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Mobiles, Bodies, and the Science of Quantified Motion: *Corpus* in Aquinas's Exposition of *Physics* VI.4 and in Early Modern Mathematical Physics

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AD IESUM PER MARIAM

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+ List of Abbreviations +

Unless otherwise noted, all Latin translations are the author's.

St. Thomas Aquinas

In De Caelo	*In Libros Aristotelis De Caelo et Mundo Expositio,* Leonine ed., t.3
Exp. Po.An.	*Expositio Libri Posteriorum*, Leonine ed., t. 1*/2
In Meta.	*In duodecim libros Metaphysicorum Aristotelis expositio*, Marietti ed.
In Phys.	*Commentaria in octo libros Physicorum Aristotelis*, Leonine ed., t. 2
QDA	*Quaestiones Disputatae de Anima*, Leonine ed., t. 24/1
SBDT	*Super Boetium de Trinitate,* Leonine ed., t. 50
I Sent.	*Scriptum Super Libros Sententiarum,* t. 1, ed. P. Mandonnet
II Sent.	*Scriptum Super Libros Sententiarum,* t. 2, ed. P. Mandonnet
Sent. De Sensu	*Sentencia Libri de Sensu et Sensato*, Leonine ed., t. 45/2
SLDA	*Sentencia Libri de Anima*, Leonine ed., t. 45/1
ST	*Summa Theologiae*, Leonine ed., t. 4-12; Ia = *Prima Pars*, etc.

The author also used Enrique Alarcón's www.corpusthomisticum.org, but refers by standard citations and page numbers to the above editions.

Aristotle

Physics	*Physics or Natural Hearing*, trans. R. G. Coughlin
	All other works cited from *The Complete Works of Aristotle*, Oxford ed.

Isaac Newton

PNPM	*Philosophiae Naturalis Principia Mathematica*, London: G.&J. Innys, 1726, 3rd ed.
Principia	*The Principia*, trans. A. Motte
The Principia	*Mathematical Principles of Natural Philosophy*, trans. I. B. Cohen, A. Whitman

Others

CommST	Thomas de Vio Cajetan, *Commentary on Summa Theologiae*
DSNP	Thomas de Vio Cajetan, *De Subiecto Naturalis Philosophiae*
Guide	I. B. Cohen, *A Guide to Newton's Principia*
Phil.Nat.	John of St. Thomas, *Cursus Philosophicus*, t. 2, p. 1, *Naturalis Philosophiae*
Physica	St. Albert the Great, *Opera Omnia: Physicorum Libri VIII*, Borgnet ed., v. 3

"His life was gentle, and the elements
So mixed in him that nature might stand up,
And say to all the world, 'This was a man!' "

— Shakespeare, *Julius Caesar* 5.5
quoted by James N. Berquist in memory of his father

Though unworthy and ill-proportioned, child's craft to a great man's regard,
allow these bits of paper and argument to honor the memory of
Mr. Marcus R. Berquist.

With the perfection of order and clarity, he taught and strengthened many in the Truth of
Reason and the Catholic Faith, to which he was unfailing witness and disciple,
following Aristotle and St. Thomas.

Requiescat in pace.

One should both say and think that Being Is; for To Be is possible, and Nothingness is not possible. . . . Nor shall I allow you to speak or think of Being as springing from Not-Being, for it is neither expressible nor thinkable that What-Is-Not Is.

— Parmenides, DK 6, 7

If Being is divided, it moves; and if it moved, it could not Be.

— Melissus, DK 10

Introduction

A Question of Order: Mobiles, Bodies, and the Science of Quantified Motion

Speaking and thinking truthfully about things in motion is difficult.[1] Consider:

> [Parmenides] Then is it not even more impossible for [the One] to *come* to be?—[Aristotle] I don't see why.—If something comes to be in something, isn't it necessary that it not yet be in that thing, since it is still coming to be in it, and that it no longer be entirely outside it, if in fact it is already coming to be in it?—Necessarily.—So if anything is to undergo this, only that which has parts could do so, because some of it would already be in that thing, while some, at the same time, would be outside. But a thing that doesn't have parts will not by any means be able to be, at the same time, neither wholly inside nor wholly outside something.—True.—But isn't it much more impossible still for a thing that has no parts and is not a whole to come to be in something somewhere, if it does so neither part by part nor as a whole?—Apparently.—Therefore it doesn't change places by going somewhere and coming to be in something, not does it move by spinning in the same location or by being altered.—It seems not.—The one, therefore, is unmoved by every sort of motion.—Unmoved.[2]

Just a few lines earlier, Parmenides has made Aristotle agree to the conclusion that, if the One is, it cannot be Many, and hence it can neither have parts nor be a whole, for having parts requires being in some way many and the whole is composed of many parts. Nor can the One be 'somewhere', for place and the placed would be an existing 'many'. As the above exchange shows, the consequence is that the One is unmoved. The otherness involved in moving requires parts ("neither wholly inside nor wholly outside") in a "something," namely place. Neither accrue to the One.

As a consequence of his premise that being is one and immobile, the Parmenidean denies that physics or natural philosophy is a possible path along his "Way of Truth." Such a science would require meaningful speech and truthful thought about "one" mobile being, its multitude, otherness and identity, and change. Yet these involve seemingly irreconcilable contradictions if Parmenides' premise is accepted. Natural philosophers endeavor to do so nonetheless, arguing about the principles of mobiles that account for the given phenomena of motion.

There are two basic ways of speaking about these mobiles. On the one hand, the motion of

[1] See Aristotle, *Physics*, 3.2, 202a1: "[Motion] is a certain act . . . difficult to know, but able to be."

[2] Plato, *Parmenides*, 138d-139a, trans. by M. L. Gill and P. Ryan, ed. by J. M. Cooper (Indianapolis-Cambridge: Hackett Publishing Co., 1997), 373. The 'Aristotle' of the *Parmenides* is fittingly, if inaccurately, named. Aristotle, discussing Parmenides, makes a note of this problem; see *Physics*, 1.2, 185b11-15, and Coughlin, ibid., 18, n.15.

a mobile or a body can be considered with priority placed on its parts—even ultimate parts such as Democritean atoms. The motion of the whole is explained insofar as one explains the motions of what it is made out of. The mathematical physics of Newton gives priority to the tiny parts or particles of a body and their "motion-paths." On the other hand, the tradition of ancient and medieval physics gives priority to the entire subject of motion before considering its parts. The proper motion of the whole requires the whole as the source of causality that informs the explanation of the parts: why the human heart beats is understood insofar as it is part of a human being. On this view, parts can still be considered in themselves in some way, given that their material causality and posteriority to the whole they compose is recognized. Therefore, among thinkers who grant reality and intelligibility to change, a basic dichotomy is between those who hold that the parts are causally prior to the whole and those who think that the whole is causally prior to the parts.

Aristotle is premier among those who take this latter approach.[3] Relegating the quandaries of the Parmenidean to metaphysics, he argues that "Every thing which changes [must] be divisible."[4] Thus, what changes is a body, for the divisible is a body. St. Thomas comments on this argument:

> Moreover, this is the treatise of *The Physics*, . . . the subject of which is mobile being simply [*ens mobile simpliciter*]. Now, I do not say 'mobile body' [*corpus mobile*], because that every mobile is a body is proven in this book. However, no science proves its own subject. Hence, right at the beginning of the *De Caelo*, which follows upon this one, a discussion of body is undertaken.[5]

Here, St. Thomas relates the conception of physics' subject to the argument of *Physics* 6.4, an argument within the book on the quantity of mobiles and motion. The Aristotelian physicist, in direct opposition to the Parmenidean, assumes the existence of moving things and shows that as moving they must have parts. The quantitative parts of a mobile, motion, and time are discussed in Book 6, and later parts of natural science discuss body as such (as St. Thomas notes of the *De Caelo*).

[3] See, e.g., Aristotle, *Politics*, 1.2, 1253a20-21: "The whole is of necessity prior to the part; for example, if the whole body be destroyed, there will be no foot or hand, except homonymously."
[4] Aristotle, *Physics*, 6.4, 234b10; and see also 186b12-13.
[5] St. Thomas, *In Phys.* I, lect.1, n.4.

This argument thus bears ramifications for how the physicist conceives of the subject of his science and the quantitative parts of the mobile. The subject of physics, most formally, is mobile being, not mobile body (as thought St. Albert, teacher of St. Thomas, or Isaac Newton as a young student).[6]

At stake here are adequate speech and thought about the mutable world. The ancient tradition of physics resolves to substantial causes (matter and form). The modern tradition, "laying aside substantial forms and occult qualities," endeavors "to subject the phenomena of nature to mathematical laws."[7] The former conceives of motion as belonging to physical bodies, the latter conceives of motion through mathematical bodies and quantities. If it is the case (as the ancients held) that the motion of mathematicals is metaphorical, then the modern view seems to offer only a partial description of motion that professes neutrality towards the physical causes at work.[8]

Since St. Thomas joins into one discussion the priority of whole to part in the consideration of moving things, the quantitative parts of mobiles, and the proper conception of the subject of physics, his exposition of Aristotle's demonstration in *Physics* 6.4 is worth understanding on two counts. First, since it grounds our understanding of the "realm" of physics, it will show the parameters of adequate scientific speech about the mutable world. Second, it will offer a contrast to the way mathematical physics conceives of mobiles, bodies, and quantities of motion.

This thesis therefore requires three parts. The first will consider St. Thomas' *Prooemium* to his

[6] See St. Albert, *Physica*, 6-7; and William A. Wallace, "Newton's Early Writings: Beginnings of a New Direction," in *Newton and the New Direction in Science*, ed. by G. V. Coyne, S.J., M. Heller, and J. Zyncinski (Citta del Vaticano: Specola Vaticana, 1988) 25-27. Wallace transcribes a portion of a student notebook of Newton's connected to a textbook written by John Magirus, the *Physiologia Peripatetica*. Newton writes, ibid., 26, (the brackets, parentheses, and superscripts reflect Newton's, and are tied to Magirus's text): "[1]Physics [which is the science of natural bodies] (and whose [3]subject is the natural mobile body) consists in the contemplation of nature [5][which is the cause of motion and rest in that in which it is primarily, essentially, and not accidentally.]" Wallace notes on this passage, ibid., 27: "Newton's entry at superscript 3 to the effect that the subject of the physics is 'the natural mobile body' represents a controversial teaching. Thomas Aquinas had held, contrary to Albertus Magnus, that this subject was mobile being (*ens mobile*) on the ground that its being "a body" could be demonstrated . . . Magirus discusses these and other opinions in his commentary, and attributes his teaching to the *recentiores* (i.e., the moderns), for which he cites Zabarella. . . . Newton, typically, shows no awareness of the problem and merely states Magirus's resolution of it."
[7] Isaac Newton, *Principia*, 3; translation emended: "mathematical laws" for "leges mathematicas"; see *PNPM*, a.
[8] See ibid., 13. See François de Gandt, *Force and Geometry in Newton's Principia*, trans. C. Wilson (Princeton University Press, 1995) 271-72. I thank Dr. Hassing for the inspiration and mode of expression in the first and last clauses.

commentary on Aristotle's *Physics*, where he connects the demonstration of *Physics* 6.4 with the subject of natural science. The second chapter will then consider the demonstration itself. The third chapter will examine in counterpoint select aspects of the Newtonian method of studying bodies in motion. The goal of the thesis is to defend this proposition: *Every (per se) mobile is necessarily divisible, and hence every mobile is a body*. It is a consequence of this, first, that the subject of physics is *ens mobile* or mobile being, which name is said analogously of those within the subject-genus of physics. Second, mathematical physics is ordered to the speculative end of physics generally conceived. It follows, third, that a thesis of reductionism proposed by mathematical physics cannot hold in the face of the analogical conception of the subject of physics.

Aristotle's demonstration in *Physics* 6.4 affords the opportunity to reflect, with St. Thomas, upon what it means to speak scientifically about mobile being, thus informing the efforts of the natural scientist and mathematical physicist to speak and think adequately about the world.

Chapter One

Finding the Subject of First Physics

St. Thomas' purpose in the *prooemium* or introduction to his commentary on Aristotle's *Physics* is to find the subject of natural science.[1] The discovery of this subject shows how it relates to the argument under investigation in *Physics* 6.4, and St. Thomas' discussion will introduce three themes integral to the argument of this thesis: abstraction, the existence of matter and motion, and middle science. These themes arise in the sections labeled [2-3], [7], and [4], respectively. I will first discuss the *prooemium* and its argument. The three themes will then be further developed.

§1 – The *Prooemium* of St. Thomas' Commentary on Aristotle's *Physics*

> [1] Because the treatise of *The Physics*, whose exposition we intend, is the first treatise of natural science, it is necessary to assign at the outset the *what it is* [*quid sit*] of its matter and subject. [2] It is to be known that, since every science is in the intellect, and through this does some thing become intelligible in act, because it is somehow abstracted from matter, insofar as things are in some way diversely related to matter, they pertain to diverse sciences. Again, since every science is had through demonstration, and moreover definition is the medium of demonstration, it is necessary that the sciences be diversified according to a diverse mode of definition.
>
> [3] Therefore, it is to be known that there are certain things dependent in existence upon matter, nor can they be defined without matter. There are also some things that, granted they are not able to exist except in sensible matter, sensible matter does not fall into their definition. [4] And these differ from each other as the curved and the snub. For the snub exists in sensible matter, and it is necessary that sensible matter fall into its definition, for the snub is a curved nose. Such are all natural things, as man or stone. But the curved, granted it cannot exist except in sensible matter, nonetheless sensible matter does not fall in its definition. Such are all mathematicals, as numbers, magnitudes, and figures. [5] But there are certain things which do not depend upon matter, whether according to existence or according to account, either because they are never in matter, as God and the other separate substances, or because they are not universally in matter, as substance, potency and act, and being itself.
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> [6] Metaphysics, therefore, is of things of this [last] sort. But mathematics is of

[1] The commentary dates to 1268-69, overlapping with the first and second parts of the *Summa Theologiae*. It is therefore a mature work of St. Thomas; see J. P. Torrell, *Saint Thomas Aquinas: The Person and His Work* (Washington, D.C.: Catholic University of America Press, 2005) 231-32, 342. The value of St. Thomas's Aristotelian commentaries for gleaning his personal thought some see as limited: see John F. Wippel, *The Metaphysical Thought of Thomas Aquinas: From Finite to Uncreated Being* (Washington, D.C.: The Catholic University of America Press, 2000) xviii-xx. My intention here is merely to investigate physics in light of what St. Thomas says in his commentary. Also, the descriptor of "first physics" for the general part of natural science contained in *Physics* I borrow from James Chastek.

those which depend upon sensible matter according to existence but not according to definition [*ratio*]. And those which depend upon matter not only according to existence but also according to definition [*ratio*], belong to natural science, which is called physics. [7] And because everything that has matter is mobile, it follows that mobile being [*ens mobile*] is the subject of natural philosophy. Moreover, natural philosophy is of natural things. However, natural things are those whose principle is nature; moreover nature is a principle of motion and rest in that in which it is. Thus, natural science is about things having a principle of motion in themselves.

[8] But because things which follow upon something common ought to be determined first and separately [from the rest], lest it be necessary to repeat them many times when treating all the parts, it was necessary that one book come beforehand in natural science, in which is treated those which follow upon mobile being in common; just as first philosophy comes before all the sciences, in which are determined those things which are common to being insofar as it is being. [9] Now this is the book *Of Physical Things*, which is also called *About Physics* or *Natural Hearing* (because it is delivered to listeners by instruction), the subject of which is mobile being simply [*ens mobile simpliciter*]. Now, I do not say mobile body [*corpus mobile*], because that every mobile is a body is proven in this book. However, no science proves its own subject. Hence, right at the beginning of the *De Caelo*, which follows upon this book, a discussion of body is undertaken. [10] Moreover, following this book are the other books of natural science in which the species of mobiles are treated—consider: in the *De Caelo* of the mobile according to local motion, which is the first species of motion; furthermore in the book *De Generatione*, of motion to form and of the first mobile things, namely the elements, as to their common transmutations; also, as to their special transmutations, in the *Meteororum*; also, of mixed inanimate mobiles, in the book *De Mineralibus*; also, of living things, in the book *De Anima* and those consequent to it.[2]

I. St. Thomas declares his intention: to find the subject of general or first physics [1]
II. He carries out his intention [2-10]
  A. Introduces two criteria for dividing the sciences and finding their subjects [2]
  B. Applies these two criteria [3-10]
    1. Applies the two criteria to find three speculative objects [3-5]
      a. The first two objects [3-4]
        i. Indicates the two speculative objects [3]
        ii. Compares these two to each other [4]
      b. The third object [5]
    2. Matches these objects to their respective sciences
      a. Matches the three speculative sciences with their objects [6]
      b. Considers physics in particular, according to his original intention [7-10]
        i. Argues for the subject of *physics generally* [7]
        ii. Argues for the subject of *general physics* [8-10]
          1st. Shows the necessity of a general or first physics [8]
          2nd. Shows the subject of first physics [9]
          3rd. Compares this subject to the other parts of physics [10]

St. Thomas' intention in [1] guides the two key distinctions he makes: that between the three

[2] St. Thomas, *In Phys.*, lib. 1, lect. 1, nn. 1-4 (Leon.2.4). I have added the bracketed numbers and outline. Latin translations my own unless otherwise noted. Also consulted: *Commentary on Aristotle's Physics*, trans. by R. J. Blackwell et al. (Notre Dame, IN: Dumb Ox Books, 1999). See St. Thomas, *Sent. De Sensu*, *pr.* (Leon.45/2.3-7) for a division after *De Anima*.

speculative sciences [6], founded on the two criteria for dividing the sciences [2], and that between the subject of physics generally and the subject of general or first physics [7-10]. The two criteria echo those given elsewhere: sciences are divided by degrees of removal from matter and motion.[3] The first criterion turns on the nature of science insofar as science is "in the intellect."[4] St. Thomas argues in a chiasmus from science being in the intellect to intelligibility to being abstracted from matter; thus, diverse relations to matter imply diverse degrees of intelligibility (an implied step), which give rise to diverse sciences. The second criterion turns on the nature of science insofar as it demonstrates through unchanging (immobile) definitions.[5] That is, since science is about necessary things, and necessary things are immobile, science is about immobile (defined) things.[6] These two completely divide the sciences: the first shows how things are in the mind, the second how the mind focuses this intention in defining things.[7]

The application of these criteria, [3-5], is straightforward and consistent with what St. Thomas holds elsewhere.[8] There are four logical possibilities according to the two criteria: an object existing in matter and defined with matter, existing in matter and defined without matter, existing and defined without matter, and existing without matter but defined with it.[9] Only the first three are

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[3] See St. Thomas, *Super Boetium de Trinitate* (*SBDT*), q. 5, a. 1, c. (Leon.50.138:113-40).

[4] See St. Thomas, *I Sent.*, d. 38, q. 1, a. 2, c., 901; *ST* Ia, q. 13, a. 12, ad3um (Leon.4.165), and q. 85, a. 1, ad1um (Leon.5.331). See also Wippel, *Metaphysical Thought of Aquinas*, 6, fn. 15, and 306-308.

[5] Hence, the two criteria here are substantially the same as in the *SBDT*, cited in fn. 3. Definition and immobility have long been tied together: see Plato, *Meno*, 98a.

[6] This seems be a problem for the natural scientist, because he studies *mobile* being. See St. Thomas, *SBDT*, q. 5, a. 2, c., and also ad4um (Leon.50.142-44), and Aristotle, *Metaphysics* 9.8, 1050b11ff.

[7] St. Thomas reiterates this second criterion in another text, see *SLDA*, lib. 1, c. 2, (Leon.45-1.12:244-50): "Et notandum quod *tota ratio divisionis philosophiae* sumitur secundum definitionem et modum definiendi. Cuius ratio est, quia definitio est principium demonstrationis rerum, res autem definiuntur per essentialia. Unde diversae definitiones rerum diversa principia essentialia demonstrant, ex quibus una scientia differt ab alia." Emphasis mine.

[8] See St. Thomas, *SBDT*, q. 5, a. 1, c. (Leon.50.138:141-54): "Quaedam ergo speculabilium sunt, quae dependent a materia secundum esse, quia non nisi in materia esse possunt. Et haec distinguuntur, quia quaedam dependent a materia secundum esse et intellectum, sicut illa, in quorum diffinitione ponitur materia sensibilis; unde sine materia sensibili intelligi non possunt, ut in diffinitione hominis oportet accipere carnem et ossa. Et de his est physica sive scientia naturalis. Quaedam vero sunt, quae quamvis dependeant a materia secundum esse, non tamen secundum intellectum, quia in eorum diffinitionibus non ponitur materia sensibilis, sicut linea et numerus. Et de his est mathematica."

[9] The fourth option is impossible: see ibid., (Leon.50.138:168-72), and Wippel, *Metaphysical Thought*, 9.

matched to the traditional speculative sciences: physics, mathematics, and metaphysics. The difference in the modes of definition of the first two is proportionate to that between the snub and the curved. The argument made in [4] through the proportion *the snub* : *the curved* :: *physicals* : *mathematicals*, is intended to manifest this distinction.[10] This same difference at once makes physics and mathematics heterogeneous and able to be relatively homogenized in a mixed science.

In accord with his original intention, in [7] St. Thomas focuses on physics and its mode to find the subject of physics taken as a whole. This includes *general physics* as its first part (see [8]). The argument in [7] flows from the mode of consideration of physics given in [6], i.e., about things that exist in and are defined with matter, and concludes from the principle that everything that has matter is mobile that the subject of physics is *ens mobile*. The last sentences of [7] flesh this argument out using Aristotle's definition of nature as an internal principle of motion and rest. For this clarification to conclude, one must assume that a mobile being is what has a principle of motion in itself.[11]

However, the study of *ens mobile* must be approached methodically through a general or first physics. The need for this is seen as follows. A subject of a theoretic science has certain properties that belong to it as a whole. The subject is common to all the parts of the science, which means that all those properties are common to the parts. A person setting out the doctrine would have to repeat these common things when treating of the parts, unless they were treated first and separately.[12] The

[10] See St. Thomas, *In Meta.*, lib. 7, lect. 4, 395-400. St. Thomas argues that the names "snub" and "curved" signify the same *essence*, and differ by what matter is implied in their definition. This implies that quantity defined with sensible matter and with intelligible matter are in different categories of consideration. (I thank David Grothoff for this reference.)

[11] See Charles de Koninck, *The Cosmos*, in *The Writings of Charles de Koninck*, vol.1, ed. and trans. by R. McInerny (Notre Dame, IN: University of Notre Dame Press, 2008) 257: "Manifestly we are dupes of a verbal trick when we say that a mobile being is a natural being. 'Natural' is after all a vague term. And to take 'nature' in a strict sense, as we will se in what follows, we must define it in terms of mobility. It is *mobile being*, not as being, but under the precise angle of mobility . . . that is the formal object of the Philosophy of Nature."

