually

exists, there is no room for anything to move, no time for an event, no time for anything whatever; and this instant of his, moreover, is always slipping away, like the point at which the rolling sphere touches the flat surface. Non-existence, then, is as essential to time as it is to movement; and you see how ready an Aristotelian is to outdo Russell in slimming down Mr. Smith. This has also been a fair instance of the kind of discussion which we no longer have time for nowadays.

You remember my mentioning two types or species of wonder, with wonder about what time is as an example of the first, and wonder about what time it would take me to ride a bicycle from here to the railroad station as an example of the second. The little taste of the first kind which Aristotle has just given us ought to make us suspect that this sort of curiosity can lead to questions far more difficult, and answers far more illuminating, than can be hoped for from any inquiry which is satisfied by measurement. Take for instance that question about what time it will take to reach the station. Since my train leaves at such and such an hour, I am answered by the approximate hour at which I should leave, and must thank you or a computing machine for having worked this out for me. Although perhaps far more exact, the answer of science to the question 'How long does it take the light of the sun to reach Hamilton, Ontario?' will be of the same nature. Such questions are necessary, and the information to which they lead may be valuable. But we must patiently insist that the first kind of question still stands, and that we should not rest until we learn whether it is relevant still.

If such questions were granted to be legitimate, and if meaningful answers to them were possible, what, it might be asked, could we *do* with the answers? In other words, is knowledge worth having which does not mean power? The state of mind which scorns the more basic kind of wonder, and the philosophy which tries to draw the teeth of Hume's sceptical attack on inductive reasoning by restricting the significance of induction to action, have much in common. Hans Reichenbach makes an interesting observation on this subject:

> The man who makes inductive inferences may be compared to a fisherman who casts a net into an unknown part of the ocean—he does not know whether he will catch fish, but he knows that if he wants to catch fish he has to cast his net. Every inductive predication is like casting a net into the ocean of the happenings of nature; we do not know whether we shall have a good catch, but we try, at least, and try by the help of the best means available.
>
> We try because we want to act—and he who wants to act cannot wait until the future has become observational knowledge. To control the future—to shape future happenings according to a plan—presupposes predictive knowledge of what will happen if certain conditions are realized; and if we do not know the truth about what will happen, we shall employ our best posits in the place of truth. Posits are the instruments of action where truth is

> not available; the justification of induction is that it is the best instrument of action known to us.[1]

[1] The method of which Reichenbach speaks, at least in the particular aspect of it which he emphasizes, is one which all of us employ. But this does not in the least oblige us to accept his general position. Indeed, he himself does not seem aware of the full implication of this position. For example, in the same section of his book (*The Rise of Scientific Philosophy*) he goes on to declare that 'All knowledge is probable and can be asserted only in the sense of posits'. Perhaps it is, but he is very certain of it, and if this certitude be accepted, it can only be by interpreting 'all knowledge' as a universal *ut nunc*, that is, of now, but subject to alteration—an ancient understanding of universality, which supposes something like what is now called a conceptual object, a supposition which modern historians of science appear to ignore. But the point is that a man holding Reichenbach's principles has no business uttering this sort of sweeping statement, which can only lead to pointless paradoxes. As for Aristotle's conduct of scientific investigation, it may rest on assumptions which now appear naïve, but it was sound and cautious in principle and method, and took full account of the uncertainty of these assumptions. We are inclined to overlook passages like those in the *De Caelo* where the Philosopher emphasizes his doubts about the foundations of his own cosmology. Aquinas shows the same perspicacity when he points out that our observation of the diurnal movement cannot put us in a position to decide whether the latter is due to what appears to be in motion or whether it is due to the motion of the observer ('Quod enim motus appareat, causatur vel ex motu visibilis vel ex motu videntis'). And, concerning the regularity of that movement, upon which Aristotle based a theory which lasted through the Middle Ages, where the Greek philosopher states that 'The mere evidence is enough to convince us of this, at least with human certainty,' Aquinas explains that 'This is not certain, but only probable. For the longer it takes a thing to change, the more time we need to observe this change. . . . Hence it is that even if the heavens were in fact destructible, the change could take so long that within the memory of man it could pass unnoticed.' Why, then, having made such reservations about them, do these men proceed as though their physical hypotheses were valid? For the same reason that Planck and Einstein feel no obligation to repeat on every page that their explanations of appearances are only provisory. As Pierre Duhem has shown, it was Aristotle after all who laid down the rule that the best we can ever hope for is 'to save appearances,' which may change as observation carries us further; and that we must always be ready to renew the principles

It must be conceded that, although we never reach speculative certitude regarding the physical structure of the world (the physicist has learned that if the universe were constructed on the lines of his theories, it would surely collapse; for, by the very laws governing his search for truth, all his laws, hypotheses, and theories about the world remain subject to alteration, and should be changed as soon as he sees the need) this speculative uncertainty can nonetheless be accompanied by shattering practical certitude. The theory of relativity, long the best available, must one day be superseded by another; even Einstein did his best to bring this about. Meantime it has worked, as you can see by the new kind of mushrooms we are growing. Again, atoms, those 'indivisibles', have ceased to be atoms since Lord Rutherford broke them down. And even today, while the name is still in use, as in 'atomic scientists', it refers to a complex structure which bears hardly a resemblance to the one our symbols conveyed only fifteen years ago. (When I first learned physics,

devised to account for them. I can see nothing outmoded in his remark that 'in general, every subject matter requires principles homogeneous with itself. But some, owing to love for their principles, fall into the attitude of men who undertake the defence of a position in argument. In the confidence that their principles are true they are ready to accept any consequence of their application [which was to happen again in the time of Galileo]. As though some principles did not require to be judged from their results, and particularly from their final issue! And that issue, which in the case of productive knowledge is the product, in the knowledge of nature is the unimpeachable evidence of the senses as to each fact.' The usual account of these things is almost enough to convince one that it was Aristotle himself who refused to look through Galileo's telescope.

atoms were still billiard balls, only much smaller; though they had already lost their Daltonian simplicity, being provided with hooks that later turned out to be too cumbersome, as I learned in the untaught appendix to our textbook.)

But of the practical value of theoretical physics, there is now certitude so fearful that the physicists themselves are dismayed at our fond hope that it can't be as bad as all that. A man needs small understanding of a munitions factory in order to blow it off the face of the earth and, similarly, the scientists have found that their meagre and doubtful knowledge of the structure of matter is quite enough to make possible stupendous feats of destruction. None of the outstanding theorists in the field ever intended anything like this brutal proof of their science. All they wanted to learn from their mathematical physics was the metrical structure of the universe and the laws that govern it, to learn all they could about these things simply for the sake of knowledge. I must mention again, however, that even the scientists, and I have given some names, are grown less sure nowadays that their inevitably limited mode of investigation is the only one relevant to the questions man may have the right to ask.

Yet the prevailing 'scientific outlook', above all in the Commonwealth, is now more than ever dominated by Hume. His critique does not affect science as a tool; indeed, mathematics is now largely recognized for what he thought it was—a tool, and a quite reliable one. But his treatment of induction and causality is now being used to snuff out that first type of wonder: wonder about what a cause is, what is cause of what, what movement is, what place and time are, and so on. His apparently cold analysis has met with considerable popular success; and its effect is to drive from the human mind that primordial curiosity, the parent of all other motives of inquiry, which Aristotle describes in his account of the beginnings of science and wisdom:

That [the most authoritative of the sciences] is not a science of production is clear even from the history of the earliest philosophers. For it is owing to their wonder that men both now begin and at first began to philosophize; they wondered originally at the obvious difficulties, then advanced little by little and stated difficulties about the greater matters, e.g., about the phenomena of the moon and those of the sun and of the stars, and about the genesis of the universe. And a man who is puzzled and wonders thinks himself ignorant (whence even the lover of myth is in a sense a lover of Wisdom, for the myth is composed of wonders); therefore since they philosophized in order to escape from ignorance, evidently they were pursuing science in order to know, and not for any utilitarian end.[1]

The context of this passage will make it clear that this wonder is of the kind now described as belonging to Stone Age mentality. It is a sentiment of this sort that we experience at some striking phenomenon that we do not understand. When the occasion is some mighty deed or work by a fellow-man, the wonder

[1] *Metaph.* I.2. Trans. Ross.

aroused may nonetheless produce contrary effects. It may lead one person to admire and praise the agent, as we admire the scientist; or, for the same achievements, may arouse scandal and blame, as in those who deride basic research as a waste of time, who disregard scientific theories because they are forever provisional, or who are irked because it is the other fellow who has earned the admiration.