[12] This concern for efficiency of presentation is not the only reason that could be offered, however. As Aristotle argues in *Physics* 1.1, it is also the case that what is more known to the human mind at first is not what is more known in itself. What is more known in itself is the specific nature of a thing, and among those specific natures, those natures that are more beings are more knowable in themselves (for being or actuality and intelligibility are convertible). See St. Thomas, *In Phys.*, lib. 1, lect. 1, n.7 (Leon.2.5). This means that what we know at first is less knowable in itself, and only in potency towards further knowledge. Thus, Aristotle and St. Thomas teach that we know the confused universal whole at first, for it contains in potency the species that are more knowable in itself. This is called the process in determination, see De

comparison of this first physics to first philosophy is intended to manifest this need. The comparison is intended to highlight only the aspect of commonality that requires such priority. St. Thomas does not intend one to conclude that first philosophy should be studied first.[13]

Having distinguished the first part of physics, St. Thomas shows its subject in [9]. The argument in the text is straightforward. After explaining the reason for the name of the book, he names its subject so that it is clear it meets the requirement of a general treatment: mobile being simply. Mobile body or *corpus mobile* cannot be the subject, since that every mobile is a body is proven by physics, and no science can prove its own subject. Rather, a science's subject and the definition of the subject are assumed, for no prior principle in the science demonstrates them.[14] The scientist's task is then to make the unknown known by means of what is more known. Now the means of demonstration is the middle term. So the middle term must be taken up as more known within the science. Hence, if the subject of physics were "body," the proof mentioned would be a vicious circle. St. Thomas supports this argument by indicating that the treatment of *corpus* starts elsewhere, in the beginning of *De Caelo*.[15] Rather, the subject of first physics is *ens mobile simpliciter*.

Koninck, "Introduction à l'étude de l'âme," in *Précis de psychologie thomiste* by Stanislas Cantin (Québec: Éditions de l'Université Laval, 1948) xxix. "Aussi, dans la science, nous considérons les choses suivant ce qui, en elles, est d'abord plus connu, pour aller ainsi par degrees vers ce qui est plus connaissable en soi; car, manifestement, homme est plus connaissable en soi qu'animal; étant animal et raisonnale, il est plus distinct, plus en act et partant plus connaissable en soi. Nous avançons donc de sujet en sujet suivant cet ordre de communauté. Dans la science de la nature, nous tâchons de savoir en premier lieu ce qui est prope à une chose en tant qu'elle est mobile, ensuite, ce qui en est vrai quant à sa mobilité selon le lieu, etc. Un dernier terme de tout ce processus, ce serait, part exemple, l'étude de la demarche caractéristique de l'éléphant. Certes, il serait impossible à un seul homme d'embrasser le vaste domaine qui sépare la consideration de l'être mobile et celle du vol de la libullule—c'est'à'dire, toutes les sciences naturelles. . . . Tel serait néanmoins l'ordre qu'il aurait à observer pour avoir une vue d'ensemble bien ordonnée." This well-ordered view comes from starting with what is first and most known to us. In physics, this is the existence of motion. This is not to say that one *demonstrates* from what is first known in this way. The experience necessary to descend in the order of generality (the process of determination or concretion) must be expanded at every step to drive the order of demonstration proper to each level; see ibid., xxx.

[13] See St. Thomas, *SBDT*, q. 5, a. 1, ad9[um] (Leon.50.141).

[14] See Aristotle, *Posterior Analytics*, 1.10, 76a31, 76b12; St. Thomas, *Exp.Po.An.*, Book 1, lect. 18, (Leon.1*/2.68:107-113): ". . . propria principia sunt quae supponuntur esse in scientiis, scilicet subiecta, circa quae scientia speculatur ea quae per se insunt eis. . . . Praedictae enim supponunt esse et hoc esse, idest supponunt de eis, et quia sunt et quid sunt." Compare John of St. Thomas, *Phil.Nat.*, 9. In this connection, St. Thomas refers to the demonstration in *Physics* 6.4, as Cajetan notes, see Cajetan, *DSNP*, 208b14-31, 210a7-23.

[15] This also obviates the apparent contradiction with Aristotle, who begins the *De Caelo*, 1.1, 268a1-6 by noting that physics studies bodies or their principles, which would include non-bodily things such as souls (as St. Thomas notes, *In De Caelo*, lib. 1, lect. 1, (Leon.3.4.)), or even light, the act of the transparent medium. Compare Andrea Falcon, *Aristotle and the*

Since the reason why body cannot be the subject of physics is a stricture provided by logic, it is not a mere quibble, but a claim about what is more knowable to us, since a scientist advances to the unknown from the more known. What is difficult to say, however, is exactly how "mobile" and "body" are related as more known to less known. This is the burden of the demonstration of *Physics* 6.4, treated in Chapter 2. To prepare for this demonstration, and the comparison to Newton's understanding of mobiles, bodies, and their science, the three themes mentioned above will now be taken up: abstraction, the existence of matter and motion, and the middles sciences.

§2 – The First Theme: Abstraction, or Thinking One Without Another

Note that St. Thomas distinguishes between "matter and subject" in [1]: "The reason for the distinction between the 'matter' and the 'subject' of a science, [lies] in the quotation from St. Thomas, 'a thing becomes intelligible in act insofar as it is more or less abstracted from matter.'"[16] While abstraction is the *reason* for the distinction, *that* 'subject' and 'matter' are distinct logically can be more easily seen, and so will be discussed first.[17] The subject of a science, generally, is the genus about which there is speech in a science.[18] Science, in the ancient and strongest sense, is knowledge of the cause, that it is the cause, and that it cannot be otherwise.[19] The cause, expressed by the middle term of a demonstrative syllogism, shows why some attribute belongs to some subject. The distinction between subject and matter is founded on this form of scientific speech.

Science of Nature: Unity Without Uniformity (Cambridge: Cambridge University Press, 2005) 31ff. That Aristotle intends a more general conception of the subject of physics as mobile being is clear from *Physics* 1.2, 185a13.

[16] Charles de Koninck, "Abstraction From Matter: Notes on St. Thomas's Prologue to the *Physics*," *Laval théologique et philosophique*, 13 (1957): 148. (This is a three part study—pt.1: ibid., 133-196; pt.2 & 3: v.16 (1960): 53-69, 169-188.) See ibid., 148ff: "matter" in "abstraction from matter" and as opposed to "subject" are uses equivocal by reason. See also Richard Berquist, commentary, in St. Thomas, *Commentary on Aristotle's Posterior Analytics*, (Notre Dame, IN: Dumb Ox Books, 2007) 414.

[17] This exposition synthesizes a traditional understanding; see St. Thomas, *Exp.Po.An.*, Book 1, lect. 41 (Leon.1*/2.151-56); Cajetan, *CommST*, q. 1, a. 3; John of St. Thomas, *Phil.Nat.*, p. 1, q. 1, a. 1; Jacques Maritain, *Philosophy of Nature* (New York: Philosophical Library, 1951) 123-135; De Koninck, "Abstraction From Matter-pt.1," 145-146; James Weisheipl, *Aristotelian Methodology: A Commentary on the Posterior Analytics of Aristotle*, ed. J. R. Catan (River Forest, IL: Pontifical Institute of Philosophy-DHS, 1958) 31-32; and Vincent E. Smith, *The General Science of Nature* (Milwaukee: Bruce Pub. Co., 1958) 21.

[18] See St. Thomas, *ST* Ia, q. 1, a. 7, s.c. (Leon.4.19): "Sed contra, illud est subiectum scientiae, de quo est sermo in scientia." Of course, in reference to a science, this "subject" has a different notion than a *grammatical* subject.

[19] See Aristotle, *Posterior Analytics*, 1.2, and St. Thomas, *Exp. Po. An.*, Book 1, lect. 4 (Leon.1*/2.17-22).

Consider the form of the strongest demonstration (all B is A, all C is B, therefore, all C is A). The scientific subject here is the genus of 'C'. To the learner, the subject of the science is the most known thing in the science (because there are no prior principles within the science to illuminate it) but also the least known (because none of its properties have been demonstrated of it).

> That there is no contradiction here can be made plain by first pointing out what is meant by *'the object of a science,' as distinguished from its subject, for the object includes the subject*. By the *object* of a science, in the strict sense of the term science, we mean knowledge acquired as the result of demonstration . . . By the *subject* of a science, we mean that *about which* we have knowledge by demonstration, viz., the very subject of the conclusion . . . In other words, the subject of scientific knowledge is both [i] what is first known, viz., that about which we seek science, and [ii] what is last known, viz., this same subject *qua* known to possess such or such a property. The subject, considered in the latter respect, is called the 'term' of the science.[20]

St. Thomas compares the transition between ignorance and knowledge to the unity of a motion: just as a motion has unity from its term (that in which the mobile rests), so a demonstration has unity from its term (the conclusion in which the intellect reaches a cognitive rest).[21] That *spoken about* in the science can be called the *subject*, and the entire conclusion can be called the *object*.

Now, the same object can be considered by two different sciences. For example, that the earth is spherical is proven both by the astronomer and the physicist.[22] These conclusions are the same *material objects*, and since their minor and major terms are the same, the difference must lie in the middle term that distinguishes the *way* the mind arrives at the material object. A scientist conceiving the minor (and major) in his own discipline conceives it *formally*, and the middle term in this aspect is called the *formal object*. Just as a visible object is seen through a medium and light is the actuality of the medium, so also the intelligible object (the conclusion) is intelligible via a medium (the middle term) that has a distinct actuality. This formal object considered in the most

[20] De Koninck, "Abstraction," pt.1, 145-46; first emphasis mine.
[21] See St. Thomas, *Exp. Po. An.*, Book 1, lect. 41, (Leon.1*/2.153:131-53).
[22] See St. Thomas, ibid., (Leon.1*/2.156:364-73); also *ST* Ia, q. 1, a. 1, ad 2um (Leon.4.7); *SBDT*, q. 5, a. 3, ad7um (Leon.50. 151:392-403); and Aristotle, *De Caelo*, 2.14. See also William Wallace, "Demonstrating in the Science of Nature" in *From a Realist Point of View: Essays on the Philosophy of Science* (Washington, D.C.: University Press of America, 1979) 146.

disciplined way is called the *formal object by which* the conclusion is known.[23]

> Yet, it is not to be understood that the unity of the first principles suffices for the unity of a science simply, but the unity of the first principles in some genus of the knowable. However, genera of knowables are distinguished according to a diverse mode of knowing [*modum cognoscendi*]. Just as those that are *defined with matter* are known differently than those *defined without matter*. Whence natural body [*corpus naturale*] and mathematical body [*corpus mathematicum*] are of different genera of knowables.[24]

Thus, the *formal subject* of a science is a genus of things spoken of in a science, which genus receives its unity from being about a single genus of *things* (the *material subject* or *matter*), whose principles are *thought about* in a certain way (the *formal object*), and the very unity of this disciplined manner is called the *formal object by which* such speaking and thinking and knowing take place. By these disciplined manners, knowable genera are distinguished, and they depend on the degrees of abstraction.

St. Thomas' doctrine of abstraction is set forth most clearly in *SBDT*, q. 5, aa. 2-4, but can only be considered briefly here. I will focus on two key points in the third article.[25] Abstraction generally names the act of the mind by which a man thinks about one thing without another. This explains the reason for the different modes of definition because the same thing can be considered in different degrees of abstraction, and these distinct aspects of thought allow for different modes of definition. St. Thomas identifies abstractions that correspond to physics, mathematics, and metaphysics (total, formal, and separation). I will outline how total and formal abstraction are possible, and then discuss both the 'matters' they abstract from (individuating and sensible) and

[23] See St. Thomas, *ST* IIa-IIae, q. 1, a. 1, c. (Leon.8.7): "Cuiuslibet cognoscitivi habitus obiectum duo habet, scilicet id quod materialiter cognoscitur, quod est sicut materiale obiectum; et id per quod cognoscitur quod est formalis ratio obiecti, sicut in scientia geometriae materialiter scita sunt conclusiones; formalis vera ratio sciendi sunt media demonstrationis, per quae conclusiones cognoscitur." See Weisheipl, *Commentary*, 32; Maritain, *Philosophy of Nature*, 127-28; and Melvin A. Glutz, "Order in the Philosophy of Nature," in *The Dignity of Science*, ed. by J. A. Weisheipl (Washington, D.C.: Thomist Press, 1961) 276-77: "There would be no reason for distinguishing sciences according to the manner of conceptualizing the subject (*obiectum formale sub quo*), if the definition of the subject were the terminal point of the science, rather than the starting point. The definition is rather the very light that reveals the necessary connection of the attributes with the subject."

[24] St. Thomas, *Exp.Po.An.*, Book 1, lect. 41, (Leon.1*/2.155:287-96). Emphasis mine.

[25] The second article considers whether physics can consider things that exist with matter and motion. The difficulty here is that this seems to directly contradict the requirement that a scientific object be immaterial and immobile. The resolution is that the natures of physical things can be considered apart from the matter that allows for motion. This is the distinction between individuating and common matter, how a particular and universal are distinguished: see St. Thomas, *SBDT*, q. 5, a. 2, c. (Leon.50.143:83-108).

those they define with (sensible and intelligible).[26]

The possibility of total and formal abstraction is based upon the intellect's first operation, "which is called the understanding of indivisibles by which it knows of any one thing what it is."[27] The first operation of the mind "looks to the very nature of the thing, insofar as the thing understood obtains some grade among beings, whether it is a compete thing, as some whole, or an incomplete thing, as a part or accident."[28] This follows Aristotle's principle in *De Anima* 2.4, that to understand the powers and activities of the soul, one must first examine the objects or the things in regard to which the powers are constituted. The constitution of reality guides nature, as it were, to determine how the mind as a power responds with certain operations to know reality truthfully. Thus, "abstraction" names that act of the mind

> by which it understands what something is, distinguishing one from another while it understands what this [one] is, understanding nothing of the other, *neither that it is with this, nor that it is separate from it*.[29]

The neither/nor qualification expresses the essence of abstraction, in contrast to judging that this is not that. One can abstract when considering things joined in reality if one can complete an act of understanding one (the abstracted) without needing to understand another (the abstracted from).[30]

St. Thomas points out two ways in which this is possible.

> . . . then, the one can be abstracted by the intellect from the other such as to understand without it . . . even if it be conjoined in the thing, whether by that conjunction by which a part and whole are conjoined, as a letter can be understood without a syllable but not conversely, and an animal without a foot but not conversely; or even if they be conjoined in the way in which form is conjoined to matter and an accident to a subject, as white can be understood without man and conversely.[31]

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[26] The abstraction called separation will not be considered here. Note that total abstraction is common to all the sciences, while formal abstraction and separation belong to mathematics and metaphysics. See Maritain, *Philosophy of Nature*, 14-21.
[27] St. Thomas, *SBDT*, q. 5, a. 3, c. (Leon.50.147:91-93). See Aristotle, *De Anima*, 3.6.
[28] St. Thomas, *SBDT*, q. 5, a. 3, c. (Leon.50.147:98-101).
[29] Ibid., (Leon.50.148:164-67). Emphasis added.
[30] St. Thomas supports this ground for the possibility of abstraction with two principles: see ibid, (Leon.50.147:121-32) and (Leon.50.147:132-36,147-50). The first is that everything is intelligible insofar as it is in act. The actuality grasped by the mind allows the mind to form the definition of the thing understood. The second adds the notion of dependency. If the definition requires the understanding of something else, it cannot be abstracted from that.
[31] Ibid., (Leon.50.147:149-58).

The first mode, the union of part and whole, allows for the abstraction known as total abstraction, or the abstraction of the universal.[32] The whole that is unable to be abstracted is that whole where "this is the case, namely, that it is such a whole because it is composed from such parts."[33] This is the case regarding the matter that enters into the definition of physical things, called common matter, which is among the "parts of the species and form."[34] This contrasts to the parts abstracted from:

> And these parts are called parts of matter, which are not placed in the definition of the whole, but rather the converse. And all the designated parts are related in this way to a man, such as *this* soul, and *this* body, and *this* nail, and *this* bone, and the like: for these parts are certainly the parts of the essence of Socrates and Plato, not, however, of man insofar as he is man, and therefore man can be abstracted by the intellect from such particulars. And such an abstraction is of the universal from the particular.[35]

This matter abstracted from is called individuating matter. The reason why the mind is able to abstract the form from individuating matter is that the latter is not part of the definition that needs to be constituted by the mind in its natural response to knowing things. Indeed, such matter is strictly speaking unintelligible. This abstraction from matter (the form of the thing from *individuating matter*) grounds the mode of definition in physics.

The second mode allows for the abstraction proper to mathematics.

> Moreover, that form is able to be abstracted from some matter, the definition of whose essence does not depend upon such matter. However, a form is not able to be abstracted by the intellect from that matter upon which it depends as to the definition of its essence. Whence, since every accident compares to a substantial subject as form to matter, and the definition of whatsoever accident depends upon substance, it is impossible for such a form to be separated from substance. But accidents come to substance in a certain order: for first quantity comes to it, then quality, then passions and motion. Whence quantity can be understood in a subject matter before sensible qualities are understood in it [the subject matter], from which it [the subject matter] is called sensible matter. And thus as to the definition of its substance quantity does not depend upon sensible matter, but only upon intelligible matter, for the accidents being removed, substance remains comprehensible only to the intellect, because the sense powers do not attain to the comprehension of substance.[36]

[32] See ibid., (Leon.50.149:282-86).
[33] Ibid., (Leon.50.148:205-206).
[34] Ibid., (Leon.50.148:208).
[35] Ibid., (Leon.50.149:229-237). Emphasis added.
[36] St. Thomas, *SBDT*, q. 5, a. 3, c. (Leon.50.148:180-200).

In the first three sentences, St. Thomas shows that no accident can be understood in abstraction from its subject. It is the order of inherence among accidents in regard to substance that allows quantity and the corresponding aspect of its subject to be understood without other accidents and the corresponding aspects of their subjects. Quantity compares to qualities as matter to form: the classic example is that color inheres in a surface. Thus, there is a certain act of a potency (quantity compared to intelligible matter) that obtains a grade of intelligibility because it can be understood without the matter of the thing as a whole (sensible matter) being taken into account. This unique abstraction from matter ("quantity and those things that follow upon quantity, such as figure and the like,"[37] from *sensible matter*) grounds the distinct mode of definition in mathematics.

Total abstraction is the abstraction of the form of a thing from individuating matter. This abstraction retains common matter, and in the definition of physical things this common matter is sensible matter. Formal abstraction is the abstraction of quantity and is concomitants from sensible matter. This abstraction retains a common matter, which in the definition of mathematicals is intelligible matter. That is, the distinction between individuating and common matter is distinct from that between sensible and intelligible matter, and the two distinctions can be combined. Individuating matter is distinguished from common matter because the former, unlike the latter, is unintelligible. Above, St. Thomas distinguished between matter insofar as it enters into the definition of a thing, and matter insofar as it is a principle of a thing's numerical individuality.[38] A mobile moving in the here and now can be pointed out, sensed, and known as an individual. The reason why it exists in the here and now is due to its material principle, which principle enters into the universal definition of a thing as common matter. St. Thomas manifests this difference by the comparison of 'these flesh and bones of this man' to 'flesh and bones of man'. Individuating matter can be seen to be unintelligible

[37] Ibid., (Leon.50:148:201-202).
[38] See St. Thomas, *SBDT*, q. 5, a. 2, c. (Leon.50.143:83-108); see also ibid., q. 4, a. 4, ad6[um].

because the intellect cannot verify the truth of a statement such as "This man exists," or "The reader of this sentence exists," apart from the senses.[39]

Matter is named sensible or intelligible in reference to the power by which their corresponding individual composite instances are known fully as individuals.[40] "Sensible matter" takes its name from the senses. For this reason, its definition is "matter insofar as it underlies sensible qualities."[41] Insofar as it names the material principle of particular sensible substances, its definition includes matter "such as bronze and wood, or whatsoever mobile matter, as fire and water, and all of this sort, and from such matter sensible individuals are individuated."[42] But if individuating matter (prime matter) cannot be sensed, why call it "sensible"? Note that

> . . . to be sensed is manifestly a mere extrinsic denomination borrowed from the sense faculty of the soul. . . . When 'sensible' and 'sense' are said to be correlatives, the true reason for referring the one to the other is to be found on the part of the sense faculty: the thing is called sensible because the sense refers to it. Hence the matter in our definitions of natural things is called sensible inasmuch as it may cause sensation, not because it is sensed. This shows, too, that the sensible matter of definitions is not confined to those things of which we can have an actual sensation, like a tree. Anything that is one in genus with what we can actually sense will be defined as made of sensible matter.[43]

Now, matter causes sensation insofar as it underlies sensible qualities. Thus, following Aristotle's doctrine of sensible objects, we can say that sensible matter is sensible *per accidens*, even though accidents sensible *per se* (such as color or shape) are included in its notion.[44]

Likewise, "intelligible matter" takes its name from the imagination insofar as it is akin to the

[39] See De Koninck, "Abstraction From Matter-pt.1," 161-62, fn.: "The mere individual can never be pinned down except by designation through an act of the senses—*this, here and now*. No amount of description ever touches the individual. To assume that it can is to assume that there could never possibly be another like this one. As we describe Alexander the Great in all that made him different from every other figure of history, we might still be talking about somebody else. And this is what is meant by the statement that the individual is ineffable: all that can be done is to point him out." See also Jacob Klein, "On Precision," in *Lectures and Essays*, ed. R. B. Williamson and E. Zuckerman (Annapolis: St. John's College Press, 1985) 291, on a description of a novel's character: "Henry James is praised for the 'extreme precision of his style,' but we might remain dissatisfied by the description of Christopher Newman. Are the 'conditions of his identity,' to use Henry James' own words, sufficiently detailed? Can this be done at all?"

[40] See St. Thomas, *In Meta.*, lib. 7, lect. 10, nn. 1493-96, 438-39.

[41] St. Thomas, *ST*, Ia, q. 85, a. 1, ad2[um] (Leon.5.331).

[42] St. Thomas, *In Meta.*, lib. 7, lect. 10, n. 1496, 439.

[43] De Koninck, "Abstraction From Matter-pt.1," 167-68. See also Aristotle, *Metaphysics*, 4.5, 1010b30ff.