But a marvel of human achievement is not the sort of case we have in mind. To appreciate the kind of debunking now so much in vogue, we must replace the person who incites wonder at his deeds or works by Nature herself. The aim nowadays seems to be to show, over and over, that Nature is 'no more than . . .'. To admire the things and processes of Nature is now almost scandalous. One of Aristotle's definitions of Nature was given earlier. It is this definition which concerns the true and thoughtful student of Nature, even though, without it, he may still do a great deal, as the history of science has shown. And anyone who observes that Nature proceeds in that orderly fashion without which there could be no science at all, can add something to this definition by an analogy. Here is how he might now reason: if the art of shipbuilding were intrinsic to wood, the ship would be produced by nature, somewhat as it is actually produced by an art extrinsic to wood; in short, it would be produced as wood itself is produced. As Antiphon pointed out, in a remark almost certain now to be called trivial, 'if you planted a bed and the rotting wood acquired power of sending up a shoot, it would not be a bed that would come up, but wood . . .' Nature, then, is intrinsic to things called natural and functions in them, not as a principle of any sort of motion but, like art, as a kind of reason, proceeding by determinate means toward a term. We come closest to this conception of nature by the further analogy of the doctor who heals himself, for his knowledge, which is the idea or principle according to which he cures himself, is intrinsic to him. In this case, of course, the curative principle is intrinsic only *per accidens*, else he would be a doctor only inasmuch as he were sick; and even *qua* doctor he cures neither himself nor any other person by nature, but by art. Nevertheless, the doctor analogy helps us to understand what is going on when living organisms, for example, close their own wounds. Taken in this sense, a nature is an instance of participation in an Art, in the Art which fashioned all natures and thanks to which they follow orderly courses. The aim of natural science, even when employing mathematical tools, then, can only be to learn everything possible about the Art that fashioned natures. For, as Aristotle said about animals,

> if some have no graces to charm the senses, yet even these, by disclosing to the mind the architectonic spirit that designed them, give immense pleasure to all who can trace links of causation, and especially to those who are naturally inclined to philosophy. Indeed, it would be strange if mimic representations of them were attractive, because

> they disclose the mimetic skill of the painter or sculptor, and the original realities themselves were not more interesting, to all at any rate who have eyes to discern the reasons that determined their formation. We therefore must not recoil with childish aversion from the examination of the humbler animals. Every realm of nature is marvellous: and as Heraclitus, when the strangers who came to visit him found him warming himself at the furnace in the kitchen and hesitated to go in, is reported to have bidden them not to be afraid to enter, as even in that kitchen divinities were present, so we should venture on the study of every kind of animal without distaste; for each and all will reveal to us something natural and something beautiful. Absence of haphazard and conduciveness of everything to an end are to be found in Nature's works in the highest degree, and the resultant end of her generations and combinations is a form of the beautiful.[1]

The Art responsible for nature is that divine Intelligence which Einstein sought in his probings of the physical world, and which left him in unceasing wonder: these are his very words.

Surely it is disheartening to reflect that we live in an age when it can be necessary, not merely to explain Einstein's speculative goal, but even to defend it against another type of mind which would have it that his time might have been better spent in the practice of plumbing. But the spirit of intellectual nihilism is gaining ground. It is frightening to think of the extent to which people are now being encouraged to banish from the minds of their children great questions as

[1] *Parts of Animals*, I. 5. Trans. Ogle.

devoid of all meaning; to dispel the wonder which is a young mind's birthright; to confine their spirit to petty problems that can be answered once and for all to the satisfaction of reasoners incapable of raising a question to begin with. We now have a philosophy to show that there are no problems but those which it has shown to be no problem; and to decree that there is no philosophy other than one that is a denial of philosophy. Under the twinkle of a fading star, Hollow Men rejoice at a hollow world of their own making.

III
The Lifeless World of Biology

In animals life is patent and obvious, whereas in plants it is hidden and not clear. . . . Life seems to consist primarily in sensing and thinking. . . . God is the supreme and everlasting animal. ARISTOTLE

THE title of this lecture may seem a contradiction, and perhaps also a piece of impertinence. Yet the judgement which it makes is both true and moderate. Indeed, it should become clear in the course of our discussion that, to the adjective lifeless, must be added the adjectives inorganic and functionless. Modern biology, if some of its distinguished representatives are to be believed, dare not call itself true science unless it avoids and ignores all that naturally comes to the minds of ordinary people when they think of familiar animals and plants. Nor have I been provoked to this general comment by one or two radical works like Mr. N. W. Pirie's *The Meaninglessness of the Terms Life and Living* (1937), or his more recent *The Origin of Life* (1953). Long before I was aware of his opinion, I had pointed out in an *Introduction a l'étude de l'âme* (1947), that neither the courses in biology followed by myself more than a quarter of a century ago, nor anything I have read since, offered any reason why the terms ' life ' and ' inanimate ' should be used at all except as ' linguistic conveniences '. The reason is that the biology I am talking about is resolved to be sternly empirical, while it can find nowhere any definite, empirically defined property able to separate, once and for all, the animate from the inanimate. Irritability, self-repair, nourishment, growth, and reproduction, as described in typical modern treatises, can be no more than provisional hypotheses, if they amount even to this. It may of course be granted that, as a matter of method, we can and should attempt to explain so-called living phenomena in terms of what we call the inanimate, as far as possible; and that, when we cannot, we should at least keep an open mind on the question. However, even this apparently broad view will lead to difficulties. ' Inanimate ', after all, is a negative term: I mean that, linguistically, it is a negation of ' animate ', so that it looks none too easy to get rid of the living when, without at least the idea of it, the ' non-living ' cannot be named.

On the difficulty of avoiding involvement with the living while pursuing knowledge of the non-living, we shall have more to say later. Meantime Professor W. S. Beck has pointed out, in his *Modern Science and the Nature of Life* (1957), that even Mr. Pirie, having made his attempt at merciless rigour, falls into the trap against which he himself has warned us by going on to use the terms living and non-living as meaningful after all. A similar unconsciously fallacious attitude is sometimes adopted by the physicist. For some unspecified reason, it is assumed that physics is about the inanimate, and that it does not deal with living things, when everyone knows that it seeks to explain things

like gravitation, and that the physicist himself is just as much subject to gravitation as a stone or a sack of potatoes. But I am distracting you with incidentals and will serve you better by getting on with my first task, which is to explain, if I can, why biologists come to this curious attitude towards their subject. Here is how they slip into it. Taking for granted our ordinary acceptance of ' living ' and ' non-living ', these writers, from the start, resolve to explain them in terms of the kind of life we know least about, that is, in terms of the so-called lowest animate forms. Once this method is adopted to the exclusion of any other, there is no escaping Professor Beck's conclusion:

> As perceptual objects, plants are plants whether we call them living or not: ' life ' is a conceptual object. In other words, Pirie is correct: ' life ' is beyond rigorous definition—but he, I, we will speak of life because we all know what it means in the large area of nonambiguity. The errors to be avoided are compulsive rigidity and failure to be happy in the company of uncertainty. When asked what viruses are and what they do, we can answer. When asked, what is life, we must reply with no more or no less than an enigmatic smile.

In his very next paragraph, however, Professor Beck, perhaps unwittingly, makes the same criticism of his method which we make:

> At the moment, I am having difficulty thinking of any use to which a definition of life could be put—*other* than to the everyday problem of recognizing death. When a

scientist manipulates a living system, it is occasionally useful to him to know if it has died. If the system is a horse, there would seem to be few problems. But we quickly discover that the ambiguity of ' life ' affects ' death ' in reverse. If it is a bacterium, a seed, or a spore, the problems may be insurmountable, and in practice we usually establish an arbitrary end point at which death, by decision, is recognized to have occurred. Quiescence and death can look very much alike and their distinction brings us straight to the bar of verbal distinctions.

In other words, we can be reasonably sure about the distinction to be made between a live Socrates and a dead one; but we cannot be anything like so sure whether this particular organism is an animal or a plant; nor whether this object, at this moment, is even a plant or something not alive at all. Now, our objection was that the man who hopes to arrive at some definition of life, enabling him to set life apart from non-life, should never begin with the study of what is alive very obscurely, if alive at all. Why not begin with horses? He can see them without a microscope. Or why not start with the kind of thing that asks what horses are? which eventually constructs microscopes and finds itself faced with the obscure forms of life?

A counter-objection to this procedure comes immediately to mind. Is it not a general principle of science that we must try to explain the complex in terms of what is at least less complex? Now a horse, or a philosopher, is more complex than an amoeba, and should not therefore be studied until the simpler

organism has been taken care of. But this difficulty is easily met. Though far more complex than the amoeba, the horse, in a sense is far more known to us; and a dead horse far more recognizably dead than a dead amoeba. What is it then, which leads a man to assume that, if the term ' life ' is to have any verifiable meaning, it must first be put to test at a level where things are most obscurely alive, if even at that level? I can accept that genes are molecules, as I do that dogs are bodies. But what the latter statement means is much clearer (though we might wrangle over what ' body ' means and, in fact, at greater length than some rigorous philosophers suspect). Faced with the question ' What is life?', are not scientists like Mr. Pirie trying to explain what man already knows well in terms of what man knows far less well? Surely the life of plants is more obscure to us *qua* life than that of familiar animals. As we first try to pin down and reflect upon the meaning of the term ' life ', why should we be requested to ignore the life already so familiar to us, and to signify which we normally use this term? That what I venture to call the more sensible and natural approach is indeed more sensible and natural seems attested by our usual manner of speaking. Take the kind of question Professor Beck wonders about: how did living organisms ' learn to repair their wounds, to resist stress, to think, feel, and reason?' Some might say that such language is no more than anthropomorphic convenience, as when we speak of the mouth of a river or the bowels of the earth.

But, even in such cases of mere metaphor, the words first imply reference to something already known, such as the mouth or internal organs of an animal—of a horse, say, or a man—though not without a generous share of vagueness. Of all our normal language it is true that, whether its words be used as metaphors, given new meanings, or meanings long worn out and now revived, they still imply reference to something already known, something that may be quite certain, no matter how fuzzy at the edges.[1] So why let ourselves be confused by questions like: 'Well, where are the mouths of the protozoa?' or 'Has the bacillus a stomach?' Surely it will be wiser to tackle the creatures we can get our hands on, and whose experience we

[1] All analogical terms are examples of what is meant. Take, for instance, the Greek 'logos', several of whose meanings are retained in our word 'reason'. Prescinding here from the historical order of its various impositions, logos first stands for the conventionally meaningful sounds or written signs produced by man for the purpose of communication: words, phrases, and speech, as distinguished from the thought they are intended to convey. Then it can mean the thought itself which the sounds are aimed to express. It was further imposed to mean what the thought names, and, again, the definition or 'what it is that the name signifies'. It may also mean proposition, argument, discussion, discourse, or treatise. Finally it has other abstract meanings such as 'notion', e.g. the notion of circle; or the reason or ground for something, as in 'Xanthippe threw a pail of water on Socrates for the reason that he came home too late', or 'the flat triangle has its three angles equal to two right angles for the reason that its exterior angle is equal to the two opposite interior angles'. The same word was again extended to mean the power of reason, the faculty; then, too, the exercise of this power, as in judgement, opinion, justification, explanation. It can also mean proportion, rule, and hypothesis. But the first imposition remains throughout important, inasmuch as the plain, unqualified, unanalysed meaning of 'word' is more known to us, while all the other meanings of 'logos' are somehow related to this first one.

share, before seeing what we can make of elusive animated particles which most of us will never so much as be able to observe.

Suppose we make a start with the most conspicuous and noisy animal here present. When I say that I am alive, how can I verify this statement to my satisfaction and yours? So far as I am concerned, 'being alive' means primarily to have sensations, such as touching, tasting, smelling and so on. If we could agree on the meaning of 'to have a sensation', we might then go on to the ancient definition of what an animal is, viz. a body endowed with power to sense. (Which aims to reveal no more than what the Bourgeois Gentilhomme already knew.) It is certainly not obvious that trees have sensations, nor that stones do not; on the other hand, there is not a scrap of evidence for sense-power in either.[1] And though one might easily point to cases

[1] In *The Foundations of Mathematics*, Frank P. Ramsey remarked that 'Where I seem to differ from some of my friends is in attaching little importance to physical size. I don't feel the least humble before the vastness of the heavens. The stars may be large, but they cannot think or love; and these are qualities which impress me far more than size does. I take no credit for weighing nearly seventeen stone.' When I pointed this out to an enthusiast of rigour I was asked: 'How do you know stars cannot think or love? Meanwhile the statement is meaningless.' Well, I do not know that the stars cannot think or love, nor how I should go about proving that they cannot. But I see no need even to try. None of this offers any reason for calling the statement meaningless. If it is utterly meaningless, how can my rigorist friend know that it is? Though I may remain in ignorance of the truth or falsehood of statements, I can still know what they mean, and would not know myself ignorant of their truth or falsehood unless I did. Thanks to Lord Russell, my friend has since matured. Now he calls the distinction utterly trivial—which it may well be, since it has always been in use and was thoroughly accounted for by philosophers thousands of years ago.

where it is not clear whether a given object is capable of sensation or not, there is no reason why these cases should disturb our own awareness of being alive as we use our senses.

Besides having sensations, I *know* that I have them; and this is of no small account. I am quite aware, when not seeing red, that I might do so, and that I might not when I do. I know that to see is not the same as to smell, and I distinguish these sensations as I receive them. I also remember having seen red when I no longer see it; and, when now observing something red, I am aware that I have seen such a thing many times before. I can try to recall when and where I last saw a red object and, when I say 'reminisce', I do not mean the same as 'to remember'; just as by 'remembering' I do not mean the same as 'to have something in mind which is not present'. I perceive and distinguish different kinds of sensations, and representations of things no longer, or not yet, present. I know that I possess these various kinds of knowledge, and know that I know this.

To all appearances, elephants have various kinds of knowledge too. I see that they can be annoyed by insects boring into their hides; that they see, hear, and smell. But I am obliged to wonder in what fashion these huge beasts know that they have sensations, and whether they ask themselves what a sensation is; or if, in any sense, they know that they know. The reasons for believing that they never wonder about such matters are more convincing

to me than the reasons for believing that they do.

Let us interview our favourite elephant in the zoo in order to establish what we first mean when declaring him to be alive. As he thrusts forth his trunk, I somehow detect that he is wondering whether he is going to get a peanut, for he remembers having been offered pellets of paper instead; and at this moment he is trying to recall whether it was this little fellow who so deceived him. Now that is what *I* mean by a live elephant as opposed to a dead one, though I can hardly imagine what it means to him, or how he could get himself into a state of wonder about what it is to be alive—or to be an elephant, for the matter of that. He never discusses such things with me, and the reason is likely that he has nothing to say on the subject. He does produce significant sounds, of course, but his vocabulary is rather limited; though I am sure it has nuances which escape any listener but a fellow-elephant. Still, I believe that, unlike Aesop's lion, the elephant has no need to say any more than he does.

All I have asserted about being alive leaves the reality in deep obscurity, and makes it clear enough that I do not know how far life extends. My certitude of touch as I sit here—my certitude of the resistance of this chair and of the warmth of my hand on this cool desk—these do not imply clear knowledge of what this sensation is, though I manage to distinguish it from other kinds. But the point is that my certitude is not diminished by ignorance of the conditions of sensation, while I do see that these are many and in some measure beyond analysis. The tactile world appears quite different when my hands are numb: tepid water, for example, feels warm when my hand is cold; and this page looks blurred when I take off my spectacles. Now, is it reasonable to argue that, because my numb hand cannot feel so well as my normal hand, that my normal hand cannot feel at all? or that my need of spectacles proves my eyes to be untrustworthy?

Yet reasoning of no higher order is now commonly used to convince us that the terms 'living' and 'non-living' are no longer of any use. It is suggested, for instance, that the word 'living' is meaningless because there are cases to which nobody knows whether it applies, that is, things of which it is not possible to be sure whether they are living or non-living. But, if the denial of the distinction is to have meaning, we must understand the terms whose distinction we deny. So far as I can see, ignorance of where life begins or ends in the world of the microscope has nothing to do with my certitude of being alive, even though I may not know much about my own kind of life. Professor Beck can bear the company of uncertainty,[1] but not, you

[1] Aristotelians bear the company of uncertainty calmly enough, of course: 'error is a state more natural to the animals than the truth, and in which the mind spends the greater part of its time' (*De Anima*, III. 3. 427[b]). Aquinas's paraphrase reads as follows: 'For error seems to be more natural to animals, as they actually are, than knowledge. For experience proves that people easily deceive and delude themselves, whilst to come to true knowledge they are in need of being taught. Again, the mind is involved in error for a longer time than it spends in knowing truth, for we barely attain to knowledge of truth even after a long course of study.'

will notice, as regards the life or non-life of a horse. 'If the system is a horse,' he said, 'there would seem to be few problems'. But, he added, 'If it is a bacterium, a seed, or a spore [and we might add, a protein molecule], the problems may be insurmountable. . . '. Of course. And since insurmountable they may well remain, it is no wonder that, if the professor is determined that the question shall not be discussed at a more intelligible level, his best answer should remain an enigmatic smile.

Let us get back to our horses. We are agreed that we know more or less what we mean when we say that a horse is alive, and that to all appearances a stone is not. It will be profitable now to make more explicit that difficulty in distinguishing alive from not-alive which was briefly pointed out earlier in our lecture. When compared to a horse, if perhaps not to a creature on the obscure microscopic level, a stone, we confidently assert, is 'not alive', or 'non-living', or 'inanimate'. Now these terms are negations. But a negation is something relative: it is the negation of something. If the negation is to be meaningful, you must know what it is that you are negating. 'Non-living' means nothing without some knowledge of what the term 'living' stands for. In short, there must have been a definite sense assigned to the term living, before any significance could appear in the negative 'non-living'. And what can this mean except that we know and name the living before we name the non-living? I certainly know far better what it is to have a sensation than I do what it is to be a stone, even when it is the very stone that causes in me a sensation as I stumble over it.

Of course, the ability to use the word 'living' in a significant way of things such as horses and men, does not mean that I know 'what life is' in the sense that I can actually define it, not merely interpret the name. It is one thing to know a thing well enough to name it; it is quite another to make fully explicit what it is that I name and to set it apart once and for all from any other kind of thing. But I think we are safe enough in distinguishing horses from stones in terms of life; and if we cannot be so definite about lower forms of life, why should this make us surrender our horses?

It was a typically Cartesian view that science must begin with what is most basic in the things under study. Many of us were raised on the 'evidence' that an atom was a much clearer thing than a stone; while in the study of life, we were made to begin by clearing away everything but the amoëba. The assumption was that whatever is less complex ought to be more accessible than the complex. In physics there are no Cartesians left: the world of mathematical physics has turned out to be far more involved than Descartes or even Newton could suspect. It has taken some centuries of experiment and symbolic construction even to approach something basic, such as what are now called atoms and quanta. And every day we learn that these are more complex than was thought yester-

day. So the fact must be faced that what we know first and foremost is not what is most basic to things themselves, no matter how much we might like to have it that way.