[44] See Aristotle, *De Anima* 2.6; see also, De Koninck, "Abstraction From Matter-pt.1," 169-174.

intellect,[45] and from the intellect insofar as it still understands a subject of quantity.[46] Hence it can be defined as "substance insofar as it is subject to quantity."[47] Its definition includes "insofar as it is subject to quantity" for this is the extent to which the mind requires such matter as a subject to think about mathematical things. Thus, in summary, the distinction between *individuating* and *common* matter looks to intelligibility, or how it is possible for things to exist *in* the mind. The distinction between *sensible* and *intelligible* matter looks more to the definition of things, or how the mind can know things in different ways through abstraction. The inclusion of sensible and intelligible matter is what makes the modes of definition in physics and mathematics constitute different genera of knowables.[48]

§3 – The Second Theme: The Existence of Matter and Motion

St. Thomas makes use of the principle that "everything that has matter is mobile" in section [7] of the *prooemium* to argue that physics studies mobile being. This might at first seem to be overstating the case. The converse, that everything that moves has matter, is more evident, for matter as an underlying principle is required to explain change. So which is it? St. Thomas states variously that "Nothing separate from matter is able to be mobile,"[49] and "wherever there is motion there must also be matter."[50] What allows for the conversion? How to guard against exceptions?[51]

Two answers can be given in outline form. First, as will be discussed in Chapter 2, the physicist assumes as evident the existence of motion and change and analyzes them to find their principles. Matter as the underlying principle of natural things is discovered by analyzing change,

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[45] See St. Thomas, *In Meta*., lib. 7, lect. 10, n. 1494, 439: "Dicuntur autem intelligibilia, huiusmodi singularia, secundum quod absque sensu comprehenduntur per solam phantasiam, quae quandoque intellectus vocatur secundum illud in tertio de anima: intellectus passivus corruptibilis est."
[46] See St. Thomas, *SBDT*, q. 5, a. 3, c. (Leon.50.148:195-200).
[47] St. Thomas, *ST*, Ia, q. 85, a. 1, ad2um (Leon.5.331).
[48] See above, fn. 25.
[49] St. Thomas, *In Meta*., lib. 3, lect. 7, n.411, 139.
[50] St. Thomas, *SBDT*, q. 5, a. 2, sc.2 (Leon.50.142:42-44).
[51] One exception is the empyreal heaven, the abode of the angels and the blessed. This does not fall under natural philosophy, but leads to a distinction; see *II Sent*., d. 2, q. 2, a. 2, obj.4 & ad4um, 73-74; see also *ST*, Ia, q. 66, a. 3 (Leon.5. 160-61). Since this is outside physics, it will not be considered. For more details see Dante Alighieri, *Paradise*, trans. A. Esolen (New York: Random House, 2007) Canto 30, and Esolen's notes, 481-82.

which analysis Aristotle performs in *Physics* 1. Hence, in the order of what is more known at first, that everything that moves and changes has matter is the more evident proposition. Thus, the proposition could be converted at least qualifiedly. However, and second, it seems likely from St. Thomas's use of the principle that he is relying on it as a proposition in the order of what is more known in itself. Perhaps one could reason thus. In order for the conversion to be false, there would have to be a material being that was simply immobile. Such a being could not have sensation, intellection, or volition, insofar as these activities are immaterial.[52] Now, motion is the act of a thing by which it attains further perfections, i.e., attains its end. Thus, such a being would either not have a *telos* or would be by its essence its own perfection, but distinct from God. These seem to be difficult requirements to come by, and that such a being exists who has them would surely require proof.

In this thesis, I will only put weight on the former reasoning. This line of argument advances from what is more known at first to what is more known in itself: it assumes that change is real to highlight the reality of matter as the principle required to make change intelligible. One can begin to understand motion and see the cause of the demonstration in *Physics* 6.4 through an understanding of motion's principles: form, matter, and privation. These will be examined in Chapter 2.

§4 – The Third Theme: *Scientiae Mediae* (*SBDT*, q. 5, a. 3, ad 5^{um}, 6^{um}, & 8^{um})

St. Thomas's doctrine on *scientiae mediae* will be gleaned here from several sources[53] and this treatment will focus on a series of replies to objections in *SBDT* q. 5, a. 3. First, several demonstrations will be presented as examples of *scientia media*. Then, the replies will be used to

[52] See St. Thomas, *SLDA*, lib. 1, c. 10 (Leon.45/1.50), lib. 2, c. 14 (Leon.45/2.127-28), lib. 3, c. 6 (Leon.45/1.234); *ST* Ia, q. 14, a. 1, c. (Leon.4.166), *ST* Ia, q. 79, a. 2 (Leon.5.259-60), q. 82, a. 3, (Leon.5.298-99); *QDA*, q. 2, c. (Leon.24/1.16-19), q. 13, c. (Leon.24/1.115-20).

[53] See *In Phys.*, lib. 2, lect. 3; *In. Post.*, lib. 1, lect. 24-25, lect. 41; *ST* IIa-IIae, q. 9, a. 2, ad3^{um} (Leon.8.75). See also Maritain, *Degrees of Knowledge*, 41-46; De Koninck, "Are the Experimental Sciences Distinct from the Philosophy of Nature," in *The Writings of Charles de Koninck*, vol.1; Weisheipl, *Commentary*, 24-26; Smith, *General Science of Nature*, 147-57; William Wallace, *The Modeling of Nature* (Catholic University of America Press: Washington, D. C., 1996) 171-72, 295-97, 324-40; Wallace, "Demonstrating in Science," 155-57; Ian Mueller, "Physics and Astronomy: Aristotle's *Physics* II.2.193b22-194a12," *Arabic Sciences and Philosophy* 16 (2006): 184-92.

analyze these arguments and understand this "science between" mathematics and physics.[54]

The following are all demonstrations (or at least arguments) in a middle science. They are presented in the schema "minor term – middle term – major term." A lack of conviction produced by merely reading the syllogism is not a strike against the demonstration.[55] Consider:

[A] The earth	is a body which casts a circular shadow on the moon in all orientations	is a spherical body.[56]
[B] The earth	is a spherical body on which a freely swinging pendulum deviates clockwise in the northern hemisphere inversely as the sine of the latitude	is spinning on its axis counterclockwise (looking down on the north pole) once every 24 hours.[57]
[C] Light	is what is reflected by the regularly eclipsed satellites of Jupiter 16 minutes later when the earth is 50,000 miles farther away	is propagated at a finite velocity of 186,000 miles per second.[58]
[D] The moon	is a spherical body illumined by an external source (the sun)	is a body that waxes and wanes through crescent phases .[59]

The general claim of a mixed science is that it demonstrates a conclusion in a genus that is less abstract using the principles of a genus that is more abstract.[60] In the above examples, mathematical principles are used to demonstrate something about physical things. Note that in [A]–[C], the middle term is a quantitative effect, and the major term is a quantitative cause (thus, converting the major premise will make these demonstrations qualifiedly *propter quid* instead of *quia*). In [D] the reverse is the case (i.e., the sphericity of the moon *explains* its phases, not vice versa). [A] relies on the geometry

[54] In general the objections argue that mathematics must consider matter or motion by raising examples of the quantity of physical things being studied, or parts of mathematics, such as astronomy, studying mobile things. The notion of a middle science preserves the purity of mathematics by showing how its principles enter into such considerations only formally.
[55] Wallace, "Demonstrating in Science," 155: "No one can acquire a scientific knowledge of a particular subject matter merely by reading a list of syllogisms . . . As was known to Aristotle, there are almost as many *per se nota* propositions in any science as there are demonstrations in that science. Therefore the only persons who can appreciate demonstrations are those who have gone through sufficient investigation to convince themselves of the truth of the premises."
[56] See ibid., 140. I have borrowed Wallace's schema for presenting the demonstrations.
[57] See ibid., 146. This will be recognized as the argument based on Foucault's pendulum.
[58] Wallace, "Demonstrating in Science," 147. See also Wallace, *Modeling of Nature*, 324-34 for an exposition of optical demonstrations of properties of the rainbow by Theodoric of Freiberg.
[59] See Wallace, "Demonstrating in Scinece," 146, and St. Thomas, *Exp. Po. An.*, lib. 1, lect. 23 (Leon.1*/2.86). This argument adapted slightly to make it argue for a property of the moon (i.e., *a priori* instead of *a posteriori*).
[60] See Aristotle, *Posterior Analytics*, 1.13, 78b32-79a15.

of light and shadow to conclude something about the earth. [B] argues from the sine properties observed in a Foucault pendulum to the rate of rotation of the earth. [C] relies on the linearity of light and the expected measurable positions of Jovian satellites to see something about the velocity of light's propagation. [D] is very similar to [A] in the geometry of light that it relies on. Note that while these arguments utilize quantity (in the case of [B] and [C] both continuous and discrete) in their middle and major terms, the subject is always a physical thing. This physicality also permeates the arguments: [A] and [D] speak of bodies casting shadows, [B] of swinging and spinning bodies, and [C] of visible light being reflected and propagating.[61] The following exposition will corroborate these initial observations as to the nature of *scientia media*.

In *SBDT*, q. 5, a. 3, ad5[um], St. Thomas makes a first distinction between physics and mathematics in regard to studying quantity and to allow for a middle science:

> Motion according to its nature does not pertain to the genus of quantity but participates something of the nature of quantity from elsewhere: according to the division of motion taken either from the division of space or from the division of the mobile. And therefore to consider motion does not pertain to the mathematician, but nevertheless *mathematical principles* [*principia mathematica*] can be applied to motion. And therefore, insofar as *the principles of quantity* are applied to motion, the physicist considers the division and continuity of motion, as is clear in *Physics* 6; and in the middle sciences between mathematics and natural science the measure of motion is studied, as in the science of the moved sphere and in astronomy.[62]

The first sentence shows why it is the case that it does not belong to the same science as mathematics to consider motion as measured, viz., the nature of motion takes part in quantity from quantified things defined by physics: space or place and the mobile body. So while the mathematician does not consider motion, the principles of his subject, viz. "the principles of quantity," are present in physics. Hence, the *physicist* in *Physics* 6 considers the quantitative division and continuity of motion, while the middle scientist, the *astronomer*, considers heavenly motion insofar as it is subject to

[61] Note about [C] that it neither requires nor concludes anything as to whether light is a body. Also, its minor term implies knowledge of where the satellites are supposed to be. Hence is already based upon an astronomical conclusion.
[62] St. Thomas, *SBDT*, q. 3, a. 5, ad5[um] (Leon.50.150:343-56).

quantitative measure. St. Thomas clarifies these two examples in the next objection.

The sixth objector assumes that astronomy is a part of mathematics according to the same mode of consideration. St. Thomas denies this in his extensive reply:

> [a] In composites, simple bodies and their properties are conserved, granted through another mode, just as the proper qualities of the elements and their proper motions are found in compounds; but what is proper to composites is not found in simple bodies. And thus it is that to the extent that some science is more abstract and considers simpler things, to such a degree its principles are more applicable to other sciences. Whence mathematical principles are applicable to natural things, but not conversely, because physics is from mathematical suppositions, but not conversely, as in clear in the *De Caelo et Mundo* 3. [b] And thus it is that concerning natural and mathematical things three orders of sciences are found: [i] for there are certainly purely natural sciences, which consider the properties of natural things insofar as they are such, as physics and agriculture and the like. [ii] But there are certainly purely mathematical sciences, which determine about quantity absolutely, as geometry of magnitude, and arithmetic of number. [iii] But there are certainly middle sciences, which apply mathematical principles to natural things, as music, astronomy, and the like. [c] Nevertheless, these are more akin to mathematics, because in their consideration that which is physical is as matter, but what is mathematical is as form, just as music considers sounds not insofar as they are sounds, but insofar as they exist according to proportionate numbers, and likewise in the others [middle sciences]. And because of this they demonstrate their conclusions about natural things, but through mathematical middle terms. And therefore nothing prevents this if insofar as they share with physics, they look to sensible matter, for insofar as they share in mathematics, they are abstract.[63]

In [a], St. Thomas gives an example from what we would call chemistry to clarify how mathematics is prior to and can enter into the considerations of other sciences.[64] A clear example here is weight. The heavy elements retain their proper motion, falling downwards, even when part of a composite, e.g., parts of an animal. However, the motion proper to the animal as a composite (walking, running) would not be found in the simple bodies. This points out both the priority of the simples to the composite and that the composite can have something proper to itself. Just as what is

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[63] Ibid., ad6[um] (Leon.50.150-51:357-91).

[64] St. Thomas refers to *De Caelo* 3.1, where Aristotle is discussing whether bodies can be composed of planes. Aristotle advances mathematical and physical lines of argument against this position: *De Caelo* 3.1, 299a12-16; and see St. Thomas, *In De Caelo*, lib. 3, lect. 3, n.4, n.6 (Leon.3.236-37), on why mathematical principles can be used: ". . . scientia quae se habet ex additione ad aliam, utitur principiis eius in demonstrando, sicut geometria utitur principiis arithmeticae: magnitudo enim addit positionem supra numerum, unde punctus dicitur esse unitas posita. Similiter autem corpus naturale addit materiam sensibilem supra magnitudinem mathematicam: et ideo non est inconveniens si naturalis in suis demonstrationibus utatur principiis mathematicis: non enim est omnino aliud genus, sed quodammodo sub illo continetur."

present in simple bodies is found in composite bodies, but not conversely, so principles in mathematics can be applied in physics, but not conversely; furthermore, just as there are properties only found in composites, so also there are considerations that only physics is able to carry out.

In [b], St. Thomas shows where this application occurs. The first two orders of sciences, pure physics and mathematics, are essentially heterogeneous, for reasons discussed above. The third order of science is middle science. St. Thomas clarifies their nature in [c], noting that the mathematical principles are the more formal element and the physical is more material. The conclusion is a physical one made through a mathematical middle term. How the middle term is mathematical will be considered shortly. This mixed quality, however, allows one to say that the argument is mathematical in its abstraction, but physical in its concretion, or as St. Thomas states in another text, physical in its end or term. Because of this end, middle sciences "remain principally natural since they terminate in the natural things that it is their goal to know better."[65] The difficulty lies in understanding the nature of the syllogism employed by a middle science. The minor premise is more physical (since the subject of the conclusion is a physical thing, as seen above), the major term and the middle term are more mathematical (as seen above), and yet the conclusion is physical. Thus, the middle term *qua* mathematical defines with intelligible matter, while the conclusion *qua* physical must define with sensible matter, in order to be truly saying something about physical things.[66] How does one avoid the objection that such an argument is in four terms? If the subject is defined with sensible matter, then must not the middle term in the minor premise be the same? But if the middle term is defined with intelligible matter, then the major term seems bound to follow suit. How can a conclusion have sensible matter in its account, when the middle term uses intelligible matter in its?[67]

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[65] De Koninck, "Experimental Sciences Distinct?" 449. See St. Thomas, *In Phys.*, lib. 2, lect. 3, n. 8 (Leon.2.63).
[66] See Cajetan, *Comm. ST*, IIa-IIae, q. 9, a. 2, ad3[um] (Leon.8.75).
[67] See Wallace, *Modeling of Nature*, 295-96, who holds that the middle terms are analogous: "Quantity as understood in physics is analogous to quantity as understood in mathematics, for, granted that the two meanings are partly the same and partly different, there is a proportionate understanding of the two terms that allows transitions to be

The eighth objection and reply help to overcome this difficulty. The objector, expanding on Aristotle's statement in *Physics* 2.7, 198a30-31, holds that mathematics studies the genus of mobile and incorruptible things, physics of mobile and corruptible things, and metaphysics of immobile and incorruptible things. St. Thomas replies that here Aristotle does not intend to divide the speculative sciences, but the things they study. Physics studies mobiles both corruptible (terrestrial, e.g., in *De Anima*) and incorruptible (celestial, e.g., in *De Caelo*). As to the middle sciences:

> . . . the mobile and incorruptible beings, on account of their uniformity and regularity, can be considered as to their motions through mathematical principles, which cannot be said of corruptible mobiles. And therefore the second genus of beings is attributed to mathematics by reason of astronomy . . .[68]

What St. Thomas says here is striking: first, that mathematical principles are applicable to motion due to the uniformity and regularity of the heavenly bodies (arising from their incorruptibillity), and second, that this applicability is not the case for corruptible bodies. The motion of corruptible bodies such as animals is not uniform or regular—an animal has many different motions at different speeds and times. Does this mean that middles sciences can only be about incorruptible things?

Consider the various middle sciences. Music considers numerical proportions found in sounds. This involves corruptible matter. Optics considers the visual line. This involves sources of light and seeing that are corruptible. The science of weights considers corruptible things. So the middle sciences are not barred from applying mathematical principles to corruptible things as such. Rather, the uniformity and regularity of quantified things determines this applicability, or, as it were, insofar as they *exist* quantitatively in the way quantity *exists in thought* in mathematics. What this means for a science of the local motions of corruptible things is difficult to say in a short span. Yet, following the above principle, insofar as their motions are regular and uniform then mathematical

made between them." The difficulty here is that St. Thomas states outright that the middle term is mathematical. Even if the uses were analogous, the argument would follow only to the extent that their *rationes* are the same.

[68] St. Thomas, *SBDT*, q. 5, a. 3, ad8^{um} (Leon.50.151:415-21).

principles can be applied to them. The substances displaying such uniformity and regularity need not be incorruptible, for the force of the arguments of astronomy did not change in themselves even after the incorruptibility of the heavens was abandoned.[69]

Therefore, it is not the case that the middle term is mathematical in one premise but physical in the other. Rather, it is mathematical in both, but is said truly of the minor term *only insofar as* that subject admits of such a mathematical character. Argument [A] above concludes that the earth is a sphere only insofar as the circularity of its shadow obtains—later arguments showed the earth to be an oblate spheroid. [B] concludes only to the extent that swings and degrees are correctly measured and the earth possesses a regular rotation. This "only insofar as" indicates two things. First, it reveals an inherent imprecision in this mode of science, both on the side of application to material quantity and in the measurement of that quantity.[70] It also indicates that the unity of this science is derived from reason, because the mind in its consideration mixes an object only obtaining in thought (mathematicals) with an object obtaining in things (physicals). Thus, the genus of scientific discourse in a middle science is only qualifiedly one. If a middle science were to be purely one, a new abstraction 'between' physics and mathematics would be needed to justify this new degree of intelligibility.[71] A common definition of middle sciences can now be proposed: *a speculative science that aims to know physical things through mathematical principles*—that is, "*quae accipiunt principia abstracta a scientiis pure mathematicis, et applicant ad materiam sensibilem.*"[72] It is important that St. Thomas mentions sensible matter, for this captures the *per se* subject of motion, and hence a middle or mixed science can know truly mobile subjects.[73] The physical aspects of such an argument enter in as material (since the

[69] See above in fn. 64; also De Koninck, "Experimental Sciences Distinct?" 450.
[70] Newton's claim will be that the imprecision on the side of measurement can be reduced without limit because there is no difference between mathematical and physical bodies—or at least one need not care about such a difference.
[71] See De Koninck, "Experimental Sciences Distinct?" 449; and see St. Thomas, *In Post. An.*, lib. 1, lect. 15 & 25; *SBDT*, q. 5, a. 4, ad1[um]. A demonstration cannot cross from one genus of consideration to another, because what is *per se* to the subject in one genus is no longer *per se* in the other, and demonstration must be had through *per se* premises.
[72] See St. Thomas, *In Phys.*, lib. 2, lect. 3, n. 8 (Leon.2.63).
[73] See St. Thomas, *In Phys.*, lib. 2, lect. 11, n. 3 (Leon.2.88): "Nam astronomia . . . ; inquantum enim applicat principia

conclusions sought are such) and the mathematical ones as formal.

– Conclusion of Chapter One

St. Thomas tells us that *ens mobile* is the subject of physics. His primary reason for this is the argument in *Physics* 6.4. Now, section [10] lists the various parts of the subject of physics. *Ens mobile* must necessarily be predicable of them all if it names the subject as a whole. Further, the genus of a science cannot be said equivocally of its various subject parts, for science does not deal in equivocations. For most sciences, the genus has one account that is univocally said of all its subject parts. Is this the case for *ens mobile*? Aristotle says that, "Motion is not beyond the things. For the thing changing always changes according to substance or amount or quality or place. But one can grasp nothing common in these."[74] That is, change is limited to certain categories, and motion strictly is limited to amount, quality, and place. There is no genus of these, because they are among the ten highest genera. "Motion" is some reality found in them all, as *Categories* 14 relates. Since a univocal definition must remain within one of the ten genera, *ens mobile* is excluded from being a genus one by univocity. What remains? Perhaps the unity found in metaphysics, a genus whose unity depends upon analogy. Perhaps just as "being" is said analogously of all the subject parts of metaphysics, so also "mobile being" is said analogously of all the subject parts of physics. This will be discussed in more detail later. St. Thomas's delineation of the subject of physics, *ens mobile*, has been presented. *Physics* 6.4, the lynchpin for this argument, must now be defended.

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mathematica *ad materiam naturalem*, circa *mobilia* considerationem habet." Emphasis mine.

[74] Aristotle, *Physics* 3.1, 200b33-35.

Chapter Two

Mobiles and Bodies in St. Thomas' Exposition of Physics 6.4

This chapter will examine the physical demonstration that every mobile is divisible. It will defend the central demonstration in *Physics* 6.4 (234b10-20), glance at the contrapositive argument in *Physics* 6.10, and show the further conclusion that every mobile is a body. This will manifest how mobility is given primacy in the scientific considerations of the physicist, for the physicist sees the divisibility of *per se* subjects of motion (mobiles) via the nature of motion. This conclusion, and certain corollaries, will help to contrast this method with Newton's method in Chapter 3. This chapter is accordingly divided into three main parts. First, remote principles from *Physics* Books 1-5 will be reviewed. Second, proximate principles from Book 6 will be reviewed, and various difficulties raised. Finally, the demonstration itself will be presented and difficulties answered.

The text of the argument in *Physics* 6.4 is as follows:

> It is necessary, however, that every thing which changes be divisible. For, since every change is from something to something, and when it is in that to which it changed, it is no longer changing, and when it is in that from which it changed, both itself and all its parts, it is not changing (for what is disposed in a similar way, both itself and its parts, is not changing), it is necessary, then, that something of what is changing be in this and something in the other. For it is not able to be in both or in neither. (By that to which it changes, however, I mean the first [term] according to the change, e.g., [in the change] from white, the grey, not the black. For it is not necessary that the thing changing be in either of the extremes.) It is apparent, then, that every thing which changes will be divisible.[1]

St. Thomas's commentary on this passage is as follows:

> Then when he says, "Every thing which changes," etc., he shows that every thing that moves [*movetur*] is divisible, by this argument. Every change [*mutatio*] is from something, to something [*ex quodam in quiddam*]. [a] But when something is in the term to which it is changing [*mutatur*], it changes no further, but has already changed, for something does not move and have moved at once, as was said above. [b] But when something is in the term from which it changes, as to itself as a whole and as to all its parts, then it does not change, for it was said that what has itself similarly, both itself and its parts, does not change, but rather rests. (But he adds "and all its parts," because when something begins to change, it does not depart the place it occupied previously at once, but part after part.) [c] Nor again can it be said that it is in both terms as to itself as a whole and as to

[1] Aristotle, *Physics* 6.4, 234b10-20. Compare this argument to its anticipation at 230b28-231a4.

> its parts while it moves [*movetur*], for thus something would be in two places [at once]. [d] Nor again can it be said that it is in neither of the terms, for we speak now of the nearest term into which it changes, and not of the last extreme, as if from white something changes to black, black is the last extreme, but gray is the nearest. And likewise if there be one line divided into three equal parts, namely the line *abcd*, it is clear that the mobile, which in the beginning of motion is in the part *ab* as in a place equal to itself, happens in some part of its motion to be neither in *ab* nor in *cd*, for at some time it is wholly in *bc*. When, therefore, it is said that that which changes, when it changes, is not able to be in neither [term], not the extreme terms but the proximate terms are understood. [e] Therefore, it remains that every thing that changes, while it changes, as to something of itself is in one, and as to something of itself is in another (just as when something changes from *ab* to *bc*, a part in itself moves leaving from the place *ab*, entering the place *bc*; and what moves [*movetur*] from white to black, a part which ceases to be white, becomes gray or pale). Thus, therefore, it is clear that every thing that changes, since it is partly in one [term] and partly in another [term], is divisible.[2]

In understanding the text, four items arise immediately. First, Aristotle is using the general word for change (*metabolê*), translated by *mutatio*.[3] This word strictly means substantial change, but it can be taken to signify both substantial changes and motions.[4] Yet how this argument follows for change in general is not immediately apparent. However, and second, it is clear what is required for the argument to follow: motion requires that a mobile while in motion be in both terms, partly in one and partly in the other. This shows the divisibility of the mobile. At the same time, this raises two difficulties: it seems that Aristotle's stipulation about the mobile in the *terminus a quo* supposes its parts, and his stipulation about "the first [term] according to the change" in the case of local motion is easily imagined as the next complete place the mobile will occupy, which again seems to presuppose parts. St. Thomas offers no immediate clarification of the two difficulties. Indeed, his comments at [b], [d], and [e] concerning the parts of the mobile seem to aggravate them. However, he highlights certain *praecognoscenda* that can be fleshed out as follows. Clearly, the entire argument

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[2] St. Thomas, *In Phys.* lib. 6, lect. 5, n. 10 (Leon.2.284-85). Two translation notes are necessary. First, St. Thomas change between the use of "*movere*" and "*mutare*" indicates that he is not attending to the difference (at least not yet). He discusses later in the *lectio* how the proof applies to species of change. Second, the use of the passive form of the verb has been rendered in the middle voice, a usage grammatically allowed and philosophically sensible. See John Lyons, *Introduction to Theoretical Linguistics* (Cambridge: University Press, 1968), 375. This issue in translation is also present in Newton, who uses passive verb forms that require middle voice renderings: see Cohen, *Guide*, 37-41, esp. n.35.

[3] See the facing Greek text in the Leonine edition, or *Aristotelis Physica*, rec. W. D. Ross (Oxonii: Clarendoniano, 1950).

[4] See Aristotle, *Physics* 5.1, 225a1-7, 225b5-9; and St. Thomas, *In Phys.* lib. 3, lect. 2, n. 4 (Leon.2.108): "Accipit enim hic motum communiter pro mutatione, non autem stricte secundum quod dividitur contra generationem et corruptionem."

assumes a mobile in motion. Thus, the existence, definition, and principles of motion are also assumed. The principle that every change is *ex quodam in quiddam* is also assumed. Parts [a] and [b] each have back references to proofs in Book 6. Part [c] makes reference to place. Finally, although not explicit, the proof is clearly dealing with a unified motion, and supposes the infinite divisibility of the continuum (since the proof does not attend to which parts of the mobile are determined by the argument). Tracking these assumed premises will show how the physicist investigates motion, from the assumption of its existence to the proof of its inherence in a divisible subject.

§1: Remote Principles for the Demonstration

In this section I will consider (1) the existence of change, (2) its principles, (3) the definition of motion, (4) place, and (5) qualitative aspects of motion. To place these in the overall order of the *Physics*, it is best to review St. Thomas' division of the text. A diligent reader of Aristotle, St. Thomas always assumes that the Philosopher's text is a unified whole.[5]

I. Aristotle determines the universal principles of the science of nature (Book 1-2)
  A. Determines the principles of the subject of the science (Book 1)
  B. Determines the principles of doctrine (Book 2)
II. Determines about mobile being in common (Books 3-8)
  A. Determines about motion in itself (Books 3-6)
    1. Determines about motion itself (Books 3-4)
      a. Determines about motion and the infinite (consequent to motion intrinsically)
      b. Determines about place, void, and time (consequents to motion extrinsically)
        i. Those which are the extrinsic measure of the mobile
          1. Place
          2. Void
        ii. Time, which is the measure of motion itself
    2. Determines about the parts of motion, or the division of motion
      a. The division of motion according to species (Book 5)
      b. The division of motion into quantitative parts (Book 6)
  B. Determines about motion by comparison to movers and mobiles (Book 7-8)

It is evident that St. Thomas sees the *Physics* as the ordered investigation of motion. The existence

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[5] See, for example, *In Phys.* lib. 1, lect. 1, n. 8 (Leon.2.6), where he dismisses an exposition of Averroes as follows: "Unde patet quod eius expositio non est conveniens, quia non coniungit totum ad unam intentionem." See also Simplicius, *Simplicius on Aristotle's Physics 6*, trans. D. Konstan (Ithaca: Cornell University Press, 1989) 15-16 (923-25.1), who argues for the unity of the *Physics* from logical dependencies, despite its consisting of *Physics* (Books 1-5) and *On Motion* (Books 6-8).

and principles of change are treated in Book 1, and motion is defined in Book 3. Place as a mobile's measure is treated in Book 4. The premises from Book 5 treat motion according to species.

(1): The existence of change is assumed by the physicist. The subject's existence and definition are assumed for no prior principles within a science can show its existence or definition.[6]

> To look into whether being is one and immobile, then, is not to look into nature. For just as for the geometer there is no longer an argument with regard to one who denies the principles, but [this job belongs] either to a different science or to one common to all, so neither [with regard] to one who denies the principles [here]. For there is no longer a principle, if there is only one thing and it is one in this way. For a principle is a principle of some thing or things. . . . It must be granted by us that either all or some natural things are moving. This is clear from induction.[7]

Aristotle refers to Parmenides, who held that being is one and immobile, "without beginning, without cease, since becoming and destruction have been driven very far away."[8] This position denies the possibility of principles, for the principle and the principled must be other, requiring multiplicity. The principle and principled in question here are nature and motion: "It is necessary that motion be supposed in natural science, just as it is necessary to suppose nature (in whose definition motion is placed), for nature is the principle of motion, as will be said below [in Book 2]."[9] Indeed, "to try to show that nature exists is laughable,"[10] since it is an attempt to show what is manifest by experience (that things exist which have intrinsic principles of motion and rest) through what is not manifest. The natural scientist assumes the existence of things moving by nature because he experiences them, not because he makes arguments for their existence apart from experience. He declares his subject to be "mobile being"—an obligatory stance if Parmenides' problem belongs to metaphysics.[11]

(2): The physicist analyzes change to find its principles: matter, form, and privation. St.

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[6] See Aristotle, *Posterior Analytics*, 1.10, 76a31-36.
[7] Aristotle, *Physics* 1.2, 185a1-5, a14.
[8] Parmenides, DK.7, in *Ancilla to the Pre-Socratic Philosophers*, trans. by K. Freeman (Cambridge, MA: Harvard University Press, 1996) 44.
[9] St. Thomas, *In Phys*. lib. 1, lect. 2, n. 7 (Leon.2.9).
[10] Aristotle, *Physics* 2.1, 193a3.
[11] "Mobile being" is preferable to "sensible being" or "natural being," because neither of these latter express how the Parmenidean problem is ignored by the physicist. Indeed, a Parmenidean (in his own way) would oblige "sensible" or "natural being". See Cajetan, *DSNP*, 211a20-33.

Thomas states elsewhere that, "Matter in itself is not able to be known adequately except through motion, and its investigation seems to pertain especially to natural science."[12] Physics investigates the principles of change based upon both the experience of change and speech about change (for speech is assumed to reflect reality).[13] Aristotle's example is a man who learns music. In speaking about this change, there is this difference, for

> a two fold mode of speaking [*modi loquendi*] is used, namely that "this becomes this," and "from this, this comes to be." For we say that "the non-musical becomes musical," and "from the non-musical, the musical comes to be." But it is not said thus in every case, for it is not said "from man, musical comes to be," but "man becomes musical."[14]

This difference in the mode of speaking about changes begins to highlight three aspects in change. Speech also points to what persists through the change:

> To these three, becoming was attributed, for it was said that "man becomes musical," and "the non-musical becomes musical," and "the non-musical man becomes musical," of which three, only the first remains complete in a production, but the other two do not remain.[15]

From these two differences, the three principles of change are more clearly seen. The subject of the change is the man, although he has a twofold account: non-musical and man (for as a non-musical man, he does not survive the change; as man he does). The "non-musical" is what is opposite and the privation of what comes to be, namely the form "musical". Together, the subject and form compose the being that comes to be.

The subject and form are *per se* principles of what they compose because the definition of the composite depends upon them. The privative form (non-musical) is only a principle *per accidens*, because it is excluded by the presence of the positive form, and thus composes what comes to be

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[12] St. Thomas, *In Meta.* lib. 7, lect. 2, n. 1285, 384. See also Wippel, *Metaphysical Thought*, 299-302, for a discussion of St. Thomas and this passage in the *Physics*. Wippel highlights that this approach to matter and form is physical, in contrast to metaphysical arguments for the subject of becoming that can be offered: see St. Thomas *In Phys.*, lib. 1, lect. 12, n. 10. The author also made use of *Commentary on Aristotle's Metaphysics*. Translated J. P. Rowan, preface by Ralph McInerny. Notre Dame, IN: Dumb Ox Books, 1995.

[13] For a like interpretation of this passage, see David Bolotin, *An Approach to Aristotle's Physics* (Albany: State University of New York Press, 1998) 15-16. See also De Koninck, "The Cosmos," 259-61.

[14] St. Thomas, *In Phys.*, lib. 1, lect. 12, n. 5 (Leon.2.41).

[15] Ibid, (Leon.2.42).

only accidentally.[16] Aristotle names the subject or underlying principle of change matter and potency:

> But as being according to potency, [material] is not destroyed in virtue of itself, but it is necessary that it be indestructible and ungenerable. For if it came to be, there must be some first underlying thing, present in it, from which [it came to be]: but this is its own nature, whence, it will be before it comes to be. For I call "material" the first thing underlying each thing, present in it, from which something comes to be, not accidentally.[17]

That is, matter as principle of change is not subject to change, because this change would again require an underlying, the nature of which was already present. The terms of all natural changes (all forms) are found in this matter in potency. For one cannot say that being comes to be from nothing or that being comes to be from being simply. Matter exists as the potency from which being comes to be (i.e., it exists in some way), but it does not exist (i.e. accidentally) because it is not form or a complete being.[18] Matter is a real *per se* principle of change.

> The consequence of all this [Aristotle's discovery of the three principles] is that we have before us the only real explanation of change. All others either deny it or simply presuppose it. Any physics, then, which does not, explicitly or at least implicitly, acknowledge the primacy of the potency of matter in the explanation of physical change is bound to go wrong, and to constitute, despite what may be appearances to the contrary, philosophical regression.[19]

(3): The physicist must know what motion is, for nature is a principle and cause of motion and rest, and thus ignorance of motion constitutes ignorance of nature.[20] Aristotle's definition of motion is "the actuality of what exists in potency, as such."[21] Aristotle uses an example from art to manifest this definition: "When the buildable, insofar as we name it such, is in actuality, it is being built, and this is building."[22] When a house is being built, it remains in potency to completion. A *completed* house is not buildable, but it has been built, and *before* construction commences, the house is

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[16] See ibid., 190b20-24; see also St. Thomas, *In Phys.*, lib. 1, lect. 13, nn.2-4 (Leon.2.45).
[17] Aristotle, *Physics* 1.9, 192a25-32. Coughlin, ibid., n.82, notes the following about the definition of the material principle given here: "'Present in it' excludes the agent cause; 'from which it comes to be' excludes the formal cause; and 'not accidentally' excludes the privation."
[18] See also Aristotle, *Metaphysics* 8.1, 1042a32-b4; 1046a4-16.
[19] Coughlin, "Matter and the Reality of the Physical World," in Aristotle, *Physics*, 227.
[20] See Aristotle, *Physics* 3.1, 200b12-15.
[21] Ibid., 201a11.
[22] Ibid., 201a16-17.

buildable, but not because someone is building it.

St. Thomas holds that "it is wholly impossible to define motion otherwise through things prior and more known, except as the Philosopher has defined it here," and that "the Philosopher defines motion most appropriately [*convenientissime*]."[23] The terms of a definition must be prior to and more known than the thing defined, for a definition explains what a thing is. A definition of motion such as *a departure from potency to act, that is not sudden*, makes use of notions posterior to motion. "Departure" is a kind of motion, and "sudden" requires time in its definition, and motion is used to define time.[24] Potency and act, however, "since they are primary differences of being, are naturally prior to motion,"[25] and hence they can manifest the nature of motion.

St. Thomas explains act and potency as placed in the definition as follows:

> It must be considered, then, that something is in act only, something in potency only, and something has itself in a middle way between potency and act. What, then, is in potency only, does not yet move. However, what is already in perfect act does not move, but has already moved. That, then, moves which has itself in a middle way between pure potency and act, because clearly it is partly in potency and partly in act, as is clear in alteration. For when water is hot only in potency, it does not move. But when it is already made hot, the motion of heating is completed. But when it already partakes something of heat, but imperfectly, then it moves to heat, for what becomes warm partakes of heat by degrees more and more.[26]

What St. Thomas is stressing here is the "middle way" between act and potency. It is the very act of the water as hot or boiling that explains how water *can be on the way to* that act in full.

This "middle way" implies two orders within the account of motion.

> The imperfect act itself, then, of heat existing in the heatable is motion. This is not the case insofar as it is in act only, but insofar as, already existing in act, it has an order to the last act. Because, if the order to the last act were taken away, the act [then present],

[23] St. Thomas, *In Phys.*, lib. 3, lect. 2, n. 3 (Leon.2.105). See also Rémi Brague, "Note sur la définition du mouvement," in *La physique d'Aristote et le conditions d'une science de la nature*, ed. by F. de Gandt and P. Souffrin (Paris: J. Vrin, 1991) 107-108, who states that ". . . je n'ai pu trouver personne qui ait réussi à donner du mouvement une définition satisfaisant à ces conditions . . ." namely, the conditions of a) being a true definition and not a provisional description, b) true for all the species of motion, c) non-circular, and d) possessing a certain originality compared to Aristotle's definition. See also Roman A. Kocourek, "Motionless Motion," in *The Dignity of Science*, 283-294.
[24] See St. Thomas, *In Phys.*, lib. 3, lect. 2, n. 2 (Leon.2.105).
[25] Ibid., n. 3.
[26] Ibid.

> however imperfect, would be the end of the motion and not motion, as it happens when something is partly heated. However, the order to the last act belongs to that existing in potency to it. Likewise, if an imperfect act is considered only in its order to the last act insofar as [the last act] has the *ratio* of potency, [the imperfect act] does not have the *ratio* of motion, but of the beginning of motion: for heating can start from cold as well as from the lukewarm. *The imperfect act, then, has the* ratio *of motion insofar as it is both compared to the last act as potency and insofar as it is compared to something imperfect as an act*. Whence, [motion] is neither potency existing in potency, nor is it act existing in act, but it is act existing in potency, such that through "act" is designated its order to the prior potency, and through "existing in potency" is designated its order to the last act.[27]

The "act" of motion is an act only in virtue of its order to the potential present in the mobile. When this order is removed the motion ceases, and the act present when the motion was interrupted is then the final act. (Think of someone catching a falling stone.[28]) Likewise, the mobile *only* in its order to a prior potency does not provide the complete account of motion, for any beginning of a motion has the same account. It is only when both orders are present that the complete *ratio* of motion is present. The latter order is named by "act" since it is the imperfect act posterior to the mere potency of the beginning, and which is prior to the former order, named by "existing in potency" due to its order to the last act. These two orders require that the mobile that has such a motion be divisible.[29]

Thus, the necessity of "act" and "existing in potency" being in the definition is clear. Now, the "as such" excludes the possibility that the act is the act of *that from which* the motion is (for that act is already present), or of the *subject* (for that act is also already present), and of the *last act* (for the act of motion is in potency to that), or of, finally, "the intermediate rest, since the potency relative to which the intermediate rest (e.g., in tepidity) is an act is not the potency to the term (hot), but rather the potency to the intermediate itself."[30] What remains is that "the act of what exists in potency as such" names only that act which is ordered to the potency of the complete, last act considered with privation. This definition also ties motion to its principles:

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[27] Ibid. Emphasis added. *Ratio* (account, definition) is untranslated to preserve the identity between the real and defined.
[28] See John F. Nieto, "Continuity and the Reality of Movement," (Ph.D diss., University of Notre Dame, 1998) 194, 197.
[29] This assumes that motion belongs to the mobile, the proof of which will be assumed here: see *Physics* 3.3.
[30] Coughlin, "The Definition of Motion," in Aristotle, *Physics*, 249-50.

> Because when the mobile is in motion it has something of the final form in it, it is no longer merely in potency; thus, relative to potency with privation, the thing moving is in act. Motion is therefore an act. But it is not only act, or simply fulfilled act, or "perfect" act, but is the act of what can have still more of that in virtue of which it is said to have a certain potency, i.e., it is the act of the potential, taken with privation.[31]

The three principles of change, therefore, are present in all the species of motion.[32]

(4): Place is next among the principles of the demonstration. Aristotle notes that place would not be investigated were it not for the experience of local motion.[33] The experience of one's own motion ("now I am in the river" vs. "now I am out of the river" vs. "now he is where I was in the river"), and the motion of other things ("Mars is now in Pisces") attunes the mind to the fact that being "in" or "out" is different from being a part in a whole: "We recognize that a thing is in place, and is not a part in a whole, because we see that the thing is separated by motion from its surroundings."[34] This experience of local motion and mutual replacement shows place as other than the placed, "just as the transmutation of forms with one matter leads to the knowledge of matter."[35]

Aristotle's preliminary investigations of place lead to four "axioms" about place ("suppositions and principles known *per se*"[36] that "we deem worthy [*axioumen*]"[37] to manifest the nature of place). These are that place contains the placed, that it is equal in size to the placed, that it is separable from the placed, and that there are natural places.[38] The candidates for place are form, matter, the dimensions surrounding the body, and the surface of the containing body.[39] The first three are eliminated because they fail to meet one or another of the axioms; e.g., place cannot be

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[31] Ibid., 248. Nature as a principle of motion would enter into this account insofar as it is form and matter.
[32] See Aristotle, *Metaphysics* 12.4, 1070b18-20: ". . . all things have not the same elements in this sense, but only analogically; i.e. one might say that there are three principles—the form, the privation, and the matter. But each of these is different for each class . . ." This notion of the analogicity of the principles of motion will be expanded upon later.
[33] See Aristotle, *Physics* 4.4, 211a12.
[34] Coughlin, "Place," in Aristotle, *Physics*, 253.
[35] St. Thomas, *In Phys.*, lib. 4, lect. 5, n. 5 (Leon.2.159). In what follows, the dialectic that manifests basic or axiomatic aspects of place will be assumed: see *Physics* 4.1-4, and see St. Thomas, *In Phys.*, lib. 4, lect. 1-6.
[36] St. Thomas, *In Phys.*, lib. 4, lect. 5, n. 2 (Leon.2.159).
[37] Aristotle, *Physics*, 4.4, 211a1.
[38] See ibid., 211a1-7.
[39] See ibid., 211b6-10.

form because form is not separable from the mobile (form "contains" the mobile in another way).[40] Place is then defined as "the first immobile limit of the containing [body]."[41] In this definition, "limit of the containing" preserves containment, "first" the equality of place and placed, "immobile" their separability (for place remains after the mobile has departed), and the notion of "body" preserves natural place. Immobility is added as a difference, distinguishing proper place from other containing bodies that can move (such as ships or jars).[42]

From this definition, I will consider two things: the difference between void-space and place, and the way in which quantity and body enters into the notion of place. Aristotle rejects the void as an alternative to place. It has good reason to seem to be place, for a void-like space underlying the placed body meets the axiom of equality, and it would be separable and immobile. St. Thomas rejects void because it would mean either that place is not other than the placed (the third axiom), or that "there would be dimensions of space existing *per se*, and yet penetrating [*subintrantes*] the dimensions of sensible bodies,"[43] which is impossible, because two bodies would be in one place.[44] Further, such a void-space does not *contain* the placed, but only *coincides* with it.

This containment springs from the quantity implied in the account of place (St. Thomas calls place an extrinsic measure of the mobile).[45] One of the first attempts to put place in a genus runs into an *aporia*: "On the one hand, then, [place] has three dimensions, length, breadth, and depth, by which all body is defined. But it is impossible that place be body, for then there would be two

[40] See ibid., 211b11-14. Aristotle has already noted previously the sense of "in" that corresponds to form: 210a22.
[41] Ibid., 212a20.
[42] See St. Thomas, ibid., lect. 6, nn. 13-17; esp. n.14 (Leon.2.165) on the complete account of this immobility, and see Coughlin, "Place," 258-59. Concerning body and natural place, see St. Thomas, *In Phys.*, lib. 4, lect. 8, n. 6 (Leon.2.171). This is related to the reason for why void (akin to Newtonian absolute space) could not provide a natural place; see ibid., lect. 11, n. 3 (Leon.2.181): "Et haec eadem ratio valet contra eos qui ponunt locum esse quoddam spatium separatum, in quod corpus mobile fertur. Non enim erit assignare quomodo corpus positum in tali loco, vel moveatur vel quiescat: quia dimensiones spatii nullam habent naturam per quam possit attendi similitudo vel dissimilitudo ad corpus naturale."
[43] St. Thomas, *In Phys.*, lib. 4, lect. 7, n. 3.
[44] This parallels the reasoning against void, which involves two bodies being in one place, introduced later by St. Thomas, see ibid., lect. 13, n. 1 (Leon.2.189-90).
[45] A measure is what makes the quantity of something known. See St. Thomas, *In Phys.*, lib. 4, lect. 1, n. 1 (Leon.2.146), and *In Meta.*, lib. 5, lect. 15, nn. 977-86, 310-12; and lib. 10, lect. 2, n. 1938, 559.

bodies in the same place."[46] This *aporia* is removed by seeing place as the *limit* of the containing body, and place is further distinguished from body by being immobile. The notion that place is a quantity (Aristotle puts it in the category of continuous quantity[47]) is preserved by the extension in three dimensions implied by the limit of the surrounding body. Thus, it is not necessary that there is a place of a point, since a three dimensional limit cannot contain a point, there being no adjacent limiting surfaces to the partless.[48] According to containment, place can only properly contain a body. However, it may be another line of argument to say that, because of what motion is, what is subject to motion according to place must be divisible.