But in biology, Cartesianism still thrives. Those of us who got their elements of biology some forty years ago will remember the doctrine that protoplasm was the universal stuff of life. Since then, the nature of protoplasm has become happily less clear, and what the stuff of life may be, is now known to be less known than ever before. Is it credible that some biologists continue to state that their science is working with physical phenomena 'entirely accessible to our understanding', by which they mean that life is explained 'in terms of physics and chemistry?' (I would be grateful if they did no more for me than to explain the distinction assumed here between physics and chemistry.) What are these physical phenomena entirely accessible to our understanding? Of the sciences of nature, physics is the most exact; but I have yet to meet a modern physicist who speaks with any such confidence about what he knows. He is in fact growing more and more baffled at the unsuspected complexity of the basal entities of the physical world. Eddington's statement, though made nearly a quarter-century ago, still stands unchallenged: as he digs to the foundations of the physical world, the scientist finds himself 'treating a universe which is the theatre of unknowable actions and operations'. How then can biology assert that it is dealing with physical phenomena entirely accessible to our understanding? In the context of an arrested Newtonian physics, in which the basic stuff of the universe and the laws governing it were assumed to be clear *more geometrico*, it would be easy enough to interpret this statement. For the great Newton wrote his *Principia* in the lucid vein of Euclid's *Elements*. But surely there is no excuse for retaining such a conception of the physical world today.

Although an immense over-simplification, the old Newtonian view was nevertheless a fascinating one. The world had the clarity and intelligibility of a machine such as man himself might build. Now there is in fact nothing more known to us, as to what they are, than the things that we ourselves have made by art or craft. True, a good deal of the material that goes into a motor, for example, is known only vaguely. But mere practical knowledge of this material is enough to get the machine to work. We know that a spark will explode gasoline vapour. With very little more knowledge than this it is easy enough to see why gasoline engines operate as they do. The works of our hands, once made—from hammers and saws, to nuclear bombs and missiles—are well known to us, as to purpose and function, because we ourselves concoct them. Now, if nature were the same kind of thing, if the whole world and each of its parts were just like a machine, we could then truly speak of physical and biological phenomena as accessible to our understanding. But it so happens that even in physics this model theory, though it worked for centuries, has now quite broken

down. Some biologists apparently survive still unaware of these developments, serenely confident that living bodies, as Descartes once thought, are just machines.

Since we are on the subject of over-simplification in biology, let us glance, for a brief moment, at how the same tendency affects the problem of evolution. Having been brought up to accept the fact of evolution, I would not find it easy now to doubt that it has happened, however uncertain I may remain about the value of any particular theory devised to explain how it happened. There seems no reason why nature, ' one mask of many worn by the Great Face behind ', could not produce living things from non-living, and higher forms of life from lower, somewhat as we build a table out of a rough piece of timber. If nature cannot accomplish something analogous to this, nature cannot be what an Aristotelian thinks it to be. But though a craftsman may make a chair, he does not thereby make ' what a chair is '. And in the same way, nature may produce this or that living being, without being in the least responsible for ' what it is to be alive ', nor for ' what it is to be this kind of living being '. As Aristotle suggested, to burn down a house is not the same as to abolish ' what a house is '. To produce a man from some ape-like creature no more destroys the *kind* of creature this was than a degenerative evolution of man into an ape-like creature could suppress ' what it is to be a man '. Believe in evolution as much as you like, you are not thereby compelled to believe that the things transformed by evolution are really only the same old thing unchanged and unchangeable. And yet it is frequently held that the various kinds of living beings are merely incidental variations of the same thing. Some people have lost all true understanding of the problem and take the attitude that it is incidental that two tons should be two tons of elephant rather than of coal. Of course, if all you want is two tons, and you get them in coal rather than in elephants, what more can you ask? It is not always realized that this kind of reduction makes of evolution a superficial phenomenon hardly worthy of the name. To give rise to more complex species now becomes something like producing those 'new' symbolic constructions of ours: 2 is nothing new over and above $1 + 1$. The idea is a venerable one, of course: the very earliest philosophers, using the analogy of human artifacts too superficially, believed that basically all things were of the same nature, as chairs, broomsticks and tables are all wood; and held, further, that these differentiations were wrought by chance, which is without reason, without 'logos'.

Life, we suggested in our opening paragraph, implies the organic, and modern biology—or so we are told—is just as ready to dispense with the one as with the other. But before going any further, let us be sure that we know what organic means. A good deal has been said about sensations as a sign of life. Of our own sensations I would like you now to notice how they quite obviously involve certain parts of our body, such

as skin, eyes, ears, and other members identifiable perhaps only after some investigation. These parts are commonly called organs, and that is in fact what they are. I do not intend to convey that, with regard to sight, for example, these eye-balls of mine, all by themselves, are the organs of vision; what lies behind them in the brain is no doubt even more important to seeing, and so must also be part of what is meant by the organ.

Now I would like to arrest your attention upon this word 'organ'. In biological writings, no word, you will agree, is more commonly used. How often, though, do we see any attempt to make plain its meaning? Organ, organic, organism, living organism —the pages of our text-books are littered with such expressions and you may be astonished to hear me maintain that, in the context of these works, their meaning can be exceedingly difficult and complex as anyone can see by reading Professor Ludwig von Bertalanffy's *Problems of Life*. Yet we simply must know what an organ means, and I think the best way to begin is to search out the first and primitive use of the word. Not that we are interested in etymology, of course, but simply that later and more cultivated meanings of a word usually grow from some original meaning.

The name 'organ' is taken from the Greek 'organon', meaning, simply, tool; something used to do something with, like a hammer or a saw. And that is what Aristotle had in mind when he described a living being as ' a natural organized body ', that is: a natural body, as opposed to an artificial one, equipped with tools. Now, to verify the meaning of this description, there is no need to hurry off to examine an amoeba for its special type of tools. If it eats and reproduces, doubtless it has the means to do so. But, if it be a good example of organ that is wanted, we had much better begin with a human hand, or an elephant's trunk. After all, it is further knowledge that we are hoping for, not ignorance of what we knew before we began (although this could apply in some cases, since most of us had to unlearn our first knowledge of things like atoms and the basic stuff of life.)

Of any organism, among the well-recognized forms, at least this much may be said: that it has structure, and that it is also an heterogeneous whole; by which I mean that a horse for example is not all legs, nor all eyes, nor all tail. But these generic qualities do not set an organism apart from structures or heterogeneous wholes that are not organisms, unless we have chosen to use organism as a metaphor or to give it wider meaning. In other words, an electron is not an organ of the atom in the sense in which the trunk is an organ of the elephant. But if it so happens that you want to extend ' organism ' to whatever has structure and heterogeneous parts you ought to be aware that you are in fact either using a metaphor, or extending the meaning of the term. If we stick to our elephant, however, and to ' organ ' as the word is exemplified by his trunk, then it is as plain as his trunk that by organ

we still mean a tool. Now, this is to mean a great deal, because a tool supposes a purpose and, when we speak of organic bodies, if we mean anything, it can only be that these bodies are equipped with the means of pursuing purposes.

You see what bad tactics it is to begin with creatures like the amoeba. With no better idea of an organ than this obscure little creature can give us, we should never know quite what we meant by the word. Later on, of course, we must investigate the world of microscopic life, but the acquisition of this new and different knowledge should not cause us to cease to know what we knew.

That many modern biologists cannot endure the thought of nature as attracted by purpose, as acting ' for the sake of something ', is evident from much of the literature on the subject. Ever since Darwin, the opinion remains prevalent that the notion of purpose in nature is unscientific and unnecessary. ' At first sight, the biological sector seems full of purpose ', admits Julian Huxley. ' Organisms are built as if purposely designed, and work as if in purposeful pursuit of a conscious aim.[1] But the truth lies in those two words " as if ". As the genius of Darwin showed, the purpose is only an apparent one.' Now, what I am attempting to establish is merely that, if an organism has no purpose,

[1] Of course there is no need to suppose, as this phrase seems to do, that, in order to be genuine, purpose requires that nature be conscious of it. All the same, Sir Julian seems to have understood that ' si natura operetur propter finem, necesse est quod ab aliquo intelligente ordinetur ' (Aquinas, *In II Physic.*, lect. 12).

it is no organism at all: it is not a body equipped with tools making possible the fulfilment of needs, but simply a mass in which appear a number of functionless appendages. Whether this last description makes sense need not concern us for the moment; the point is that there is no avoiding it when purpose is denied.

Sometimes, of course, a scientist is entirely right in excluding all thought and mention of purpose from his work. This is the case whenever his methods are formally mathematical. Thousands of years ago, it was recognized that, when applying mathematics to the study of the physical world, we prescind from the good and from purposeful activity in nature, no less than we do in pure mathematics. It is not because it is good for the triangle that this figure of parabolic geometry has its three angles equal to two right angles; nor is Galileo's quadratic law a plausible equation because it is better for it to be so. So if by the scientific outlook is meant the mathematical vision of things, it will be necessary to agree with Bertrand Russell when he says that '"purpose" is a concept which is scientifically useless': and that 'in science it is the past that determines the future, not the future the past. "Final causes", therefore, do not occur in the scientific account of the world'.

But good was pursued and evil avoided in this world long before there was any scientific account of it, nor does the scientific account in this respect seem to have made any great change in things. The natural world

is so mysterious, so stupendously complex, that a science has every right to prefer the simplest possible methods, and so is entitled to shun many problems that cannot yield to these tactics. But we do not have to accept what is left as the world in which we have got to live. Whatever the nature of cancer, for example, we continue to call the subjects of it 'victims' and, leaving aside worse evils, we may truly declare that it is simply good not to have this one. We shall also agree that it is good for a man to have both his legs, both his eyes, and water when he is thirsty. Nor should it appear any less reasonable nowadays to state that it is good for him to be endowed with the proper electrical charges, molecules, cells and tissues. Certain chemical combinations make obviously living beings possible; and why should these not be called good, at least with respect to the beings which need them? Purpose is of course clearer to us in what we are wont to call the higher animals, not to mention man himself: it is plainly good for an elephant to have his trunk, and a whale his tongue. The good is obscure to us in the case of plants and, when, if at all, we penetrate to still lower regions of being, final causality fades out of sight. Where we can no longer relate things to recognizable living creatures, we can no longer be sure of what is good or bad. All this is the plainest common-sense, and we shall be silly indeed if we abandon any of it for no better reason than the impossibility of translating it into formal equations.

Aristotle had his troubles with that account of the world which Russell calls scientific, whereby the past determines the future; a doctrine which aims to explain in terms only of causes that exist *before* whatever it is that comes to be. One of these causes is the stuff (a conveniently vague term) of which that which comes to be is composed, another is the agency giving rise to that which comes to be. Instances of these two causes would be the stone and the sculptor, both of which exist *before* the statue does. Causes that come to be *after* these first two,[1] would be, for example, the shape of the statue, by which it differs from the unhewn stone, as well as what the sculptor had in mind while selecting the stone and hewing it, namely the statue. Now, although the completed statue is what comes about last of all, it was what the sculptor intended before it actually came to be. This finished shape, and that which he intended while hewing this stone, are in fact one and the same thing, though considered in different respects: the shape now in existence makes and reveals the statue as different from the unhewn stone, and it is also what the sculptor intended.[2]

[1] As Russell has repeatedly observed, 'The "efficient" cause is what we should call simply "the cause . . .".' While it is the one which was first named 'cause' it is, in fact, as Hume's critique has shown, quite difficult to defend outside the domain of human making and acting. But once we have defined cause as 'that upon which something depends in being or becoming', the notion of material cause is the most obvious and certain, such as the wood of a wooden table; then that of form, e.g. the shape of the table; and any critical discussion of causality should begin with these.

[2] An infinity of different purposes may be prior or concomitant to his purpose as a sculptor, such as his commission, obedience to his patron, and so on. But these are incidental to the statue as such—

Now, according to the doctrine which Aristotle had to refute, the analogy between art and nature would hold with regard to the stuff that things are made of, and in some small measure for the agency as well. But the analogy already begins to break down in the latter respect, inasmuch as agency is reduced to aimless process determined only by the material, in the way a landslide occurs. There is no agent intending a good so that, if some good does in fact come about, such as the erect posture of man and the attendant size and quality of his brain, this happened for no other reasons than the kind of stuff of which he is composed and the process which left him composed of it: there was nothing prior to these that had anything to do with what came about. Here is how Aristotle describes the theory:

Why should not nature work, not for the sake of something, nor because it is better so, but just as the sky rains, not in order to make the corn grow, but of necessity? What is drawn up must cool, and what has been cooled must become water and descend, the result of this being that the corn grows. Similarly if a man's crop is spoiled on the threshing floor, the rain did not fall for the sake of this—in order that the crop might be spoiled—but that result just followed. Why then should it not be the same with the parts in nature, e.g. that our teeth should come up *of necessity*—the front teeth sharp, fitted for tearing, the

which faces us again with that distinction between *per se* and *per accidens*. If this distinction is ignored, we can say just as readily that the sculptor's purpose was to escape his mother-in-law's bad temper, and so vindicate that extreme form of sophistry which exploits the kind of non-being found in the infinite *per accidens*.

molars broad and useful for grinding down the food—since they did not arise for this end, but it was merely a coincident result; and so with all the other parts in which we suppose that there is purpose? Wherever then all the parts came about just what they would have been had they come to be for an end, such things survived, being organized at random in a fitting way; whereas those which grew otherwise perished and continue to perish, as Empedocles says his 'man-faced ox-progeny' did.[1]

If the notion that there is purpose in nature can be made to look so outlandish, as in the instance of rainfall and the growth or destruction of grain, the reason may be an undue haste in relating an effect to a cause out of all proportion to it, as in the case of 'his house burned down because he was on time for dinner'; which seems easy enough since, even when convinced that there ought to be such a cause, we can in fact rarely put our finger on it. But whatever this cause may be, the grain is good and nourishing, and its destruction will be regretted when there is need for food. The point is that so long as we do not care whether a product of nature is good or bad, the question whether—even in the frightful contrarieties upon which the so-called balance of nature depends—there is action for a purpose or not, is irrelevant. If we allowed all the same that nature does in fact produce good results—such as the proper kind of teeth; and regrettable deviations too, such as blindness and cancer—but without purpose, so that even failures

[1] *Physics*, II. 8. 198^{b}16. Trans. Hardie and Gaye.

have no proportionate cause, such results would have a reason to account for them, but not in respect of their goodness or badness. Nonetheless, we could hardly put down all effects of this to chance, since chance events can scarcely be held to occur without supposition of action for a purpose—unless of course we impose a new meaning on the word, as in 'laws of chance', which in their mathematical form are wholly determined. Nor can we escape the conclusion that a distinction between what is by nature and what is by chance will be irrelevant if there is no distinction between *per se* and *per accidens*.[1]

If in science it is the past alone that determines the future, by reason of what the past was and of the consequent necessity of what shall be; if that which, though last to exist, but first intended, is not a true cause; as if the house as actually built is in no sense responsible for the builder's choice and arrangement of materials, or the spider's web as actually finished does

[1] That chance events occur in action for a purpose is plain in the case of human actions. If a man, digging for water, strikes oil, this event, so far as *his* explicit intention is concerned, is a piece of good fortune. But if in digging a well for water, a man frequently strikes oil, then, provided he knew this, the discovery of oil would not be attributed to chance. A pregnant woman took the bus to go to market, caught the German measles, and eventually gave birth to a crippled child. So far as she was concerned, there was no connection between what she intended and what happened as an unforeseen side-effect. In nature, chance can be recognized in the case of the lioness which, having lost her cubs during an elephant raid, finally gives up the search when she loses their scent at the stream they had fled across; then there appears an antelope which she pursues for the sake of food; the prey leaps across the stream, and the lioness in pursuit is suddenly faced with her cubs. If this discovery can be called a good, it is a chance event in nature.

not suppose something analogous to intellect producing something like the purposeful structure of our machines,[1] then the scientific outlook on nature will be

> as if one were to suppose that the wall of a house necessarily comes to be because what is heavy is naturally carried downwards and what is light to the top, wherefore the stones and foundations take the lowest place, with earth above because it is lighter, and wood at the top of all as being the lightest. Whereas, though the wall does not come to be *without* these it is not *due* to these, except as its material cause; it comes to be for the sake of sheltering and guarding certain things. Similarly in all other things which involve production for an end; the product cannot come to be without things that have a necessary nature, but it is not due to these (except as its material); it comes to be for an end. For instance, why is a saw such as it is? To effect so-and-so and for the sake of so-and-so. This end, however, cannot be realized unless the saw is made of iron. It is therefore necessary for it to be of iron, *if* we are to have a saw and perform the operation of sawing. What is necessary then, is necessary *on a hypothesis;* it is not a result necessarily determined by antecedents. Necessity is in the nature of matter, while 'that for the sake of which' is in the definition [of what a saw or what a house is].[2]

But to hold that nature acts for an end does not imply that we can in every instance tell what that end is: just as it is one thing to say that there is such a thing as a living being, and quite another to be able to distinguish

[1] When we speak of the purpose of machines, we do not mean that the machines themselves act for an end, but that we, the principal agents as distinguished from these tools, built them for a purpose.

[2] *Ph.* 9200[a].

the living at all levels of life. There are large areas of science, as we have seen (indeed all of mathematical physics), where no object or activity is to be recognized as good and where search for purpose would be wholly vain. In biology proper, however, while one can deny purpose, it is not easy to do it and remain consistent.[1] For instance Professor Beck himself banishes all purposeful production in and by nature, but then goes on to write:

> It is by connecting coat color with the idea of visibility and the concept of protection from predatory animals that permits us to say the polar bear is white because white fur enhances his chances of survival. By going behind what is observable, we have explained his color.