(5): Certain things about the qualitative division of motion are the last of the remote principles to be explained. (A): Aristotle states in *Physics* 5.1 both that, "every motion [*kinêsis*] is from something and to something,"[49] and, "every change [*metabolê*] is from something to something."[50] That this is the case is clear from the definition of motion and the implied order in motion from the current act (the *that from which*) to the last act (the *that to which*). This nature of motion manifests that these *termini* are other than the subject of motion.

> And this is clear through this, that that in which motion is, moves. However, the *terminus* of the motion neither moves [another] nor is moved, whether the *terminus* of motion is the species (as in alteration), or the place (as in local motion), or the size (as in growth and diminution). But the mover moves the subject that is moved to what it is moved, i.e., to the *terminus ad quem*. Since, therefore, motion is in the subject that moves, but not in the *terminus*, it is manifest that the mobile subject is other than the *terminus* of the motion.[51]

Aristotle has already argued that motion is in the mobile. Hence the subject and terms of the motion are other, for the one moves, the other doesn't. This shows that the *ratio* of the mobile (the subject

[46] Aristotle, *Physics*, 4.1, 209a5-7.
[47] That place is a quantity, distinct from place as "where," see *Categories* 6, 4b22ff; place is a quantity to the logician because it is a sort of measure, see *In Meta.*, lib. 5, lect. 15, n. 986, 312: "Locus autem ponitur ibi [*Praedicamentis*] species quantitatis, non hic [*Metaphysica*], quia habet aliam rationem mensurae, sed non aliud esse quantitatis."
[48] See Aristotle, *Physics*, 4.5, 212b24-25; and see St. Thomas, *In Phys*., lib. 4, lect. 8, n. 3 (Leon.2.170-71). This relies on one of the proofs that the continuum cannot be composed of points, considered below.
[49] Ibid., 5.1, 224b1. St. Thomas's Latin translation has "*motus*."
[50] Ibid., 225a1. St. Thomas's Latin translation has "*mutatio*." This variation makes sense on St. Thomas' reading, for the first case lists four things required for motion, and the second distinguishes change generally from motion strictly.
[51] St. Thomas, *In Phys.*, lib. 5, lect. 1, n. 5 (Leon.2.229).

of motion) is other than the *ratio* of the *termini* (which includes *place* as a term).

(B): Aristotle treats of the unity of motion in *Physics* 5.4. St. Thomas notes that both the indivisible (e.g., the unit) and the continuous (e.g., a line) can be called one simply:

> Motion cannot be called simply one as the indivisible, since no motion is indivisible. Whence it remains that it is called one in this way, as a continuum, and that it is this way that motion is one simply, that it be a continuum, and the very continuity of motion suffices for its unity. For it follows that if it is continuous, then it is one. Therefore whatsoever are required for the continuity of motion, are required for its unity.[52]

Motion is not one as a unit because it is divisible. This appeals to the quantitative divisibility of a motion. The rest of the argument connects a motion's unity and its continuity. These requirements for continuity of a motion are part of the formal aspect of a motion. So, the order of consideration here is from what makes a motion one as a continuum to the nature of that continuum (in Book 6). Motion is one simply when it is one in number and not just one in kind,[53] for which three things are required: unity in species, unity in subject, and unity of time. Unity in species is required lest the motion be two contiguous motions, "for a man running might fall sick right away, and, like the torch relay race, the locomotions are contiguous, but not continuous."[54] The motion accruing to the runner is not one because it is divided in species. Nor is running back and forth in the relay race one motion for the variation in direction varies the species.[55] Rather, continuity is assured when the possible extremes within a motion are one, which occurs if all these extremes are the same in species and have one final terminus. Unity in subject is required because diverse subjects can be contiguous but not continuous. A subject of motion (considered as the runner and the torch) changes at each relay station.[56] Unity in time is required for "in the time in which [the motion] is interrupted, it is necessary

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[52] St. Thomas, *In Phys*., lib. 5, lect. 7, n. 1 (Leon.2.252).
[53] See Aristotle, *Physics*, 5.4, 227b21ff; St. Thomas, *In Phys*., lib. 5, lect. 6, n. 5 (Leon.2.249).
[54] Aristotle, *Physics*, 5.4, 228a27-29.
[55] See Themistius, *On Aristotle: Physics 5-8*, trans. R. B. Todd (London: Duckworth, 2008) 36, and St. Thomas, *In Phys*., lib. 5, lect. 6, n. 4 (Leon.2.248): ". . . ad hoc quod motus sit idem specie, non solum requiratur identitas termini secundum speciem, sed etiam identitas eius per quod transit motus."
[56] See St. Thomas, ibid., lect. 7, n. 3 (Leon.2.252); see also John Philoponus, *On Aristotle's Physics 5-8*, trans. P. Lettinck (Ithaca, NY: Cornell University Press, 1994) 59-60 (566.1ff).

that [the mobile] rest."[57] If and only if these three are present is the motion one and continuous. A runner running towards the finish line in a stretch of time has one continuous motion.

(C): The last remote prerequisites are the definitions of "together according to place," "separate," "touching," "between," "in succession," "contiguous," and "continuous," in *Physics* 5.3. St. Thomas notes that these definitions are laid down "as also in the beginning of Euclid definitions are posited, which are the principles of the subsequent demonstrations."[58] These definitions are used through the rest of the book (as in the consideration of contiguity or continuity of motions above) and they are also used at the beginning of *Physics* 6.

The first three definitions form a group. Things are "together according to place . . . which are in one first place," and they are "separate" or apart "which are in a different place."[59] The notion of first or proper place specifies those things that are together. Two men in a room are "together" but only in a common place, whereas a body and its parts are together in the same proper place.[60] Separate is the opposite of together. Together is placed in the definition of "touching," which is "those of which the ends are together."[61] The next four definitions also form a group, for "between" is placed in the definition of "in succession," which is in the definition of "contiguous," which is in the definition of "continuous." The "between" is "that [into] which the moving thing is naturally apt to reach before it changes to the extreme, changing continuously according to nature," which is explained by, "The thing between is one of at least three things: for the contrary is the extreme of the change."[62] That is, since every motion is from something, to something, what is between is that

[57] Aristotle, *Physics*, 5.4, 228b4. Note that this requires that the division of the time of a motion be an actual division.
[58] St. Thomas, *In Phys*., lib. 5, lect. 5, n. 1 (Leon.2.244). Compare Philoponus, *On Aristotle's Physics 5-8*, 51 (551.8ff).
[59] Aristotle, *Physics*, 5.3, 226b21-22.
[60] See Philoponus, *On Aristotle's Physics 5-8*, 46 (538.4-19); and H. G. Apostle's commentary in Aristotle, *Aristotle's Physics*, trans. H. G. Apostle (Bloomington: Indiana University Press, 1969) 273.
[61] Aristotle, *Physics*, 5.3, 226b23.
[62] Ibid., 226b24-25. Translation slightly emended with "that [into] which" to reflect St. Thomas' "in quod." Sachs has "that at which" and Apostle "that to which," which reflect more of the accusative sense. See Aristotle, *Aristotle's Physics: A Guided Study*, trans. J. Sachs (New Brunswick, NJ: Rutgers University Press, 2004) 139; and Aristotle, *Aristotle's Physics*, 95.

which is neither of these extremes. So in a change of color from white to black, gray falls in between. Those are "in succession . . . which, being after the beginning either by position or in species or in something else thus determined, has nothing among things in the same genus between itself and that to which it is in succession."[63] In a row of houses, one house succeeds the previous despite the trees or yard that might intervene, for these are not of the same kind. Many examples can be given here that meet the notion of orders in position (succeeding towns or lines), in species (two succeeds one, a species succeeds its proximate genus, and earth succeeds water in the order of natural places), or something else (as things ordered by art, such as the parts of a book, as the introduction and exposition, or in the orders of power, dignity, or knowledge).[64] Succession falls into the definition of "contiguous," which "is what touches, being successive,"[65] that is, succession is a genus for contiguous, the difference being "touching." Two physical bodies are contiguous when their extremities touch. The continuous adds to the notion of contiguous, for

> The 'continuous' is what is indeed something contiguous, but I call a thing 'continuous' when, in those which touch, the limit of each comes to be one and the same, and, as the name signifies, are held together.[66]

This definition of the continuous is an expanded version of the one given at the beginning of *Physics* 6, "that of which the extremes are one," which is also implied in the *Categories*.[67] It contrasts with the other definition of the continuous, that which is divisible into ever divisible parts.[68] It is the potential extremes of the parts of a line, surface, or solid that are one, as are the parts of a living thing or parts of living things that grow together, as a grafted tree limb.

What is successive need not be contiguous, but the contiguous must have successive parts,

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[63] Ibid., 226b35-227a2.
[64] See Philoponus, *On Aristotle's Physics 5-8*, 48-49 (543.6-544.5); St. Thomas, *In Phys.*, lib. 5, lect. 5, n. 6 (Leon.2.245)
[65] Aristotle, *Physics*, 5.3, 227a9.
[66] Ibid., 227a11-13.
[67] Ibid., 6.1, 231a22; see *Categories* 6, 5a1-5.
[68] See Aristotle, *Physics*, 3.1, 200b20 and 6.1, 231b17.

and likewise for the contiguous and the continuous.[69] Now, the contiguous and the continuous differ in that their limits are actually or only potentially two. It is a question, then, whether two mathematical solids can be said to be contiguous. The answer brings to the fore the fact that these definitions are within physics.[70] Not only are these definitions framed in terms of place and hence examples given in terms of "placeable" things, but the diversity of contiguous extremes is something that can only be realized in physical things. This physical nature of contiguity, the distinction of adjoined limits, is then used (in *Physics* 6.1) to eliminate certain claims about the continuum. Two polished plates of glass pressed together in an air pressure experiment are contiguous, for their surfaces, while in the same place, are still actually distinct. The case is otherwise for a limb grafted onto a tree: there the division between the extreme of the limb and the tree exist only potentially. However, if two mathematical solids are said to be contiguous, then they will have extremes that are in the same place. That is, they will be joined at a single surface. However, this surface belongs to one solid no more than to the other. Therefore, the definitions of *Physics* 5.3 are physical.[71]

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[69] This is Aristotle second sense of before or prior, by nature or in being, see *Categories* 12, 14a29-35.

[70] See Apostle, in *Aristotle's Physics*, 273 (#1): "These attributes are considered by physics rather than by mathematics because place enters into their definition; and the nature of place is considered by physics, for it is defined in terms of a body, which is considered by physics." See also 274 (#12). See also James Jope, "Subordinate Demonstrative Science in the Sixth Book of Aristotle's Physics," *The Classical Quarterly* 22 (1997) 282-283.

[71] Due to space, I must pass over two difficulties. The first is that the definition of "between" contains "continuously," yet the notion of continuous depends upon contiguous, which depends upon that of successive, in whose definition "between" is placed. Some commentators answer this by distinguishing between a vague and definite notion of a continuous motion: the definite notion is developed in *Physics* 5.4. See Themistius, *Aristotle: Physics*, 32 (173.8): "But what is in and of itself continuous must be separately defined with greater precision." Also, Nieto, "Continuity and the Reality of Movement," 170, who holds that, "continuously" is used here "in a pre-scientific sense and for the purpose of turning our attention to the experience in which we become aware of the 'between'." St. Thomas notes the preference given to time in the explanation of between, which seems to make the consideration less rigorous: see *In Phys*., lib. 5, lect. 5, n. 4 (Leon.2.245). The second difficulty is that the notion of "place" used in the definition of "together" is in the definition of "touching," which implies that the limit between touching surfaces is in place. David Bostock raises this difficulty: see "Aristotle on Continuity in *Physics* VI," in *Aristotle's Physics: A Collection of Essays*, ed. by L. Judson (Oxford: Clarendon Press, 1991) 181, n.5. Some answer it by replacing "together" with "coincident". See Philoponus, *On Aristotle's Physics 5-8*, 46-47 (539.2-5); and Apostle, in *Aristotle's Physics* (translator's commentary) 273: "The whole boundaries of two or more things cannot be together; and since boundaries are not bodies, they are not in a place except accidentally, i.e., as attributes of bodies which (bodies) are in a place essentially. Perhaps 'coincident' is better than 'together'; anyway, 'together' must not be confused with 'together in place.'" The difficulty, however, is merely shifted to coincidence (*co-incidere*, to fall together). Consider Aristotle, *Physics*, 4.5, 212b27: ". . . place is also somewhere, but not as in a place, but as a limit in the limited." See also Nieto, "Continuity and the Reality of Movement," 168: "This would not be the most proper, but an analogous, sense of place, one appropriate to surfaces."

§2: Proximate Principles for the Demonstration

In this section, I will consider (1) the composition of the continuum, (2) the proposition that nothing at once moves and has moved, and (3) that all motion takes time. I will also raise certain difficulties concerning the quantitative treatment of motion. While Book 5 treats of motion as to its species (looking to the whole motion—its indivisibility), Book 6 treats motion in its quantity (looking to motion's divisibility). St. Thomas orders Book 6 as follows. The opening of the book is dedicated to showing that no continuum is composed of indivisibles, nor is itself indivisible, which preserves the continuity of motion. Then, before treating of the division of motion, two preparatory measures are necessary: showing that there is no motion or rest in an instant, and showing that what moves is divisible. Thus, granted that no continuum is composed of indivisibles, these show that there is no motion to investigate within the now, nor motion of a point. The remainder of the book then demonstrates a series of theorems about the quantitative parts of motion. St. Thomas again finds a unity within Book 6 in terms of its place within the *Physics* as a whole.[72]

(1): It belongs to physics to show that the continuum is not composed of indivisibles.[73] The question is this: for the continuous wholes of common experience, which have parts with unified extremes and are divided into what they are composed of, are they composed of ever divisible or indivisible parts?[74] Aristotle presents four arguments, two of which (the first and fourth) will be examined here. The first is based on the definitions of "continuous" and "together." To compose one quantitative whole, parts must be joined either by way of continuity or contiguity. However, [a]

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[72] This contrasts with other, more recent interpretations of *Physics* 6. See Sarah Waterlow, *Nature, Change, and Agency in Aristotle's Physics* (Oxford: Clarendon Press, 1982) 131-32, concludes that the definitions of motion in Book 3 and 6 are incompatible; also Bostock, "Aristotle on Continuity," 179-80 and 204, finds the book riddled with inconsistencies and mistakes, and again a disconnect between the treatment of motion in Book 3 and 6 is the culprit (180): "A simple hypothesis suggests itself: Book VI was written before the doctrine of Book III was worked out."

[73] See St. Thomas, *In Phys.*, lib. 1, lect. 7, nn. 3-4 (Leon.2.22-23); also, *Exp. Po.An.*, lib. 1, lect. 5, (Leon.1*/2.25:144-52): "Sunt enim quaedam propositiones, quae non possunt probari nisi per principia alterius scientiae; et ideo oportet quod in illa scientia supponantur, licet probentur per principia alterius scientiae. Sicut a puncto ad punctum rectam lineam ducere, supponit geometra et probat naturalis; ostendens quod inter quaelibet duo puncta sit linea media."

[74] See Simplicius, *On Physics 6*, 17; where he presents this division based on this "common notion" about composition.

continuity requires that the limits of the parts be one, and [b] contiguity requires that the limits of the parts touch by being together.

> [a] But the limits of points cannot be one, because a limit is said with respect to some part. However, in the indivisible there is not something that is a limit, and something else that is a part. [b] Likewise, it cannot be said that the limits of points are together, because nothing can be the limit of a partless thing, since the limit and that of which it is the limit are always other. However, in the indivisible there is no such otherness.[75]

Hence, neither by way of continuity or contiguity can the continuum be composed of points. St. Thomas is relying here on a definition of point like that of Euclid's: the point is that which has no part.[76] The point, without otherness in part and limit, is excluded from both modes of composition.

The fourth argument uses a *reductio* based upon one definition of the continuum (what is divisible into parts always divisible). This argument is supported by proving this definition by the other (what has parts the extremes of which are one).[77] The argument itself is as follows:

> From whatsoever a line or a time is composed it is divided into. If, therefore, either of these is composed of indivisibles, it follows that it is divided into indivisibles. But this is false, since no part of a continuum is divisible into indivisibles, for thus it would not be divisible *in infinitum*. So, no continuum is composed of indivisibles.[78]

The argument by *modus tollens* is clear enough. Aristotle defends the definition as follows:

> It is apparent, too, that every continuous thing is divisible into things which are always divisible. For if [it were divisible] into indivisibles, indivisible will be touching indivisible. For the extreme of continuous things is one and touches.[79]

A divisible continuum requires parts that are *here* and *there*: parts outside of parts. (Note that no distinct notion of place is required to understand this definition.) So composing indivisibles must at least touch each other, for the limits of the parts of a continuum are one. This requires contiguity (touching) and hence being together in place. But the first argument denies them touching and contiguity. Thus, they fail to meet the second definition of continuity, which thus defends the first. In

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[75] St. Thomas, *In Phys.*, lib. 6, lect. 1, n. 3 (Leon.2.268).
[76] See Thomas Heath, *Mathematics in Aristotle* (Oxford: Clarendon Press, 1949) 89; Aristotle, *Metaphysics* 5.6, 1016b24-26.
[77] Aristotle allows this mode of defense. See *Posterior Analytics*, 2.8, 93a10-14; *Physics*, 231b19 and Coughlin, 140, n.5.
[78] St. Thomas, *In Phys.*, lib. 6, lect. 1, n. 6 (Leon.2.268-69). See Aristotle, *Physics*, 6.1, 231b11-12.
[79] Ibid., 231b16-18.

this case, the formal definition of a continuum (that which has parts the limits of which are one) shows that the material definition (what is divisible into ever divisibles) is true. St. Thomas compares these two definitions both in terms of composition and resolution as well as formal and material.[80]

Finally, concerning these arguments it should be noted that they apply to the composition of any species of continuum by its lower dimension (i.e., lines of points, surfaces of lines, solids of surfaces). For all of these are *partless* and hence *indivisible* with respect to the higher dimension in question (i.e., a line is *breathless* length, a surface *depthless* breadth and length). Hence the middle terms of the two arguments apply in the lacking or handicapped dimension(s).

(2): It is also "necessary that what moves whence and whither be not at once moving and have moved whither it was moving when it is moving."[81] In the argument of *Physics* 6.1, this is the second of two preliminaries for the argument that "if a magnitude is composed of points, then motion will not be composed of motions, but from 'moves' [*ex momentis*]."[82] When someone is walking to Thebes, one cannot be walking and have walked to Thebes at the same time. St. Thomas notes that this proposition is assumed by Aristotle "as clear through itself [*quasi per se manifesta*]," but still defends it by *reductio*. If one denies this proposition, one must say that two times are

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[80] See St. Thomas, *In Phys.*, lib. 3, lect. 1, n. 3 (Leon.2.102); *In De Caelo*, lib. 1, lect. 2, n. 2 (Leon.3.6). Thus, moving from the compositive to resolutive (or synthetic to analytic) notion of the continuum is not an unsupported generalization as Hans-Joachim Waschkies holds, "Mathematical Continuum and Continuity of Movement," in *La physique d'Aristote*, 153 and 175, n.81. The definition of the continuum need not be the conjunction of both; see Bostock, "Aristotle on Continuity," 183-84.

[81] St. Thomas, *In Phys.*, lib. 6, lect. 2, *vetus*, text 7 (Leon.2.270): ". . . necesse est quod movetur unde et quo, non simul moveri et motum esse, quo movit quando movet . . ." See Aristotle, *Physics*, 6.1, 231b29-30.

[82] St. Thomas, *In Phys.*, lib. 6, lect. 2, n. 3 (Leon.2.271). See Aristotle, *Physics*, 232a7, and Coughlin, 140, n.6, on the translation of "*momentis*" or "*kinêmatôn*" as "moves": "In chess, the actual motions of the pieces between their squares are of no interest; the players are only interested in where the pieces end up and where they started. The translation 'moves' is an attempt to use a word with the same root as 'motion' (because the Greek words too share a root), while indicating that the change is sudden and there really is no motion between the terms, but the mobile is now here and then there." This argument is close kin to the argument of *Physics* 6.4. The former argues that, a mobile moving through a path composed of indivisible places having indivisible moves corresponding to each place, would violate the prior principle that a moving thing cannot be moving and have moved at once. For a motion of moves could have no duration in any of its places, for then they would not be divisible. It then only has moved in each of its places, but then it can no longer be moving. So, from indivisibility one argues to the destruction of the structure consequent to motion. That is, *one motion no longer exists*. The latter argues from the structure consequent to motion that the mobile is divisible (and hence does not move through indivisibles except accidentally). This preserves the *unity* of motion.

simultaneous. One would have to say "it is" and "it was" simultaneously and in the same respect.[83]

(3): It is also the case that all motion and rest takes time, or conversely, that there is neither motion nor rest in an instant, i.e., "in the now." The argument about motion will be considered.[84] This argument assumes three prior theorems: that the now is indivisible, that a slower mobile traverses a lesser space in the same time as a faster mobile, and that a faster mobile traverses the space a slower mobile does in less time.[85] It follows from these

> that in the now nothing can move. Because if something could move in the now, it will happen that in the now two mobiles move, of which one is faster, the other slower. Let the now be *n*, and some faster body move in *n* through magnitude *ab*. But the slower moves through less in an equal [time], therefore the slower in this instant moves through a smaller magnitude which is *ag*. But the faster traverses the same space in less [time] than the slower. Therefore, because the slower body moved through magnitude *ag* in the whole now, it follows that the faster moves through the same magnitude in [a time] less than the now: therefore the now is divided. But it was shown that the now is indivisible, therefore something cannot move in the now.[86]

The *reductio* of this argument is clear enough. To deny the immediate premises of the argument (concerning the faster and slower mobiles) would be to deny faster and slower altogether.