Now this oddity could arise from the mere fact that Professor Beck is using words such as 'because', 'enhance', and 'survival'. When taken together these terms would normally convey that it is a good thing for the polar bear to have a white fur, the reason being that it enhances his chances of survival, which, for the polar bear at least, is apparently a good. All the same, 'because', or 'the reason for', are ambiguous terms, as can be readily seen in 'The landslide occurred because of abundant rains', 'Mr. Smith built a house because he had the means to do so', or

[1] I must not convey the impression that Professor Beck's negation of purpose in nature is the common view amongst biologists today. Quite the opposite was held by Guyénot and Cuénot, who, in their later years, could reconcile random mutations with finality; and by C. H. Waddington, a scientist who is fully aware of the limitations of mathematics in biology, as may be seen from a reading of his *The Strategy of Genes.*

'he built it because he wanted shelter'. It is the meaning of the term in the last example which is crucial. Does there exist in nature a 'because' which offers an intended good as explanation? The modern theory of random mutations, anticipated by Empedocles, proposes a final account of the origin of all types of organisms which dispenses with ends or goals as causes. But can such a theory stand? How does it differ from the assertion that the duck was brought down because this particular pellet of shot happened to strike a vital spot? that the bird was therefore downed by a random missile, since any other pellet might have done as well? Will this do as an ultimate explanation? Why does the hunter cultivate a random distribution of bird-shot? What would happen to mushrooms if they did not produce their enormous superfluity of spores? Or to humanity if there were but one sperm for each ovum? The duck-hunter uses shot instead of a bullet because it enhances the probability of hitting his target. In fact, his hope is that most of the pellets will miss the duck. But the 'scientific account' of a successful shot would suggest that the bird fell because one of the pellets happened to strike it in a vital organ. This is true; but is this all there is to it, when the plain purpose of a charge of many pellets was to ensure that one should strike?

A certain reluctance upon the part of biologists to introduce final causes into their explanations is some-

times quite understandable. This type of cause, if first and foremost, is also the most obscure. To see how it works, even in the most vivid examples, is not easy and, as we have seen, in large domains of research it cannot so much as be identified. Yet, if it be final cause which establishes an intelligible connection between the other causes in nature, as it does between causes in art, it remains true that to banish finality completely will be to imply that nature is basically unintelligible. The scientific account of things would consequently be obliged to shut out all reference to reason as explanatory of anything. Now many things can in fact be accounted for without reference to intelligence, but the crux of the matter is that final cause is not one of them. Are we to conclude that the scientific account of the world cannot be a reasonable one? that science is hostile to intelligence?

Now, it is a curious fact that the writers most ready to make this suicidal rejection can accuse those who disagree of falling victim to anthropomorphism. Not only a belief in purpose, but even the conviction that nature is more than a machine, or that certain creatures, like mammal and tree, are radically different, or that general laws governing the whole universe can never explain everything in it, are all scornfully dismissed as 'organismic concepts', beliefs on a level with pan-animism. But might not the opposite be nearer the truth?

It will be agreed by all, except those perhaps who are determined that nature shall behave as they think best, that mental constructions, though submitted to experience and converging towards nature, may never be equated with what they are thought to approach. To so equate them would be anthropomorphism of the most preposterous kind. It would be like saying that nature is just what we happen to know of it; or that what a thing is in itself exists after the manner in which we come to know something about it; that our symbolic constructions are after all neither symbolic nor constructions; or perhaps that nature, which we reach out to in our constructions and idealizations—such as inertia, true spheres, stars taken as points, and ideal gasses—is itself no more than symbolic construction or idealization. Such a claim would indeed be far more anthropomorphic, or organismic, in the pejorative sense usually intended, than the belief that everything in nature is alive in the way a man is, that thunder and lightning are signs of nature in a rage, or that the moon truly smiles upon the waters. But it is just as easy to be anthropomorphic by rash denials as by rash assertions. The assumption that, to be valid, the word 'good' must have a single meaning, or that 'purpose' must be confined to human affairs, may well be another instance of anthropomorphism, though of a more sophisticated kind. This word 'good', like any other analogical term, such as 'life', 'power', 'cause', has many, distinct meanings, as in 'a good steak', 'a good man', 'a good saw', 'a good house', 'a good mind', and so on. What the single word 'good' stands for is not a single concept, but a whole

group of more or less interrelated concepts, all of which refer to different kinds of things or properties.

The idea of final causality in nature is based, of course, on the assumption of an analogy between art and nature; it being understood that analogy means proportion, not identity. Now, if the analogy holds good: if what comes about by nature is *that for the sake of which* the process resulting in the structure of an elephant or a man occurs, in a way somehow comparable to human purpose in making or in acting, then, to ignore that nature acts for an end would place us in the position of a man who is ready to explain a house without any reference to that for the sake of which houses are built. The difficulty of the question 'What is a house?' would be side-stepped by the narrower questions, 'What is a house made of?' or 'How was it made?' Only the latter type of question would now belong to the scientific outlook. And, of course, if it were indeed impossible to learn what a house is for, there would remain plenty to occupy us in finding out what it is made of, and in understanding what would happen if its materials were arranged otherwise—if the foundations were of wood, the roof of stone, or the walls too high.

Justification of the analogy is not an easy task, and I do not intend to attempt it here. The point I wish to make is that, if the analogy is true, if there is such a proportion between nature and art, it follows—as a tautology, if you wish—that the unqualified negation of nature's action for an end implies that nature, as compared to art, is finally unintelligible, that the bird building her nest functions as we might if we produced a house with no purpose in view, or made something that just happened to be a house. I venture to say that this seems to be the way some people want it nowadays: a methodology which takes us back to the beginnings of science, to the *antiquissimi philosophi*,[1] to whom it appeared that things must be accounted for entirely in terms of whatever they are made of: which is what these earliest philosophers meant by nature, substance, essence, or matter. And this was far from being a bad start, for, as a matter of principle, we must account for as much as possible with as little as possible.

Nevertheless, it is not so easy to hold nature and intelligence apart. In some of his pursuits, man finds himself almost watching and listening for some sign of nature's intentions, because it can be so much to his advantage to learn them. What is our agriculture, for example, but an attempt to make nature improve upon herself? to achieve more efficiently and rapidly what our investigations have convinced us she must be trying to achieve? We have discovered short-cuts to objectives which nature could attain only imperfectly, if at all. Medicine and surgery supply familiar instances. The fruit, vegetables and live-stock we feed on are largely the products of nature enhanced by art—*ars cooperativa naturae*. Indeed we may be at last on the verge of rousing life itself in a test-tube. But when we do bring to life some inanimate brew (a trifling

[1] Thomas Aquinas, *Quaestiones Disputatae de Veritate*, q.5, a.2.

achievement, notice, if there is really no difference between them), surely the least we may hope for is that we shall do so knowingly, with full appreciation of what we are about. And should this come to pass, dare we then maintain that this is the first time that something natural was brought about by reason and not by chance? Or will the fact that reason at long last has managed to do what Nature does, prove rather that Nature is very like us? that Nature, though not possessed of the kind of reason that we have, does nevertheless share in some kind of Reason, and in one the power of which we may well envy since, if we commanded it, we could make wood *grow* into a ship?

The trouble, it seems to me, is that the attempts to account for the living entirely in terms of the general laws of mathematical physics are the result of the artificial barriers which have been set up between the sciences of nature, so that there is nothing left for the isolated worker but to explain everything in terms of his own department (although in this he at least bears witness to the scientist's instinctive desire to attain the whole, and thus to philosophize). But such a procedure is defensible only when adopted as a mere working hypothesis. Nature is a heterogeneous whole, in the exploring of which various methods must be used. Dissect it as we may, the subject under investigation somehow persists in remaining one. Take, for example, Democritus's 'small world' that is a man: he can be cooked down to his molecules, and even to less, to sheer radiation. But what is the effect of this rendering process? Does it enable us to pronounce that now at last we have got hold of what a man is? It is familiar knowledge that a house is composed of brick, cement, boards, plaster, nails, wire and pipe. But does this knowledge ever impel us to think or state that a house simply is brick, cement, boards, plaster, and so forth? The difference between a heap of building materials and a house is plain enough. Surely there is an even greater difference between a heap of molecules or atoms and a man.

The truth is that, by the sciences of nature, we should not mean physics and biology (including psychology) only, as these are now understood. Professor Pascual Jordan seems aware of this when (in *Physics of the 20th Century*) he regrets that 'The increasing independence of natural scientific branches from philosophy from Aristotle's time to the present has simultaneously also emptied philosophy of its original content and problems.' For whatever may be the tactics of this or that science of nature, it remains true that all should converge upon the single, though infinitely varied, whole which is their subject. Now, to hold this general objective steadily in view, and in its light, to pass judgement on the conclusions of specialized branches of research, is the business of natural philosophy—which should be the concern of each and every scientist. The fatal consequences of abandoning all thought of the subject as a whole, to become absorbed

and lost in independent investigation of single aspects of it, is illustrated everywhere. The absence of co-ordination between the sciences, the failure of each to reflect constantly upon the scope and significance of the others, have brought all to a state of hollowness and shapelessness, like the grin without the cat or the cat without an outline. In earlier sections of this lecture, I had occasion to emphasize relatively clear cases of what we mean by being alive, and to suggest that it is a strange sort of science which cannot bring itself to allow them. But I do not contend for a moment that knowledge of nature has accomplished much by identifying such cases, or by offering a general definition of what they represent. To recognize something as an animal may be just a little better than nothing; to recognize it as a horse or a rabbit, is to make a real step, though only the first, towards our true goal. Knowledge of nature that would rest in something so vague and general as what is conveyed by 'animal' would be as empty in a way as the kind which I am criticizing. My purpose in all such examples was simply to show that, to recognize the horse or the cat at least as living things, is far less fictitious than to dismiss them as machines.

The problems of philosophy, when distinguished from those which Bertrand Russell calls scientific, will remain forever in debate. Should the day ever come when Leibniz has his way: when, to settle their problems, philosophers will merely have 'to take their pens in their hands, to sit down to their desks, and to say to each other (with a friend as witness, if they liked), " Let us calculate " ', there will be no more problems, for there will be no one to raise them. Meantime, the calculators have their use, while philosophers are forever in need of being debunked, a thing no one knew better than Socrates. Nevertheless, as Aristotle suggested, no one can deny philosophy without at least implying a philosophy of his own, and his own may prove to be a very foolish one.

What did we know of man before we found out that he is a throng of electric charges? and that he is composed of multitudinous cells? and that the circulation of his blood is an exquisite piece of chemistry and mechanics? Is it possible that, having learned all this, we may remain far more ignorant of him than Sophocles, or Shakespeare? or the people who believe they know what these writers meant? Over the dead body of his faithful Cordelia, the aged Lear had no more need of what is now offered to mankind as science than you and I will feel, if it ever becomes our lot to know such grief.

> She's gone for ever.
> I know when one is dead, and when one lives;
> She's dead as earth.

Epilogue

Reckoning with the Computers [1]

If the word 'thought' be taken in the meaning conferred upon it by the late A. M. Turing,[2] there is no doubt that our computing-machines think; just as, according to a certain meaning of ' select ', our potato-sorting machines do select tubers of various sizes. There is no reason to find fault with writers like Turing over such use of words. When computer-men agree to call the operations performed by a calculating-machine ' thinking ', and to speak of them as ' logical ', they are exercising the technician's right to impose new meanings on old names, or perhaps to revive worn-out meanings recorded only in etymological dictionaries. *De nominibus non curat sapiens.* So we may easily receive Turing's opinion:

> The original question, ' Can machines think? ' I believe to be too meaningless to deserve discussion. Nevertheless

[1] This little essay was occasioned by a comic version of a theological argument offered by a writer typical of the ultra-scientific outlook which we have been studying. So ready was this calculator to tell what he thought of theology, that it seemed only right to attempt to show what theology might think of him. The article was composed some years before its author received the invitation to give the Whidden Lectures, but remained unpublished partly because its allusiveness seemed to make unfair demands on the reader. Since the three lectures which now precede the little paper contain the information which should help to make it intelligible, I venture to present it here as their not inappropriate conclusion.

[2] ' Computing Machinery and Intelligence,' in *Mind*, 1950, vol. LIX, no. 236, pp. 433–460.

> I believe that at the end of the century the use of words and general educated opinion will have altered so much that one will be able to speak of machines thinking without expecting to be contradicted.

But we are perhaps closer to the end of the century than Turing knew. As the word 'thought' is now used frequently enough by philosopher or scientist, it is difficult to see why its meaning cannot be verified of things other than man. Carrots, apparently, 'know how' to grow, and even wheelbarrows how to be pushed around. The human mind may be more complicated than UNIVAC, and UNIVAC more complicated than a wheelbarrow, but the position of the computer-men is that all three are machines in the same sense, with operations equally mechanical. After making this point, Turing might have been expected to let the matter rest. He does not. Certain difficulties may be raised against the identification of 'thinking' with both the mental act of man and the mechanical movement of the machine, and these difficulties he is determined to face. The first opinion opposed to his own he calls the 'theological objection', and he formulates it as follows: 'Thinking is a function of man's immortal soul. God has given an immortal soul to every man and woman, but not to any other animal or to machines. Hence no animal or machine can think'. To this argument he appends a footnote: 'Possibly this view is heretical. St. Thomas Aquinas (*Summa Theologica*, quoted by Bertrand Russell, *A History of Western Philosophy*, Simon and Schuster, New

York, 1945, p. 458) states that God cannot make a man to have 'no soul. But this may not be a real restriction on His powers, but only a result of the fact that men's souls are immortal, and therefore indestructible.'[1]

Turing observes that he himself is

> not very impressed with theological arguments whatever they may be used to support. Such arguments have often been found unsatisfactory in the past. In the time of Galileo it was argued that the texts, 'And the sun stood still . . . and hasted not to go down about a whole day' (Joshua, x, 13) and 'He laid the foundations of the earth, that it should not move at any time' (Psalm cv, 5) were adequate refutations of the Copernican theory. With our present knowledge such an argument appears futile. When that knowledge was not available it made quite a different impression.

The argument which he professes to cite is of course a hopeless one. But the first question is why he should concern himself with theological objections at all. Even when all appeal to authority is ruled out, as it ought to be in philosophy as such, his theory, one would think, might still be discussed in terms of what we already know and understand. However, it is precisely here that the difficulty lies nowadays. The new conceptions of scientific rigour are such that no acceptable account of what we already know and understand seems possible.[2] Lord Russell, Ludwig Wittgenstein, and

[1] *Ibid.* Russell's reference is to the *Summa contra Gentiles*, Book II.

[2] The new conception of rigour, simply precludes human speech. Exactness demands that we behave like our computers and say nothing at all. The trouble with words is that, even when they are not

Kurt Goedel constitute empirical proof of this retreat, or advance—for it may be looked upon as either. An advance it is at least in this respect that it reveals, once for all, what happens when certain types of question are entirely put aside. These I would describe as the kind which, historically, have led to endless controversy; I mean philosophical questions, and all of them, at least in the Platonic or Aristotelian sense of philosophy. This initial exclusion of all the classic problems was plainly formulated as a principle by the late Richard von Mises when he said: 'It is impossible to accept as the basis of mathematics merely statements that seem self-evident, if only because there is no agreement as to which statements actually belong

ambiguous, they never convey all that they refer to. The sounds *horse*, *stone*, *wheat*, may yield all that we need for practical purposes by recalling what these objects are for the breeder, the builder, and the baker. But can they tell what these things are in themselves? Can anybody? From the ultra-scientific point of view, then, which means that of the more exact sciences, like mathematical physics, there is a sense in which our normal use of words must appear a gigantic fraud. The very word *atom* is a good enough example, for originally it meant *indivisible*. One might also mention the words *word* and *language*. The reason for the scientist's dissatisfaction, and even irritation, with what he terms *natural* language, and for the fury of some philologists at the notion that language can actually convey what is knowable and true, may reside in the nature of our mind and the fashion in which it assigns significance with dependence upon words as artifacts. For words first refer to what we know of what is named, and stand only indirectly for the thing itself. Since primitive meanings rest on what we first know and since this, however surely apprehended, is very little, it is inevitable that, as our knowledge widens, so will the significance of our names, often by stages which go quite unnoticed. No wonder that, in the end, *atom* comes to stand for what is entirely divisible; and no wonder the scientist recoils from the confusion which may result if *atom* is used like *apple*.

to this class'. This of course means the immediate rejection of mathematical science as understood by the ancients, who considered it a branch of philosophy and the very model of what they meant by a *disciplina*. And, though it may be objected that the mathematics which von Mises is thinking of has almost nothing to do with that of Plato or Aristotle, this objection is actually irrelevant. The distinction between science and calculus, like every similar distinction, is ultimately based on some self-evidence or other, about which, according to the attendants of the new mechanical brains, there may always be disagreement. Well, what line of inquiry is left to us? Where do we go from here? This question too has been taken care of, not by the computer-men, but by thinkers equally up-to-date. It seems that, to find out where to go, or even where to start, would be to deprive oneself of the spirit of adventurous inquiry. People who want to be sure of something—or even, more modestly, of nothing—have no place in our time. There exists a type of psychoanalysis to show that they are victims of a morbid craving to return to the vegetative night of the embryo. As for debate about the soul and its destiny, there is reason to fear that, if the men behind the calculators take part, the result can only be a comedy of errors. For instance, if by soul is understood what the latest critics of ancient philosophy mean by this word (a meaning easily derived from the context of their criticism), any Aristotelian, whether Averroist, or Thomist, or whatever his dissension, will realize that he owns no such thing, personally or collectively, and will shudder at the prospect of ending his days with an 'immortal' one. The modern interpretation of this term, for instance, is as remote from Aristotle's *psychè* (originally 'breath', 'wind', 'breeze') in Book III of the *De Anima*, as would be 'exhaust', used of the exhaust of a modern engine. So we cannot make out, any better than Lord Russell can, why the dissolution of that tedious bundle of events, Mr. Smith, should be trailed by some kind of perpetual exhaust. No living thing stands in need of such a soul.