I will now consider several objections to the proof of *Physics* 6.4. The first is a general difficulty with Aristotle's method. Bostock finds several of Aristotle's conclusions unpalatable, e.g., that nothing moves and has moved at the same time and in the same respect. He holds that this arises from the fact that "our usual vocabulary for talking of movement is not well adapted to [the opposite] possibility,"[87] namely that motion is made up of 'moves,' and hence a mobile can be moving and have moved in such a way. Likewise he objects to Aristotle's exclusion of motion from the now, for "We may surely say that the body is moving in any instant that falls within the one sub-

[83] See St. Thomas, *In Phys*., lib. 6, lect. 2, n. 4 (Leon.2.271), and Aristotle, *Physics*, 4.10, 218a9-21.
[84] The first argument about rest is from rest as a privation of motion, and hence follows upon the argument about motion.
[85] See, respectively, Aristotle, *Physics*, 6.3, 233b33-234a24, 6.2, 232a28-32, and 6.2, 232b6-14.
[86] St. Thomas, *In Phys.*, lib. 6, lect. 5, n. 8 (Leon.2.284). See Aristotle, *Physics*, 6.3, 234a24-31.
[87] Bostock, "Aristotle on Continuity," 189. Bostock's critique is difficult to compare with Aristotle's account since he assumes a Dedekindian account of continuity, and hence motion is better conceived as the continuity of a function between time and space: see 188, fn.13 and 179, 185-87, and 208. This follows Bertrand Russell: see *Principles of Mathematics* (Cambridge: University Press, 1903) 473; and Nieto, "Continuity and the Reality of Movement," 26-55.

stretch [of its motion], . . . All that is needed is a *decision* as to what to say about these special instants of change."[88] He adds that "it is of course perfectly common usage to speak of a thing as moving with a given speed, or as being at rest, *at* this or that instant, and it is usage which Aristotle himself falls into without noticing it."[89] Bostock concludes that Aristotle was mistaken to argue to such an empirical truth based merely on our speech, "not only because the notion of an instantaneous velocity is of course crucially important to Newtonian physics," but also because "it is a perfectly common-sense notion, and one which it is difficult to avoid when stating Aristotle's own position."[90]

Bostock also raises an objection directly against the argument in *Physics* 6.4. He pinpoints Aristotle's qualification that the argument relies on the first term (the "next") of the change:

> But here of course we may object that there need not be any such "first" state, and there cannot be if, as with motion, the states intermediate between A and B form a continuum. Indeed, Aristotle himself is going to argue for that very point in the next two chapters. So this argument . . . must be dismissed as worthless.[91]

Bostock refers especially to the argument in *Physics* 6.5 (236a7-26). He also objects against the contrapositive argument in *Physics* 6.10, that a point cannot move. He notes:

> Although [Aristotle] calls such a thing a point, it is perfectly clear that his argument does not in fact show that a genuine point, which occupies *no* spatial distance, is incapable of moving. At best it shows that if an extended thing can move (in the direction in which it is extended), and if motion is continuous rather than 'cinematographic', then that thing must be divisible, at least in thought. But if we consider a genuinely unextended thing, such as the point at the tip of an arrow, it is perfectly evident that this can move, even though it has no parts. (Similarly, a surface of a cube can move in a direction in which it is not extended.)[92]

The reply that the tip of the arrow is in motion *per accidens* Bostock dismisses by saying that motion *per se* was not assumed as a premise. The difficulty of *conceiving* the possibility of the motion of a point he construes as actually about the difficulty of *perceiving* the motion of a point:

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[88] Bostock, "Aristotle on Continuity," 193.
[89] Ibid., 195.
[90] Ibid.
[91] Ibid., 201. Note that St. Thomas also raises this objection: see *In Phys*., lib. 6, lect. 5, n. 19 (Leon.2.287).
[92] Bostock, "Aristotle on Continuity," 203.

> But then the correct conclusion to draw seems to be that an unextended thing cannot be identified *per se*, though there would be no difficulty in supposing it to move *per se*. *For there is not, as Aristotle apparently claims, anything in the concept of motion that requires a moving thing to have parts*. (Thus if, for example, the material world were wholly made up of unextended centres of force—as Boscovich suggested—there would be no conceptual impediment to their moving.)[93]

This is the key issue: does the nature of motion require something of the mobile in which it inheres?

For the sake of the intended comparison to the Newtonian conception of motion, mobiles, and bodies, the following two difficulties can also be raised. First, one could grant Aristotle the fact that the mobile cannot be a point. However, what about a line or a surface? Are they excluded from moving? Yet if so, then what is it about the three-dimensionality of a mobile that allows it mobility? Indeed, one could press the issue and ask what is it about a physical body such that it can move, whereas a mathematical body, a *stereon*, cannot? Second, one could grant that a mathematical body cannot really move and that only bodies existing in matter can move. Yet, just as quantity can be considered abstractly otherwise than it exists in matter, so also a body in motion could be considered otherwise than it exists: merely in its quantitative aspect. There is a unique degree of abstraction that renders sheer quantity in motion intelligible. Consider Newton's own proposal:

> Moreover, since body is here proposed for investigation not in so far as it is a physical substance endowed with sensible qualities, but only in so far as it is extended, mobile, and impenetrable, I have not defined it in a philosophical manner, but abstracting the sensible qualities (which philosophers also should abstract, unless I am mistaken, and assign to the mind as various ways of thinking excited by the motions of bodies), I have postulated only the properties required for local motion. So that instead of physical bodies you may understand abstract figures in the same way that they are considered by geometers when they assign motion to them, as is done in Euclid's *Elements*, Book I, 4 and 8.[94]

It is a question, however, whether by "the properties required for local motion" are meant those which make motion able to exist or able to be understood. Yet if either were true, *Physics* 6 as physics would be (at best) superfluous, improper, or (in the worst case) false.

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[93] Ibid. Emphasis added.

[94] Isaac Newton, "De Gravitatione" in *Philosophical Writings*, ed. A. Janiak (Cambridge: University Press, 2004) 13; a revised translation from *Unpublished Scientific Papers of Isaac Newton*, ed. by A. R. and M. B. Hall (Cambridge: University Press, 1962).

§3: The Demonstration of *Physics* 6.4 and Conclusions About *Corpus*

From the text of the argument as St. Thomas comments on it, quoted above, the consequence of the argument can be seen in light of the principles reviewed. Every change is from something and to something. This is necessary from the nature of motion, a specification of the "structure" implicit in motion due to the order to the last act and the order to the current potency. Given this structure, five options are logically possible for the mobile. The mobile can either be wholly in the *terminus ad quem*, wholly in the *terminus a quo*, wholly in both terms, in neither term, or partly in one and partly in the other term. The question is, which options allows for the full *ratio* of motion, such that the mobile is actually moving? St. Thomas notes about the first four options that

> Aristotle in his demonstration does not use as a principle that every change is divisible, since rather, conversely, from the division of the mobile he proceeds to the division of motion, as is clear below. In the same way he later says that divisibility is in the mobile prior to the motion or change. But he uses principles *per se* known which are necessary to concede in whatsoever change: namely, that which changes, when it is according to whole and parts in the term from which it changes, does not yet change according to that change, and when it is in the term to which, it does not change but has changed, and that it cannot be wholly in both nor in neither, as was explained. Whence of necessity it follows that in whatsoever change, what changes, while it changes, is partly in one term and partly in the other.[95]

It is not the divisibility of motion that proves that the mobile must be divisible. Rather, what is known *per se* upon the nature of motion drives the demonstration. This is simply what the structure of motion dictates about the mobile in regard to the *termini* of the motion. That is, the first four options are eliminated because they exclude the possibility of motion, while the fifth allows for it.

If the mobile is wholly in the *terminus a quo*, it cannot be in motion for this excludes the order to the last act as such. If the mobile is wholly in the *terminus ad quem*, it cannot be in motion for this excludes the order to the current act of a potency as such (i.e., the mobile would be resting, or violating the principle that one cannot be moving and have moved simultaneously and in the same

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[95] St. Thomas, *In Phys.*, lib. 6, lect. 5, n. 14 (Leon.2.285).

respect). The mobile cannot be wholly in both terms, for it would then have two acts that negate each other—it would be in two places at once. (This also denies the premise *ex quidam in quoddam*, for "if it were at once in both termini, it would not be moving from one to the other."[96]) Finally, the mobile cannot be in neither term, "for we are now speaking of the next term to which it changes,"[97] i.e., the change from white to gray, or:

> Likewise, if one line is divided into three equal parts, namely the line *abcd*, it is clear that the mobile, which in the beginning of motion is in the part *ab* as in a place equal to itself, happens in some part of its motion to be neither in *ab* nor in *cd*—when it is wholly in *bc*. Since, therefore, it is said that that which changes, when it changes, cannot be in neither [term], the last term is not taken, but the next [term].[98]

Thus, only the fifth option remains. St. Thomas presents this as the minor term: "Every thing that changes, while it changes, as to something of itself is in one, and as to something of itself is in another."[99] This can be rephrased as *every thing that is changing is partly in one and partly in another*.[100] The fifth option makes the predicate *per se* (without a middle term) due to the very nature of motion. Thus motion's nature is the cause of the conclusion. The implied major term is *every thing that is partly in one and partly in another is divisible*. This premise is *per se* because the divisible is what has part outside of part. St. Thomas, reiterating the middle term, then draws the conclusion: "Therefore, it is manifest that *every thing that is changing*, since it is partly in one and partly in another, *is divisible*."[101]

To see the consequence of this argument, the most obvious objection must be answered. It can be put in two ways. First, it seems that the stipulation that the mobile in the *terminus a quo* rests there as a whole with all its parts begs the question. Second, since a next place is assumed, this also seems to beg the question. St. Thomas puts it slightly differently: "It does not seem possible to take a

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[96] St. Thomas, *Sent. De Sensu*, c.15, (Leon.45-2.82:56-58). I owe these references to the *De Sensu* to David Grothoff. See also *Commentaries On Aristotle's On Sense And What Is Sensed And On Memory And Recollection*, trans. by K. White and E. M. Macierowski (Washington, D.C.: Catholic University of America Press, 2005).
[97] St. Thomas, *In Phys*., lib. 6, lect. 5, n. 14 (Leon.2.285).
[98] Ibid.
[99] Ibid.
[100] The necessary specification "while it changes" allows the use of the present progressive aspect.
[101] Ibid. Emphasis added for uniformity.

first into which it changes because of the infinite divisibility of magnitude."[102] How does the argument not beg the question by this 'next'? Note that this objection is restricted to local motion.

There are two parts to the answer.[103] First, is motion possible at all without a next place? In other words, can one leave a place *simply by ceasing to be at that place*? It is certainly the case that when one leaves a place one ceases to be there, just as one is no longer a quality, shape, or kind of thing when one ceases to be such. Yet in fact, one does not come to be somewhere else by ceasing to be where one is, but rather ceases to be where one was by coming to be somewhere else: "Leaving a place is temporally simultaneous with entering another place, but entering is prior to leaving in notion: a mobile leaves insofar as it enters some other place."[104] This is in accord with the principles of change. The form or last act of the motion is prior by nature in the motion. The beginning of the motion is a privation of this form, and so it is only a principle accidentally. One denying a 'next place' makes matter non-being, for one is then forced to say that insofar as one ceases to be one comes to be, but matter is ungenerable and indestructible.[105] Therefore there must be a next place, otherwise motion is impossible. Now, one could continue and argue that a point cannot be ordered to a next place, for there is no next point-place for a point.

> For a divisible mobile, on the other hand, there is a next place which is equal to it, and the ordered parts of the mobile can account for the ordering of the parts of the motion in going from one place to the next, and then to the next, etc. When the divisible mobile moves, it is only moved as a whole when it attains to such a next place; before that, it has only moved partially, and such a partial motion is in notion posterior to the motion of the whole, for a partial motion is partial because the whole is what it is, and the parts are only potential. Since a point has no parts, there would be no order in the parts of its motion, and it could never undergo a partial movement, but would instead be suddenly and all at once in a place distant from the one it was in.[106]

If one denies a next place, one denies motion by removing its principles, so there must be a next

[102] Ibid., n. 19 (Leon.2.287).
[103] My answer draws upon a reply to this objection in the lecture "The Proof for the Existence of God in Book VII of the *Physics*," given by Mr. Marcus Berquist at the 2010 meeting of the Society for Aristotelian Studies.
[104] Coughlin, translator's notes in Aristotle, *Physics*, n.19, 141. Compare to Aristotle, *Physics*, 6.5, 235b10.
[105] See Aristotle, *Physics*, 1.9, 192a26-29.
[106] Coughlin, translator's notes in Aristotle, *Physics*, n.19, 141.

place. While there cannot be a next place for a point due to the composition of the continuum, there can be a next place for a body. Therefore, what moves must be divisible.

Rather than present this argument (which does not show the cause), Aristotle *assumes* that there must be a next place (lest one deny the existence of motion), and proceeds to show that the mobile must be divisible. How, then, does the argument avoid circularity? This is the second part of the answer. One must not confuse what the imagination contributes to the argument with the argument itself, which only the mind can attend to. St. Thomas himself provides us with the *image* of the line *abcd*. However, the *reason* why a mobile must be divisible is not the nature of the *terms* of the mobile's motion, but the nature of *motion*. Recall that the mobile is other than the *termini*. Hence, one can require something of the mobile in which motion inheres *qua* independent of the determinate nature of the *termini* to which the nature of motion refers. Now, motion as an imperfect act requires that the mobile be in a way *here* and in a way *there*. "Here and there" need not be determinate to the nature of places as *termini* (i.e., limits of containing bodies), but merely import a recognition that this change is according to place conceived as quantity. It is *motion* as an imperfect act that requires the same numerical subject-mobile to have *both* a here *and* a there—otherwise it is not in motion. This is to be divisible, to be partly in one, and partly in another—having part outside of part.

Therefore, every mobile insofar as it moves must be divisible. Since the divisible cannot be composed of indivisibles, the mobile must be a continuum. Now, if a mobile lacked a dimension, it would be handicapped in that dimension insofar as it would be indivisible in that dimension. Hence, a perfect or complete (*teleion*) mobile must be divisible in every possible way. As Aristotle argues in the beginning of the *De Caelo*, what is divisible in every way, and hence a *teleion* magnitude, is a body.[107] Therefore, every thing that moves is a body. This harmonizes with what St. Thomas asserted

[107] See Aristotle, *De Caelo*, 1.1; St. Thomas, *In De Caelo*, lib. 1, lect. 1 (Leon.3.6-8); on "completeness" see Aristotle, *Metaphysics*, 5.16, and as this relates to "body," Falcon, *Aristotle and the Science of Nature*, 33.

in his *Prooemium*. Body is not the subject of physics, because mobile being is proven to be bodily, due to the nature of motion. This confirms that mobile being is the subject of physics.

Yet does this argument show the divisibility of the mobile through all species of change? St. Thomas answers that this demonstration, like others in *Physics* 6, "pertain to local motion perfectly, while to the other motions not completely, but insofar as they share in something of continuity and regularity."[108] Alteration and growth require qualifications to connect the account of their motion with divisibility.[109] So the proof is "true of local motion simply and absolutely, but of alteration in some way."[110] What is really at stake, however, is how this qualification can be made without making a general conception of motion and the general subject of physics scientifically useless and trite.

Aristotle proposes in *Physics* 6.2 (232b23) that everything moving can be moving both faster and slower. St. Thomas objects that, "This proposition seems to be false. For there are determinate speeds of motions in nature, for there is some motion so fast, that nothing can be faster, namely the motion of the first mobile."[111] One would expect this: the determinate nature of a thing bounds its accidents to what is best for that thing. To solve this, St. Thomas distinguishes between the nature considered as a common nature [*ratio communis*], or as applied to its proper matter or subject.[112] He gives an example of the sun. Nothing in the nature of the sun prevents many suns from existing, but it happens that the entire species is only contained in one subject instance.[113] So, nothing in the common nature of motion prohibits various degrees of speed, although this will be limited given

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[108] St. Thomas, *In Phys*., lib. 6, lect. 5, n. 16 (Leon.2.286).
[109] See ibid.; and St. Thomas, *Sent. De Sensu*, c. 15 (Leon.45-2.87).
[110] St. Thomas, *In Phys*., lib. 6, lect. 5, n. 16 (Leon.2.286). The alteration only reveals something about quantity "in some way" insofar as the quality inheres in quantity, having quantity *per aliud*: see Aristotle, *Physics*, 6.5, 236b5-10.
[111] St. Thomas, *In Phys.*, lib. 6, lect. 3, n. 9 (Leon.2.276). Today one would give light as an example.
[112] See Richard F. Hassing, "Thomas Aquinas on *Physics* VII.1 and the Aristotelian Science of the Physical Continuum," in *Nature and Scientific Method,* ed. D. O. Dahlstrom (Washington, DC: Catholic University Press, 1991) 128-29, and 111, fn. 2.
[113] One could update this example by considering an endangered species of animal with only one surviving member. See also Aristotle, *De Caelo*, 1.9, 278a13: "There is a difference, then, between this heaven and heaven without qualification." Also, St. Thomas, *In De Caelo*, lib. 1, lect. 19, n. 6 (Leon.3.77), and De Koninck, "Abstraction From Matter," 165: "The science of nature cannot be science about this universe of ours qua *this*."

determinate movers and mobiles. He concludes that "frequently such propositions are used in Book 6, which are true according to the common consideration of motion, while not according to its application to determinate mobiles."[114] So, for the common nature of change, every change is from something and to something. This requires that what is changing be partly in one and partly in another terminus. How this pans out in determinate species of motion requires further argument in some cases but not others. Another example would be terrestrial and celestial bodies. The *ratio communis* of local motion in this proof applies to them both equally, despite the fact that they have different proper natural motions and are corruptible and incorruptible substances. Since "there is not the same matter of heavenly bodies and the elements, except according to analogy, insofar as they agree in the *ratio* of potency,"[115] these bodies are not even in the same real genus, and are called bodies analogously.[116] Yet such is the breadth of the scientific genus of mobile being.[117]

Physics 6.10 complements *Physics* 6.4: the indivisible cannot move, except *per accidens*. While the indivisible can move *per accidens* insofar as it is in a body, the physicist cares only for motion *per se*.[118] The second argument in this chapter, particular to local motion, proceeds thus:

> For everything that moves according to place, it is impossible that it first cross [*pertranseat*] a magnitude greater than the mobile itself, rather than an equal or a less. But the mobile must always cross a magnitude equal to or less than itself, rather than a greater. If therefore this is so, it is clear that a point, were it to move, would first cross something less than or equal to itself, rather than a length greater than itself. But it is impossible that it cross something less that itself, because it is indivisible. It remains, therefore, that it cross something equal to itself. And thus it must number every point in the line, since a point always, when it moves with a motion equal to a line, because of this moves through the whole line. It follows that it measures the whole line by numbering every point. Therefore it follows that a line is composed of points. If, therefore, this is impossible, it is impossible that the indivisible move.[119]

This *reductio* points out that mobiles, divisible ones included, first cross lengths equal to or less than

[114] See St. Thomas, *In Phys.*, lib. 6, lect. 3, n. 9 (Leon.2.276). This is physics' *processus in determinando*, see Ch. 1, fn. 12.
[115] St. Thomas, *ST*, Ia, q. 66, a. 2 (Leon.5.156-57); and see *In De Caelo*, lib. 1, lect. 6-7.
[116] See St. Thomas, *ST*, Ia, q. 66, a. 2, ad2^{um}.
[117] See Ch. 1, fn. 71: St. Thomas notes in *SBDT*, q. 5, a. 3, ad8^{um} (Leon.50.151) that physics studies both.
[118] See Aristotle, *Physics*, 5.1, 224b26. Bostock's objection that this does not enter into the premises of the argument fails.
[119] St. Thomas, *In Phys.*, lib. 6, lect. 12, n. 6 (Leon.2.318).

themselves. However, a point cannot cross something less that itself. The point must pass through places equal to itself along its path. Thus, the moving point would count out all the points on the line, and hence would count the points composing the line, were a line composed of points.

Thus it is clear that Bostock's assertion, that there is nothing in the concept of motion that implies the divisibility of the moving thing, fails on two counts. First, negatively, because the motion of an indivisible requires the composition of the continuum from indivisibles, which is impossible. Second, affirmatively, because the nature of motion, as seen in *Physics* 6.4, does require that one subject of motion have both a here and a there.

Physics 6.4 allows for the proof that there is no first in motion. But if it requires a first in motion (the "next" place), how can one show there is no first? The answer is a distinction:

> However, "first in which it has changed" is said in two ways: in one way, as the first in which the change has finished (for then it is true to say that it has changed); in another way, as the first in which the mobile began to change. What is first according to the end of the change, then, both is present and is. For a change can be finished and there is an end of change, which end, then, was shown to be indivisible through its being a limit. But the first according to the beginning does not exist in any way.[120]

The first meaning attends to the *terminus ad quem* of a motion. What has changed has reached the last act of the motion, which means that the mobile has reached the last act as terminus, which terminus is indivisible (otherwise the motion would not yet be complete).

By contrast, the second meaning of a "first in motion," the "in which" or "into which" looks to a first "just after" the *terminus a quo* of a motion which has nothing prior to it. This does not exist in any way, whether in time, in the mobile itself, or in the places moved to, for the continuum is infinitely divisible and not composed of indivisibles. The argument regarding the mobile explicitly uses *Physics* 6.4.[121] Let the mobile be *AB*. Since every mobile is divisible, let there be some part *AC* of it that has moved first (in the second sense), and let *EF* be the time that measures the motion. If

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[120] Aristotle, *Physics*, 6.5, 236a7-14. Compare 6.8, 238b23-239a24: there is also no last of a coming to rest.
[121] See ibid., 236a27-37, and see St. Thomas, *In Phys*., lib. 6 , lect. 7, n. 7 (Leon.2.296).

AC has changed in the whole time *EF*, since both the continuous time measuring the motion and the continuous mobile are divisible, there is a time and part of the mobile the half of *AC* and *EF*, and so on endlessly. Thus, there is no "first part" of a mobile and its motion in the second sense.

The difficulties raised above can now be addressed. Bostock claimed that the nature of speech about motion does not give insight into the nature of motion, which is rather an empirical question. Yet, as evidenced by the exposition of the principle that what is moving cannot have moved simultaneously and in the same respect, our speech about motion, involving changes in tense and an experience of time, is founded on the nature of motion. Speech about motion is thus a sign and aid. The like can be said about the other difficult cases. Speech about these situations is a guide that looks to the nature of motion as measure. Each of these cases is seen to be true because motion is the act of what exists in potency as such. What follows from the definition of motion confirms what speech indicates. Further, Bostock's difficulties with the demonstrations in *Physics* 6.4 and 6.10 were resolved in the course of the demonstration. The nature of motion does indeed demand the mobile's divisibility. There is no conflict with the assumption of a "next place" and the argument that there is no first in a motion (in the second sense), because the next place corresponds to a first in motion in the first sense. The last difficulty, the motion of lines and planes, was also answered in the course of the demonstration. The motion of a mathematical solid, however, and the possibility of a unique abstraction for thinking about its motion will be taken up in the next chapter.

– Conclusion of Chapter Two and Transition

This chapter showed, first, that every thing that is moving is a body, the cause of which is the nature of motion insofar as it makes demands upon its subject. The mobile, considered indistinctly as some thing, is seen to require quantitative parts as the physicist proceeds from the indistinct consideration of one motion and one mobile (a mobile whole) to the parts of that whole, thus seeing it more distinctly as a whole. Second, this consideration of mobile being was seen to be broad enough to include mobile beings called body only analogously. Stepping back from these considerations of the physicist, one can see the evidence that the physicist's subject is mobile being. Analyzing motion leads to knowledge of its principles of being, its definition, place, the unity requisite to motion, and the nature of the subject of motion. This supports St. Thomas's assertion in the *prooemium* that mobile being is the subject of physics. The moments of physics from the assumption of motion's existence to its definition, are the beginning moments, and properly speaking not demonstrated. The moment of *Physics* 6.4 234b10-20 *is* a conclusion reached by scientific discursion from the nature of the subject of physics.

To lead into Chapter 3, one last question about *Physics* 6 must be answered. Are its considerations part of physics or of a middle science? Jope argues for the latter.

> Since, as we have noticed, continuity and extension are mathematical concepts while motion and time are physical ones, the study of motion and time as continua commensurate with extension must be just such a subordinate science.[122]

His reasoning here is that while mathematics studies its object (quantity) apart from a subject (existing in matter), physics must study its object (natural things) as existing in a subject. Thus, "It must be the subordinate sciences . . . that do study abstract mathematical properties *qua* inherent in certain physical subjects."[123] He concludes that *Physics* 6 studies the continuity of time and motion, but only qua inherent in motion and time." His second reason is that the definitions used in Book 6, from *Physics* 5.3, "are carefully selected to include both physical and mathematical reference to continuity and to exclude other aspects of both physics and mathematics."[124] A mean in definition seems to imply a middle science.

However, this claim does not follow for many reasons. First, there is the authority of St.

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[122] Jope, "Subordinate Demonstrative Science in Aristotle," 289. See above, p. 40.
[123] Ibid., 289-90.
[124] Ibid.

Thomas, who places *Physics* 6 entirely within natural science.[125] Second, as Jope realizes, the demonstrations in a middle science do not give the reason, but only the fact. However, it was shown above that the proof of *Physics* 6.4—which Jope does not mention—gives the cause of the conclusion: the nature of motion. This proof, at least, cannot be by a middle science.

Further, Jope's first reason is subject to a category error. He reasons that mathematics studies quantity, and physics studies natural things. If physics is found studying quantity, however, it must be in some way mathematical. Aristotle has said this way is a subordinate science, therefore, etc. However, physics can study quantity by including sensible matter in its definition, which mathematics abstracts from. These modes delineate different genera of consideration.[126] That Jope does not see this is clear when he says that physics studies quantity insofar as it inheres in motion and time. This is a logical kind of inherence, i.e., when quantity and continuity are *said of* motion and time as a subject. The real inherence is that of an accident in a substance. In this order, motion inheres in a divisible subject, and time draws its divisibility from the divisible motion that it measures, which motion is divisible due to the divisible subject. All of these are defined physically, and hence not part of a middle science.

Jope's second reason fails because the definitions of *Physics* 5.3 are in fact physical. Above, this was most manifest in the definition of contiguity. However, since certain of these definitions apply to mathematics if one abstracts from sensible matter, it can seem as if both physical and mathematical things are being discussed at once. Jope brings in the authority of Simplicius, who supposed that *Physics* 5.3 gives only definitions which are relevant for physics, yet then points out that even Simplicius notes that "together" is considered only as in place, and not in time or nature, which are included in the *Categories*. His conclusion is that, since Aristotle is eliminating certain natural

[125] Besides the commentary, see St. Thomas, *SBDT*, q. 5, a. 3, ad5^{um} (Leon.50.150), above in ch. 1, fn.63.
[126] See St. Thomas, *In De Caelo*, lib. 1, lect. 3, n. 6, cited in Ch. 1, fn. 65.

things (time and nature) to consider the more quantitative (place), this brings the physical closer to the mathematical. Yet it does not follow that it is a middle science because of mode of definition. Aristotle highlights the quantitative, but the quantitative is not convertible with the mathematical. Physics proper deals with quantity in motion in *Physics* 6. Note that the cause for this propriety is that it both defines with sensible matter (this concerns principles of knowing) and it explains the nature of the mobile as mobile, divisible, and continuous through its material subject (this concerns principles of being). It now remains to compare this approach—the quantity of a mobile as seen through its motion—to Newton's approach.

Chapter Three

Bodies and Motion in Newton's Mathematical Physics

Newton's "Preface to the Reader" announces as the goal of the *Principia* the investigation of natural forces through their mathematical principles. Newton's observation, that, these "forces being unknown, philosophers have hitherto attempted the search of nature in vain,"[1] is akin to Aristotle's, that ignorance of motion is ignorance of nature.[2] The previous chapter showed Aristotle's approach: knowledge of what motion is, besides clarifying nature as a mobile's internal principle of motion and rest, shows the physicist that the subject of motion is a divisible body. Newton's approach to and understanding of "nature" markedly differs. No meager or mean handful of thinkers has striven to determine the precise character of this difference. This chapter will only point out in a more dialectical fashion several key contrasts between these approaches by utilizing the three themes introduced in Chapter One and developed in Chapter Two. Their Newtonian analogs can serve as a basis for contrast. First, Newtonian abstraction will be examined, second, Newton's account of the existence of bodies and motion,[3] and third, Newton's science of moving bodies and the justifiability of the position that it is a middle science.

§1: Newtonian Abstraction

While Newton abstains from extended physical argumentation in the *Principia*, he nonetheless must take a stand on the existence and definitions of motion, body, place, and time to be able to defend the application of his mathematical physics. These definitions have a certain manner of being conceived or mode of definition. This mode is discussed explicitly in *De Gravitatione*.[4]

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[1] Newton, *Principia*, 4.
[2] See Aristotle, *Physics*, 3.1, 200b14-15.
[3] It will become clear why I have changed this from "the existence of matter and motion."
[4] The incomplete and unpublished *opusculum* whose first words are "De gravitatione et aequipondio fluidorum," was composed by Newton c. 1684-1687, just before the first edition of the *Principia*. See Cohen, *Guide*, 47 and 100-101, and Betty Jo Teeter Dobbs, *The Janus Faces of Genius: The Role of Alchemy in Newton's Thought* (Cambridge: Cambridge University Press, 1992) 148. This is a revision of Koyré's date of 1670, see his "Newton and Descartes," in *Newtonian Studies* (Cambridge, Mass.: Harvard University Press, 1965) 82-83. Its revised date brings it closer to the *Principia*, and therefore despite its being unpublished, the essay can be used to expand upon Newton's ideas about general physics as stated in the

In *De Gravitatione*, Newton proposes a unique abstraction applicable to the motion of quantities. The first text concerned with this "abstraction" was quoted above.[5] Newton states that through this abstraction he postulates only those properties of bodies required for local motion: extension, mobility, and impenetrability. In this way one can "understand abstract figures in the same way that they are considered by geometers when they assign motion to them."[6] Is this a possible "middle level" between mathematical and physical abstraction that considers just enough of the principles of a real body to see the mathematical properties of its mobility?

First, what exactly does Newton mean by "mobility"? This seems to beg the question. However, from the next section, it will be clearer what Newton means by the conditions of mobility and extension of a body. If this account fails, so also does his account of this abstraction. Second, "impenetrability" seems to involve a sensible quality in its account—namely the contact between limits of bodies—and this seems to import sensible matter. However, if one reduces impenetrability to the inability of two bodies to be in the same place, then this reduces to the same answer as the first objection, namely how one accounts for the unity and individuality of the mobile. That is, if Newton wishes to find a middle "abstraction" to consider motion, he must establish how, without sensible matter (which implies prime matter as the subject of change[7]), one is to account for there being one mobile subject. That he claims there is one mobile subject seems to follow when he holds that motion is used to demonstrate Euclid I.4 and I.8 (two triangle congruency theorems) or XI.Def.10 (the definition of equal and similar solids) by superposition.[8] Here Newton follows one

Principia. The conceptual kinship between the two works also stands to reason because (1) the twofold theme of the motion of bodies and the behavior of fluids mirrors the first and second books of the *Principia*, and (2) the manner of presentation that Newton sets for himself at the beginning of the *opusculum* ("I shall not be reluctant to illustrate the propositions abundantly from experiments as well, in such a way, however, that this freer method of discussion, disposed in Scholia, may not be confused with the former, which is treated in Lemmas, propositions, and corollaries." Hall 121, Janiak 12) matches that of the *Principia*.

[5] See above, ch. 2, p. 46.

[6] Newton, *De Gravitatione*, Hall ed., 122.

[7] See above, Ch. 1, p. 16.

[8] See Newton, *De Gravitatione*, Janiak ed., 13, editor's n. 3: "Some [including Newton] take it to be demonstrable as a theorem through the method of superposition," and hence in the same way as Euclid I.4 and I.8.

of his teachers, Isaac Barrow, who held that the whatness and existence of geometrical objects is properly shown through motion, and hence the subject of geometry is co-extended and adequate with the subject of physics.[9] The most famous examples include Euclid's definition of the cylinder or the right cone, and Apollonius' of the conic surface. Yet, it is not necessary to take such motions as entering into mathematics strictly, but merely as means of illustration. Apollonius' definition of a conic surface involves the rotation of an infinite line about the circumference of a circle through a point not on the circle. It could be reformulated as 'the surface defined by the circumference of a circle and a point not on that circle.' Likewise, envisioning the generation of a straight line by the motion of a point may work in the imagination, but not if one asks about the whatness of the *path* that the point supposedly followed. This cannot be defined again through motion. Thus, the quiddity of mathematicals is not properly disclosed to the mathematician through motion, which is rather an imaginative aid outside the disciplined mode of the science. From these it appears that Newton's attempt to find a middle abstraction falls short.

Among the properties required for motion is impenetrability. Near the end of the *De Gravitatione*, Newton returns to this abstraction when he considers the definitions of denser and rarer, elastic and nonelastic, and hard and fluid bodies. He has previously defined body as 'that which

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[9] See Isaac Barrow, *Lectiones Mathematicæ*, in *Mathematical Works*, ed. W. Whewell (Cambridge: University Press, 1860) 197: "No one of the geometers is not led to define in a like fashion what magnitudes soever (lines, surfaces, solids) from circular, right, parallel, and mixed motions as far as he thinks is allowed. Which definitions are not only legitimate, but are the best of all, for they do not only explicate the nature of the magnitude defined, but reveal [*commonstrant*] the possibility of its existence. The manner of its construction manifestly indicates not how it is to be described, but they show by exhibition [*experimento*] that such can exist." In his work *The Usefulness of Mathematical Learning* (London: Stephen Austen, 1734) 20-28, Barrow discusses what are (materially) middle sciences. His conclusion (ibid., 26-27) is that "Mathematics . . . is adequate and co-extended with *Physics* . . . In reality those which are called *mixed* or *concrete* mathematical sciences, are rather so many examples only of geometry, than so many distinct sciences separate from it: for when once they are disrobed of particular circumstances, and their own fundamental and principal hypotheses come to be admitted (whether sustained by a probable reason, or assumed *gratis*) they become purely geometrical." Note how Barrow's comments reflect the structure of proof used by a middle science, and the structure of the 'genus' of a middle science (a genus only qualifiedly one, constituted by mathematical principles and assumed physical matter). See ch.1, p. 24. As Niccolò Guicciardini notes (*Isaac Newton: On Mathematical Certainty and Method* (Cambridge, Mass.: MIT Press, 2009) 171), insofar as Barrow influenced Newton, his conception of geometric quantity allows for conceiving limiting procedures in terms of the continuity of generating motions, which same continuity of motion "allows mathematics to be envisaged as a language applicable to the study of the natural world." I owe Guicciardini's work for references to Barrow.

fills place,' and place as 'a part of space which something fills completely,' but does not bother to define space, it being "too well known to be susceptible of definition by other words."[10] Qualitative kinds of bodies are then defined in terms of the manner in which they fill space. Denser and rarer bodies have fewer or more porous (void) spaces, elastic and nonelastic bodies can or cannot suffer compression, and hard and fluid bodies have parts that either do not or do yield to pressure.

Newton is concerned first to allow for a mode of conceiving of a complex or composite body as a uniform body. He tells one to imagine or feign (*finge*) that there is in composite bodies an "absolutely perfect mixture of parts and pores thus infinitely divided" in whatsoever part, a manner that "suits mathematicians."[11] Mathematical calculation requires regularity when considering in summation the forces on and motions of the parts of hard or fluid bodies, "for one cannot ratiocinate mathematically concerning ones partially [hard or fluid], on account of the innumerable circumstances affecting the figures, motions, and contexture of the least particles."[12] Newton again feigns (*fingo*) a clean division into opposites by abstracting from heterogeneity and physical causes. A fluid body is "uniformly divided at all points," whereas a hard body "is a single undivided and uniform body which preserves its shape most resolutely."[13] Newton closes with an analogy about this mode of consideration. Just as the geometers "abstract" from physical irregularities and "do not accommodate their definitions of figures to the irregularities of physical bodies,"[14] so also the mathematical physicist "abstracts" from the bodily irregularities. His examples from geometry, however, are not those of pure geometry, but of applied geometry—surveying (measuring fields) and astronomy (the sphericity of the Earth). That is, this abstraction is defined in terms of measurable precision, an empirical limitation. This was noted of the middle sciences above. The

[10] Newton, *De Gravitatione*, Hall ed., 122.
[11] Ibid., 150.
[12] Ibid., 151.
[13] Ibid.
[14] Ibid., 152

middle term applies only insofar as the quantity obtains in the physical terminus of the argument. However, here it is not insofar as it exists but insofar as it is measured.

A like mode of conception is also present in the *Principia*. One would expect this from the nearly identical language Newton uses to describe space, place, and motion in both works, stating that "in philosophical disquisitions, we ought to abstract from our senses."[15] Now, two aspects mark Newton's abstraction, besides an abstraction from all sensible qualities: a removal of the consideration of physical causes and a removal of the heterogeneities and irregularities of physical bodies. Newton's famous text for the former is the following:

> I here design only to give a mathematical notion of these forces, without considering their physical causes and seats. . . . I likewise call attractions and impulses, in the same sense, accelerative, and motive; and use the words, attraction, impulse or propensity of any sort towards a centre, promiscuously, and indifferently, one for another; considering those forces not physically, but mathematically: wherefore, the reader is not to imagine, that by those words, I anywhere take upon me to define the kind, or the manner of any action, the causes or the physical reason thereof, or that I attribute forces, in a true and physical sense, to certain centres (which are only mathematical points); when at any time I happen to speak of centres as attracting, or as endued with attractive powers.[16]

This is the "secularism" that De Gandt notes as characteristic of Newton's mathematical physics.[17] The natural, physical difference between pushing and pulling, impulse and attraction, is ignored—they are mathematically identical by the third law.[18] The specific reasons for omitting physical causes

[15] Newton, *Principia*, 15. Compare to the quote from *De Gravitatione*, above, ch. 2, p. 46-47.
[16] Ibid., 13 and 130. Compare the closing scholium of Book 1, Section 11, 153-54.
[17] De Gandt, *Force and Geometry*, 271-73: "The theory of the *Principia* is neutral to an astonishing and exceptional degree. This neutrality has become a habit over the three hundred years of mathematical physics. . . . The *Principia* was written to discover the true causes and to advance towards the supreme cause, but the task was still incomplete . . . [It] constitutes the columns of an unfinished edifice, but posterity will little by little become habituated to this incompleteness. According to Koyré's formulation, 'the thought of the eighteenth century thus reconciled itself with the inexplicable.' Through Newton's work, a certain level of theory became autonomous. Once the mathematical physicists have determined the center of forces, once they know the law of the variation of the force and have succeeded in unifying families of phenomena, they can, on their own accounts, reflect on 'causes' or 'physical reasons' (to use Newton's term); but that will have no effect on the unfolding of their reasonings in mechanics or dynamics. . . . The term 'secularism' characterizes this situation very well. Scientists can have all sorts of private opinions as to the ultimate realities (or even have none at all), yet a certain common cultural life is possible—with procedures for purring to the proof, and rules governing the confrontation of ideas at a certain level. Mathematics plays a privileged role in the 'neutralization' of the study of force." See Newton *Principia*, 442: ". . . we have not yet assigned the cause of [gravity]."
[18] See ibid., 130. When Newton finally puts the third law of motion into play (in Book 1, Section 11), he states: "I shall therefore at present go on to treat of the motion of bodies mutually attracting each other; considering the centripetal forces as attractions; though perhaps in a physical strictness they may more truly be called impulses. But these propositions are to be considered as purely mathematical; and therefore, laying aside all physical considerations, I make use

are different in *De Gravitatione* and the *Principia*, yet both texts attribute to the conception of the quantity in question a remotion of some aspect present in the physical quantity, which aspect could provide *physical* knowledge, but which makes no difference as to calculating the effect of forces.

The second aspect, the removal of the irregularities in physical bodies, is found in Book 1, Section 12, titled "Of the attractive forces of spherical bodies." Newton has previously only needed the "bodies" in his propositions to be represented by points. Even in Section 11, where the third law comes into play and the centers of force in bodies are no longer considered as immobile but mobile, and hence dependent upon their quantity of matter, it sufficed to look to point-centers of mass. In Section 12, Props 70-76, Newton expands the limited mode of representation to spheres in order to consider the mutual attraction of spherical bodies in orbits. He builds his case by considering the attraction exerted by "hollow" spherical surfaces on corpuscles inside and outside such surfaces, and the attraction of external corpuscles by all the points of a "solid" or "filled" sphere. He inserts a scholium after Prop 73 about the parts of these spheres:

> By the superficies of which I here imagine the solids composed, I do not mean superficies purely mathematical, but orbs so extremely thin, that their thickness is as nothing; that is, the evanescent orbs of which the sphere will at last consist, when the number of the orbs is increased, and their thickness diminished without end. In like manner, by the points of which lines, surfaces, and solids are said to be composed, are to be understood equal particles, whose magnitude is perfectly inconsiderable.[19]

That is, bodies are subjected to laws of force—and hence to laws of motion—only insofar as each of their infinitesimally small parts are integrally subject to the same proportion or function.

Newton then moves on to show (in Prop 74) that, given an attracting force acting inversely as the square of the distance ($1/d^2$), the whole solid sphere acts on an external corpuscle by a force

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of a familiar way of speaking, to make myself the more easily understood by a mathematical reader."

[19] Newton, *Principia*, 157. Again, Newton follows his teacher, Barrow, in a guarded neutrality to the problem of the composition of the continuum. See Isaac Barrow, *Lectiones Geometricæ*, cited in De Gandt, *Force and Geometry*, 110: ". . . to every indefinitely small particle of time (I say instant or indefinite particle, for just as it matters little whether one supposes the line composed of innumerable points or of indefinitely small lines, so it amounts to the same to suppose time constituted of instants or of innumerable very small times; also, for the sake of brevity, let us not fear to employ instants or points in place of indefinitely short times)."

according to $1/d^2$, where 'd' is the distance from the corpuscle to the center of the sphere. He accomplishes this through an integral summation of all the composing spherical surfaces within the volume of the sphere. Now this also means that, as long as the corpuscle is external to the sphere, the attractive force of the whole solid sphere acts as if the mass of the whole sphere were concentrated at its point-center, i.e., the spherical body can be taken as a mass point. Newton then expands upon the properties of homogenous spheres in Prop 75, showing that, given a $1/d^2$ attraction to the points of a sphere, another sphere will be attracted with a force inversely proportional to the square of the distance between the centers, and, vice versa, the same is true of the attracted sphere, if it attracts. Since Prop 74 shows that the spheres and their centers are equivalent, and all previous considerations required only point-bodies, Newton concludes: "Those several truths demonstrated above concerning the motion of bodies [i.e., points] about the focus of the conic sections will take place when an attracting sphere is placed in the focus, and the bodies move without the sphere."[20]

Finally, Newton expands Prop 75 to heterogeneously composed spheres in Prop 76. If spheres have parts dissimilar in density and attractive force, but similar in that these parts are parts of concentric spherical surfaces within the sphere and having an attractive force varying as $1/d^2$, then the attractive force of the whole sphere on another sphere will be as $1/d^2$ between centers. To prove this proposition, Newton composes the sphere of "filled" and "void" spherical layers and, to offset the loss of density due to the void layers, "attraction-less matter." The density of the solid sphere can then vary according to any law, and its attraction according to $1/d^2$ is assured. In this way, heterogeneity is simulated via homogeneity.[21]

About these considerations in Book 1, Section 12, note that they are the key mathematical

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[20] Newton, *Principia*, Proposition 75, cor. 3, 158.
[21] See ibid., 159.

theorems that Newton needs to discuss the motion of the planets and sun in the solar system.[22] He has "built up" volumetric bodies with the method of the calculus—added "new conditions to his mathematical construction by introducing further *aspects* of the world of nature."[23] What applies to the *least parts* of those bodies—the force laws and consequent motions—applies to the *whole in sum*. The application of univocal laws to any part whatsoever requires the assumed uniformity of these parts. This homogenizes the real heterogeneity of planets. Newton also simulates heterogeneity through a calculus of homogeneity, and even here, assumes a homogeneity (the perfect "layers" of spherical surfaces) that may not exist in physical bodies. That is, this property of varying density may not be distributed with "mathematical" perfection, although it could be "close enough" given certain levels of imprecise measurement. Finally, the "fudge" of non-attractive matter is reminiscent of the "perfect mixture" of void and filled spaces Newton asks one to imagine when considering porous bodies. Newton is relying on his "abstraction" in the *Principia*.

§2: Newton on the Existence of Bodies and Motion

I will now examine Newton's account of bodies and motion. His account of space and body guides his account of motion. I will therefore examine the former first and then the latter.

In the *Principia*'s "Preface to the Reader," Newton homogenizes the distinction between physical and mathematical body—or is at least neutral to whether or not a difference will matter when conducting mathematical physics. The "Preface" has a single overall aim: to introduce the general aim of Newton's mathematical physics by showing how the phenomena of nature can be appropriately treated through mathematical laws. He first argues for the inclusion of geometry within universal mechanics.[24] This section makes use of two middle terms: the notion of perfect accuracy and the subordination of one science to another.

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[22] See ibid., Book 3, Props 7-9, 333-36.
[23] Cohen, *Guide*, 159. My emphasis.
[24] See Newton, *Principia*, 3.

The argument to include geometry within universal mechanics begins with the ancient division of mechanics into the rational ("which proceeds accurately [*accurate*] by demonstration"[25]) and practical kinds. Newton then notes the distinction between geometry and mechanics:

> But as artificers usually work with insufficient accuracy, it comes to pass that mechanics is so distinguished from geometry, that what is accurate is referred to geometry; what is less accurate, to mechanics. Yet the errors belong not to the art, but to the artificers. He who works with less accuracy, is the more imperfect mechanic; and if one could work most accurately [*accuratissime*], he would be the most perfect mechanic of all.[26]

The key idea is that error which arises from inaccurate work on the part of a mechanic belongs "not to the art," that is, does not arise from the nature of the art itself and the matter of the art, but from the artificer. Hence, it is itself an error to divide mechanics from geometry due to a lack of accuracy. This is clear from the conclusion Newton intends to draw: that if the highest level of accuracy could be obtained, one would be a perfect mechanic. This assumes that there is no essential discrepancy between the accuracy possible with the physical bodies of mechanics and the mathematical bodies of geometry that arises from the nature of either.[27] To admit of this same level of accuracy, the subject matters themselves much coincide in their ability to admit of such accuracy.