So, to pursue the rigour demanded by the bundle-of-events-and-computer-philosophy is to be led to this utter impasse. No questions can now be asked or answered; no statement made which is not a tautology; no mental act performed which cannot be matched by a machine. Since we are at a full stop anyhow, we may as well spend our time trying to understand what we have done, and why the result should be such a dead end. The attention paid by Turing to an argument supposedly taken from theology invites us to inquire what this science, which is not so easily silenced as philosophy, might be able to teach about the reasons for our plight. In the same ancient Literature to which he makes reference, there are several passages which throw a curious illumination upon that bundle of dust and that mechanical calculator which man has made of man. 'It is an old tale now', proclaims the prophet Jeremiah, 'how thou didst break in pieces the yoke of my dominion, didst sever all the bonds between us,

crying out, *I will not serve!*' (ii.20). And Ezekiel adds the warning: '. . . Such a fire I will kindle in the heart of thee as shall be thy undoing, leave thee a heap of dust for all to gaze at' (xxviii.18).[1] Now St. Thomas, upon whom Turing draws for his specimen of theological reasoning, explains that 'the activity [of him to whom these words were addressed], averted from what is one and first, was bent upon the many of inferior things, and it was *their* primacy that he coveted'.[2] This mysterious personality directed all his thought and action towards the scattered and sheerly multitudinous, his aim being to reduce everything to that alone. A lesser creature, like man, can pursue such a goal only in thought and, when he does so professedly, is called a sophist. Because the sophist, in his elaborate mental traffickings, uncontrolled by any standard of truth, has the infinite store of what is merely incidental, of *ens per accidens* (as well as an infinity of logics) to draw upon. He may thus make the worse reason appear the better, that which is least seem greatest, whatever there is most of seem synonymous with that which most truly is and, in the end, lead his hearers to conclude that nothing is but what is not.

What about poor Mr. Smith, then? 'Dust thou art and unto dust thou shalt return', the ancient writings tell him, in a phrase which acknowledges the 'thou' that he is, as well as the dust that he is made of. But the ancient writings care no more about a scientific

[1] Trans. Knox.
[2] *Q.Q. Quodlibetales*, qdl. 5, a.7, ad 1.

attitude than they do about Copernican astronomy. The modern choice is to be scientific and, scientifically, Mr. Smith is a bundle of occurrences and that is all that can be known of him. The choice is simple enough, in all conscience: *him* is cast aside in favour of a 'mere bundle' of fleeting 'occurrences' which happen to nobody, as being far more a sheer many opposed to the one than the particles of dust helping to make up the *per se* whole of a human being. To meet this new scientific standard of reality, to announce the primacy now desired, how can one do better than to present the rational animal as a mere bundle rather than as a substance? If, even of those particles of dust, Mr. Smith holds in himself a number greater than the number of men on earth, as a bundle of events, he is multitudinous beyond imagination. So, with substance forever banished as 'a hopelessly muddle-headed notion', Mr. Smith, 'a collective name for a number of occurrences', no matter how you look at him, is Legion. Every single one of us is a crowd of something or other. 'I' always stands for a bundle, like the pronoun 'we' in 'We, the Cricket Club'. And everything else is Legion too. If a house could speak for itself, for example, without falling into the snare of mere linguistic convenience, it would say something like 'We, the boards, bricks, mortar, nails, &c.'; while each board in turn would cry 'We, the woodfibres, cells, &c.'; each brick in turn 'We, the molecules of calcium, silica, &c. . . .', and so on, with not a thing knowing where to stop. Besides, whether

there is a 'thing', and a 'where to stop', are now distressingly meaningless questions (unless stated in language exhibiting how meaningless, and then quite unimportant). It is some consolation to note that even the old *non serviam*, if mere linguistic phenomena be ignored, is now quite emptied of its ego. If 'ego' is only a collective sign for a bundle of non-egos, there just isn't anybody there to utter a defiance.

Heaps of dust, disconnected thoughts, bundles of occurrences with no string to tie them, all these are bad enough; but there is worse to come. We have not yet measured the full scope of the perverted desire to accord primacy to the things which have most of all the nature of the many. This appetite is not sated until it has triumphantly embraced the order of a multitude which is negative—its final feat, as it were, and one which can procure man's deliverance from anything at all that he chooses to be delivered from. In his discussion of the classic arguments 'professing to prove the existence of God', Russell offers the frightening spectacle of a thinker become the contented captive of a logical fiction, namely, of the kind of infinity which can have no meaning whatsoever outside the art of calculation. Here are his own words:

> All of these [arguments], except the one from teleology in lifeless things, depend upon the supposed impossibility of a series having no first term. Every mathematician knows that there is no such impossibility; the series of negative integers ending with minus one is an instance to the contrary.

So, in the order of pure science, of speculative activity, man is asked to find ultimate satisfaction in seizing an infinity which is only a negation.

In the order of operation, as we have seen already, the new science asks man completely to renounce thinking as a power peculiar to him, and to persuade himself that he stands on the same level with his own tools, that is, of the complex tools called machines. We have come a long way from *non serviam*, at any rate, since the only thing tools actually can do is to serve, and in the strict sense of the word, as a hammer serves to drive nails; for a tool, like any instrument, is of its nature *movens motum*. Hence, Aristotle held that we ourselves, in one way or another, are the agent and final cause of artificial things. This opinion is no longer acceptable to science, of course, not only because it distinguishes between art and nature, but more especially because it distinguishes between instrument and principal agent. For the computers, if we can believe their interpreters, present us with an utterly new kind of tool: an instrument divorced from any principal agent, like a sign that does not signify, or a relation adrift without terms. The point is that the tool-maker, the agent, in whose interest the machine was built, has faded out of sight, he is 'refined out of existence'. And he has been deprived of all grounds for existing by the possibility of one computer giving birth to another, a notion which Russell might accept, since he has proved something to the effect that we may have tools of tools and nothing but tools, to no end,

without end, like a series without a first term. *Non serviam* seems a sinister kind of boomerang.

Whatever the ultimate value of those venerable scriptures to which Turing referred, it is clear that they utter a strophe which he ought to find deeply significant, a strophe for which we have been attempting to provide an antistrophe. We have thought it our duty to expose the implications, both of *non serviam*, and of the accompanying desire that primacy be granted to the sheer many. And we have seen that the new 'light-bearers' do not express their rejection of ultimate order in the romantic fashion of Marx quoting Prometheus; nor by simply making over some of their own servitude to the machines by using these for 'the more repulsive drudgery' of science; but by actually identifying knowledge and science with the mechanical process which may attend them, by insisting that what goes on in the computer is the same as what goes on in man's more abstract thinking. If man could accept this identification, he would never again need to face any question with power to disturb him, so that it is not surprising that those who make this reduction do so with an air of triumph. As for the primacy of the many, its most entitative expression becomes the mere bundles that are Mr. Smith, Russell, and so on. Logically, of course, the dispersion must be carried on and extended to all things, and ultimately to the universe itself—that supreme bundle out-bundling all.

Lord Russell warns us that we may some day blow up our pieces of the cosmic bundle. The curious thing is that he is appalled at the prospect. If the grand scattering be surveyed from his scientific point of view, what reason can there be for emotion? For Russell, the time is neither in nor out of joint, so that it is difficult to see what significance he can attach to an eventual dislocation. Under his tutelage we ought to have learned, surely, that for all living creatures without exception, destruction implies only that it shall be as if they had never been, and that to attach any significance to the annihilation of such things is to betray a foolish devotion to that ' organismic conception of nature ' which furnishes the world with substances, animate and inanimate, rational and irrational. Further, if Mr. Smith, like the good scientist he is, can accept his own impending dispersal as he would the scattering of a set of ninepins, one is at pains to see why his equanimity should be disturbed where the whole bundle of humanity is concerned. Such inconsistency looks suspiciously like nature thwarting theory. And even though Mr. Smith does not accept the destruction of all mankind as calmly as he does the effect of a well-aimed ball on the ninepins, what can it matter? The final catastrophe would take place in strict fidelity to law, and with merciful suddenness. Those who expected the event would be neither surprised *nor* mistaken, as Russell once explained:

Suppose you are walking in a thunderstorm, and you say to yourself, ' I am not at all likely to be struck by lightning.' The next moment you are struck, but you experience no surprise, because you are dead. If one day the sun explodes,

as Sir James Jeans seems to expect, we shall all perish instantly, and therefore not be surprised, but unless we expect the catastrophe we shall have been mistaken.

Suppose we did precipitate that transformation of all matter into pure radiation, that ' stupendous broadcast ' which was Arthur Eddington's version of a possible end of the world, why call it a catastrophe? Upheavals in the universe are an everyday occurrence. Life thrives on them. On our little planet, is anyone troubled by the fact that the sun is in continuous explosion? And any loss incurred may not be irremediable since, according to some people, when thermodynamic equilibrium is reached, the universal process of degradation will sweep into reverse, so that eventually the whole farce will be acted out again. And an almighty farce it has been and would be a second time, if it is a world where the distinctions between living and non-living, between rational and irrational, are to be rejected as evidence only of man's basic vanity, and even of his unfeeling cruelty towards what Turing calls ' the rest of creation '; where man is accused of ' brutality ' because he cares more for the living than for the lifeless, and for man than for beast; a farce to which no words will ever do justice, if machines are brought to prove that rationality, the ' specific difference ' of the human being, finds its proper home at last in its opposite.