"Accuracy" is difficult to pin down: correctness, exactitude, precision, and rigor are often used interchangeably for it.[28] Newton eventually concludes that "geometry . . . is nothing but that part of universal mechanics which *accurately* proposes and demonstrates *the art of measuring*."[29] A

[25] Ibid., 3.
[26] Newton, *Philosophiae Naturalis Principia Mathematica* (*PNPM*) a. Translation my own.
[27] I owe this point to Dr. Richard Hassing.
[28] The following discussion draws on Jacob Klein, "On Precision," 290-91. See also *The Principia*, Cohen ed., 381. In his translation, Cohen uses "rigor" and "exactness" in translating the first two instances of "*accurate*" in the "Preface." Zev Bechler comments in his "Introduction" to *Contemporary Newtonian Research*, ed. by Zev Bechler (Dordrecht: D. Reidel, 1982) 16-17, that "two concepts are assembled here under the single term "accurate"—the accurate as the logically rigorous, and the accurate as the non-approximate." Bechler's interpretation seems inconsistent once one realizes that Newton relies upon accuracy in the second sense to give accuracy in the first sense meaning. Bechler asserts that the accuracy which geometry has as opposed to what mechanics seems to lack is non-approximation or ideal objects. This already brings in the notion of measurement—what an artisan does with regard to the matter of his art. However, he then asserts that "there is nothing wrong with it as an art, for as such it is as perfect as geometry." In what does this perfection consist? Not logical rigor, for Newton never says this, and furthermore, because it is *the very work of the artisan* that Newton calls inaccurate in regard to a possibly more accurate (exact, correct) measurement which, *given the matter*, admits of perfect accuracy. Yet this was the perfection of accuracy Bechler denies the *matter* of the art (and hence the art) previously.
[29] Newton, *Principia*, 3. My emphases.

tailor, carpenter, or machinist must be *accurate* in their acts of measuring, cutting, joining, fitting, etc. They must *carefully* execute these acts. If they do not follow the rules of their art carefully enough, following them *incorrectly,* their product will be imperfect. The suit will not fit, the joints with not line up, nor the machined parts match *exactly* or *precisely*. Newton seems to assume that the geometer and mechanic will follow the rules correctly. What he points out, however, is that the mechanics do not yet work as *carefully* as they could, because they have not yet realized the *precision* or *exactitude* with which they could execute those rules—a precision available in the very nature of what they work on.

Now, as Newton claims, there could be an artificer who works *accuratissime*. Who would he be? Several authors hold that Newton is referring to God in this passage.[30] A human artificer could, perhaps, realize such accuracy, but nature and nature's God have already done so.[31] The mathematical physicist is the one who will attempt to achieve this level of accuracy—to reach the absolute quantities of nature through the relative, sensible quantities.[32] Newton goes on in the "Preface" to explain why the most accurate artificer would be the most perfect mechanic: "for the description of right lines and circles, upon which geometry is founded, belongs to mechanics."[33] The most perfect mechanic would close the gap between an inaccurate practice of measurement such that his practice of mechanics could found geometry—he would realize in exercise (an identity between measured and mathematical objects) what exists in them to be discovered. It follows, since accuracy in practice is only a defect of the artificer, that geometry need not be excluded from mechanics—indeed, it is part of universal mechanics, and proposes the art of measuring all quantities. Natural rational

[30] See Bechler, ibid., 15-17; Guicciardini, *Mathematical Certainty*, 299-301, 313-15; James W. Garrison, "Newton and the Relation of Mathematics to Natural Philosophy," *Journal of the History of Ideas* 48 (1987): 611-12. Guicciardini cites in support passages from the beginning of *Geometriae Libri Duo*, an incomplete treatise of Newton's, from Isaac Newton, *The Mathematical Papers of Isaac Newton*, v.7, ed. D. T. Whiteside (Cambridge: Cambridge University Press, 1967-81) 287, 289.
[31] As Guicciardini notes, God has already done so—generated natural things and their forces with mathematical accuracy. See Guicciardini, *Mathematical Certainty and Method*, 314-15. He cites from *De Gravitatione.*
[32] See Newton, *Principia*, 17: "Relative quantities are not the quantities themselves, whose names they bear, but those sensible measures of them (either accurate or inaccurate), which are commonly used instead of the measured quantities themselves."
[33] Newton, *Principia*, 3. See also Garrison, "Relation of Mathematics to Natural Philosophy," 609-27, and Mary Domski, "The Constructible and the Intelligible in Newton's Philosophy of Geometry," *Philosophy of Science* 70 (2003): 1114-24.

mechanics may use then mathematical principles in natural philosophy of mobile bodies because their character is already mathematical. This character is most clearly expressed in *De Gravitatione*.

Newton's motivation for his discussion of space and body in *De Gravitatione* is to argue against Descartes' claim that there is no real distinction between them.[34] This debate is not of present concern. Newton's account gives us insight into the ground between the relationship between a body and its motion. After his arguments distinguishing space and body, Newton gives an account of space, and then of body.[35] He notes six properties of space "not only to show that it is something, but what it is."[36] I only need to examine two of these.

First, the parts of space are actually distinguished into an infinity of all possible parts, from all variety of figures to their boundaries (surfaces, lines, and points). Newton reasons as follows: extended volumes of space are distinguished by surfaces as common boundaries, surfaces by lines, and lines by points. Each of these species of boundary is partless with respect to the next:

> Surfaces do not have depth, nor lines breadth, nor points dimension, unless you say that coterminous spaces penetrate each other as far as the depth of the surface between them, namely what I have said to be the boundary of both or the common limit; and the same applies to lines and points.[37]

Since space in all directions is composed of such parts, it follows that there are "everywhere

> common boundaries of continuous parts . . . everywhere surfaces . . . everywhere

[34] See Koyré, "Newton and Descartes," 82. See Descartes, *Principles of Philosophy*, II, 11, in *The Philosophical Writings of Descartes,* vol. 1, trans. by J. Cottingham, R. Stoothoff, and D. Murdoch (Cambridge: Cambridge University Press, 1985) 227-28; hereafter CSM. Compare St. Thomas, *In Phys*., lib. 4, lect. 13.
[35] See Newton, *De Gravitatione*, Hall ed., 132; Newton's argument centers around the fact that one can conceive (imagine) extension without "the dispositions and properties of a body." Yet, this is the same argument used by Descartes, as Newton notes in the same place, from which Descartes concludes that body and spatial extension are the same.
[36] Ibid.
[37] Ibid., 132-33. See J. E. McGuire, "Space, Infinity, and Indivisibility: Newton on the Creation of Matter," in *Contemporary Newtonian Research*, 148ff. McGuire recognizes that the figures and surfaces of Newton's space are present actually, ibid., 149. McGuire is stymied, however, by the attribution of dimensionless-ness to the boundaries. He argues (historically) from *Quaestiones quaedam philosophicae*, a section from Newton's "Trinity Notebook" he composed as a Cambridge undergraduate, where Newton argues that lines, surfaces, and solids cannot be composed of indivisibles, but must rather be composed of indivisible minima. McGuire is relying on the early composition of *De Gravitatione*, c. 1668 (see ibid., 145), nearly 20 years before Dobbs' revised date. This puts the essay much closer to Newton's "Trinity Notebook," c. 1661-65. Therefore, historically, McGuire's arguments seem unable to support the former. Philosophically, furthermore, it is unnecessary to posit indivisible minima. Rather, the structure of space can be composed of indivisibles and understood through the method of the calculus. This is what Newton does in the *Principia* (e.g., see the text to fn.19 above).

> lines . . . and everywhere points in which the continuous parts of lines are joined together. And hence there are everywhere all kinds of figures, everywhere spheres, cubes, triangles, straight lines, everywhere circular, elliptical, parabolical, and all other kinds of figures, and those of all shapes and sizes, *even though they are not disclosed to sight*.[38]

It is clear that any figure or boundary is *possible* in space due to the nature of the parts of space. However, that there is an actual infinity of all possible figures and boundaries follows from the last observation, ". . . even though they are not disclosed to sight." Now, sensible qualities are not really present in things.[39] Thus, the presence of such figures in space is independent of observation.

> For the delineation of any material figure is *not a new production of that figure with respect to space*, but only a corporeal representation of it, so that what was formerly insensible in space *now appears to the senses to exist*. For thus we believe all those spaces to be spherical through which any sphere ever passes, being progressively moved from moment to moment, even though a sensible trace of the sphere no longer remains there.[40]

The space is spherical because this is required in order that it receive the body. Thus, this position concerning the parts of space follows from the distinction between space and body (what extension bodies can have, space must already be), looks forward to the "true nature" of body, and dovetails nicely with what the *Principia* states.[41]

Second, the parts of space are, furthermore, motionless. In both the *De Gravitatione* and the *Principia*, Newton reasons that, otherwise, the parts would move "into themselves" and cease to be individuated. Here are both chains of argumentation:

> For just as the parts of duration are *individuated by their order*, so that (for example) if yesterday could change places with today and become the later of the two, it would lose its individuality and would no longer be yesterday, but today; so the parts of space are *individuated by their positions*, so that if any two could change their positions, they would change their individuality at the same time and each would be converted numerically into the other. The parts of duration and space are understood to be the same as they really are because of their mutual order and position; *nor do they have any hint of individuality apart from that order and position* which consequently cannot be altered.

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[38] Newton, *De Gravitatione*, Hall ed., 133. My emphasis.
[39] See above, ch. 2, p. 46, fn. 92.
[40] Newton, *De Gravitatione*, Hall ed., 133.
[41] See Newton, *Principia*, 13: "Absolute space, in its own nature, without regard to anything external, remains always similar and immobile. Relative space is some movable dimension or measure of the absolute spaces; which our senses determine by its position to bodies." Relative space is only a rational construction due to sensible bodies.

> Absolute and relative space, are the same in figure and magnitude; but they do not remain always numerically the same. . . . As the order of the parts of time is immutable, so also is the order of the parts of [absolute] space. Suppose those parts to be moved out of their places, and they will be moved (if the expression may be allowed) out of themselves. For times and space are, as it were, the places as well of themselves as of all other things. *All things are placed in time as to order of succession; and in space as to order of situation [situs]*. It is from their essence or nature that they are places; and that the primary places of things should be moveable, is absurd.[42]

Newton, in both places, argues from a comparison of the order of the parts of time. In *De Gravitatione*, Newton places more emphasis on numerical identity, and in the *Principia* on the essence of place. Yet, while the wording of his application of this principle of individuation to space varies, each expresses essentially the same thing: moving primary places would "be converted numerically into [each] other," or "be moved . . . out of themselves." However, this order provides individuation and their character as place, and so cannot be disturbed.

Before considering body, something must be said about the principle Newton uses to individuate the parts of space, the mutual order of its parts. Here any part must be meant—solids, surfaces, lines, or points—although as all are actually divided, the consideration is reduced to points. A scholastic would respond that relation alone cannot individuate. An order of parts (*situs*) involves the relation of parts of extension to each other, and these parts must be individuated already.[43] Thus, this order is not a principle, although perhaps a sign, of individuation. As discussed in Chapter One, formal differences (such as those of order) cannot individuate; rather, one must rely on a sheer material principle.[44] However, since Newton would reply that this line of argument requires the parts of space to be substantial, which he denies,[45] an argument *ad hominem* is required. Nerlich shows that, even granting this mutual order, the argument fails, for every point in absolute space (and hence any

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[42] Newton, *De Gravitatione*, Hall ed., 136, and *Principia*, 13, 15. My emphasis. For a discussion of these passages, see Graham Nerlich, "Can Parts of Space Move? On Paragraph Six of Newton's 'Scholium'," *Erkenntnis* 61 (2005): 119-135.
[43] See St. Thomas, *In Phys*., lib. 3, lect. 5, n. 15 (Leon.2.114-15).
[44] See above, ch. 1, 15-16.
[45] See Newton, *De Gravitatione*, Hall ed., 131-32, 140.

possible figure) has the same order, and sameness affords no difference of individuality.[46] He argues as follows. The relations of all points to an origin point *p* in absolute space can be described as to their order via coordinates. However, if *F*(*p*) expresses the totality of these relations, and the general form of this expression is *F*(*x*), since *any* point in space would satisfy the expression (i.e., the character of "origin" belongs to every point in space), the same order holds for all points. Therefore, "[o]rder and relation without some hint of individuality independent of that order is powerless to identify—to distinguish any point from any other."[47] Newton cannot account for the individuality of the parts of space. This inability bears significant consequences for his account of the relationship between a body and its motion.

Newton presents a thought experiment to manifest the nature of body. He states that its explanation must be more uncertain, because the existence of body is dependent upon God's will. Therefore, "although it scarcely seems credible," Newton chooses the following method:

> I am reluctant to say positively what the nature of bodies is, but I rather describe a certain kind of being similar in every way to bodies, and whose creation we cannot deny to be within the power of God, so that we can hardly say that it is not body.[48]

Newton then argues, from an analogy to the human will, that God, who is infinite and has infinite power, can exert an influence on any part of space such that it will become impenetrable, reflective of light and sound, i.e., be "$body_2$," behaving indistinguishably from real "$body_1$."

God could thus sustain in space impenetrable particles of $body_2$, as of yet motionless, and,

> If we further imagine [*fingamus*] that that impenetrability is not always maintained in the same part of space but can be transferred hither and thither according to certain laws, yet so that the amount and shape of that impenetrable space are not changed, there will

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[46] See Nerlich, "Can Parts of Space Move?" 122-25.
[47] Ibid., 123. Nerlich also discusses in this article the similarities of Newton and Leibniz' position on this issue. However, Newton has no "principle of sufficient reason" to rescue the effects of the "principle of the indiscernibility of identicals." As Nerlich points out (133, n.12), even God cannot "move from a complete description to a particular instantiation," with regard to the parts of space because their very individuality is dependent upon that complete description of their mutual order. Newton's position excludes God from creating matter.
[48] Newton, *De Gravitatione*, Hall ed., 138. This echoes Descartes' *Le Monde*, CSM 92: "And my purpose is not to explain . . . the things which are in fact in the real world, but only to make up, as I please, a world . . . which nevertheless could be created exactly as I have imagined it." See Koyré, "Newton and Descartes," 70.

> be no property of body which this does not possess.[49]

That is, $body_2$ will lack no property of $body_1$. In this fashion, Newton conceives that the whole corporeal system of the world can be simulated. Newton then defines bodies "as *determined quantities of extension which omnipresent God endows with certain conditions*."[50] These conditions are mobility (according to a law), impenetrability and interaction according to a law, and the ability to excite sensation. Thus, sensible body is the impenetrable, measurable phenomena of parts of space related by certain laws. The stage is set for Newton's physics.

Newton makes several observations regarding these $bodies_2$.[51] They obviate the need to discuss prime matter and substantial form. Space stands in for matter and shape for form. The 'composite' is thus the product of God's will and guided by His law for it: "the form or formal reason of the body denoting every dimension of space in which the body is to be produced."[52] This 'composite' receives its individuation from this numerically one form:

> Between extension and its impressed form there is almost the same analogy that the Aristotelians postulate between the *materia prima* and substantial forms, namely when they say that the same matter is capable of assuming all forms, *and borrows the denomination of numerical body from its form*. For so I suppose that any form may be transferred through any space, and everywhere denote the same body.[53]

That is, Newton attempts turns the Aristotelian principle of individuation on its head.[54] Not only does the formal reason of a body make it numerically one but the continuity of the body's motion is thus preserved by the *same* form holding through space according to a law of motion:

> Continuity of motion is not a result of God's continually *recreating* the same conditions in successive regions of space; but rather of his maintaining, through the laws of his *potentia ordinata*, *the same formal reality in different parts of space through successive times*.[55]

However, what grounds the individuation of the 'forms' of these bodies through the different parts

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[49] Ibid., 139.
[50] Ibid., 140.
[51] See ibid., 140-41.
[52] Ibid.
[53] Ibid.
[54] See McGuire, "Space, Infinity, and Indivisibility," 179.
[55] Ibid. My emphasis.

of space and time? McGuire makes a helpful distinction: it is one thing to account for the criteria of *identification* of a body, but another to account for the criteria of *identifying* the same body. The principle of numerical sameness in being is other than that of knowing it, a principle Newton was familiar with.[56] The "denomination" above is an identifying one. The principle of identification or individuation is extension, for "'this' particular instantiation of the form of matter has as its referent 'this' determinate figure in space."[57] Now, Newton's attempt to individuate of the parts of space fails, a fact McGuire recognizes also. Thus, a moving body cannot gain numerical unity given the principles stated. Furthermore, nor can it gain unity from the laws of motion.

Newton mentions such laws briefly in *De Gravitatione*. They are the conditions which God implants in bodies so that they interact naturally in space—the "formal reason" of these bodies.[58] This *ratio formalis* or law expresses in the locus of space all possible locations of the body:

> The identification of the body, by means of its "formal reason," is reminiscent of the body uniting the ultimate velocities provided by the calculus in the *Principia*. There the method of limits identified the points that constitute the curve. Here the form denominates "all dimensions of space in which the body is to be produced." This demands that the motion is posterior to the succession or "range" of dimensions "in which the body is to be produced." For God's intention is to produce the same form in these infinitely many places. The same will necessarily be said of time. The movement of a body will follow God's determination in the form of the spaces in which the body is to be produced at the appropriate times.[59]

This "appropriateness" is dictated by the law of motion in question. Now, this law of motion is discovered through the method of the calculus, and it is "caused" through the forces of the body analyzed. This analysis also makes use of the principle of inertia. It will become clear that these principles cannot provide a real unity to motion and the mobile, for, as the above quote points out, motion is posterior to the individuated spaces and times "through" which the body "moves."

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[56] See ibid., 179-80. McGuire cites the *Principia* in support: "This is surely what lies behind his claim, that in distinguishing absolute from relative space we must not confound real quantities with their relations and sensible measures.'"
[57] Ibid., 179.
[58] See above, fn. 52.
[59] Nieto, "Continuity and the Reality of Movement," 91-92.

Inertia and the first law are bound together. Inertia is the closest thing to a physical principle that Newton proposes, despite his avowals to the contrary.[60] Indeed, inertial force as defined in *De Gravitatione* is the closest Newton comes to formulating a replacement for Aristotle's definition of nature.[61] As others point out, Newton never denies that it is a physical cause: it is a primary and unchanging property of matter (unlike gravity) for it does not exhibit more and less, given a determinate quantity of matter, and through it Newton is able to apply mathematical principles to physical bodies. "Even a universal gravitation denuded of its causal suggestions," is indemonstrable of bodies, "unless inertia is the essential power of matter."[62]

Inertia, or by its now infrequently used name, *vis insita*, is defined as follows:

> The *vis insita*, or innate force of matter, is a power of resisting, by which every body, as much as in it lies, endeavors to persevere in its present state, whether it be of rest, or of moving uniformly forward in a right line.[63]

As Newton explains, this *vis insita* is proportional to quantity of matter, and differs only in notion from *inertia*. Inertia names the inactivity of the body, or the difficulty of changing its state of motion or rest, while the innate force names the power of resisting extrinsically impressed forces and preserving its state of motion. This maintenance is of a forward, uniform, right line motion. It is an intrinsic force of the body, but not one directed to a determinate end, but rather towards the preservation of a right line 'out of' its present place *ad infinitum*.[64] The power of inertia sees any intervening body as an obstacle whose interaction via impressed forces elicits the action of inertia.[65]

This extrinsically impressed force is an "action" within an instant. An impressed force is

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[60] See ibid., 93ff.
[61] See Newon, *De Gravitatione*, ed. Hall, Definitions 5 and 8, 114: "Vis est motus et quietis causale principium. Estque vel externum quod in aliquod corpus impressum motum ejus vel generat vel destruit, vel aliquo saltem modo mutat, *vel est internum principium quo motus vel quies corpori indita conservatur*, et quodlibet ens in suo statu perseverare conatur & inpeditum reluctatur. . . . *Inertia est vis interna corporis* ne status ejus externa vi illata facile mutetur." My emphases.
[62] Nieto, "Continuity and the Reality of Movement," 95. See also Koyré, "Appendix C – Gravity an Essential Property of Matter?" in *Newtonian Studies*, 149-163.
[63] Newton, *Principia*, 9.
[64] Recall above, ch. 2, pp. 48-49.
[65] See Nieto, "Continuity and the Reality of Movement," 97ff, and 114.

defined as the "action exerted upon a body, in order to change its state, either of rest, or of moving uniformly forward in a right line," which action is one in the present, for it "remains no longer in the body, when the action is over."[66] However, as the Second Law states, "the alteration of motion is ever proportional to the motive force impressed . . . whether that force be impressed altogether and at once, or gradually [*gradatim*] and successively."[67] Thus, impressed force is instantaneous—whether at a single 'blow' or *gradatim* (in a gradated manner, distributed over a series of 'blows'[68]). Newton does not speak of the impressed force in the second law as acting over time.[69] Thus, the "transition from impulsive (instantaneous) to continually acting forces essentially bids us conceive of these forces as a sequence of infinitesimal impulses,"[70] through the least parts of space ("dx") and time ("dt"). This transition to a unified continuum with respect to the action of impressed force must be paralleled by a transition with respect to the continuous motion of the body, arising from the exchange between (intrinsic) inertia and the (extrinsic) impressed force.[71] Can inertia play its part?

The rule according to which the inertial power "acts" is the First Law, now called the law of inertia: "Every body perseveres in its state of rest, or of uniform motion in a right line, unless it is compelled to change that state by forces impressed thereon."[72] The Second Law details the nature of this intervention and the Third Law the conditions of their interaction. Speaking very generally, a law of motion of a body is founded on the concert of these laws that detail the interaction of inertial

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[66] Newton, *Principia*, 10.
[67] Ibid., 19.
[68] Cohen, *Guide*, 112: "A source of confusion to today's reader may be Newton's use here of the adverb 'gradatim,' which means 'gradually' or 'by degrees.' Today, the English adverb 'gradually' has lost its original sense of 'step by step,' but in Newton's day, 'gradatim' or 'gradually' still meant 'by degrees,' by stages or by steps."
[69] See Cohen, *Guide*, 111.
[70] Ibid., 116.
[71] For a discussion of the transition Cohen refers to on the side of the Second Law and impressed force, see Michel Blay, "Force, Continuity, and the Mathematization of Motion at the End of the Seventeenth Century," in *Isaac Newton's Natural Philosophy*, ed. By J. Z. Buchwald and I. B. Cohen (Cambridge, Mass.: MIT Press, 2001) 225-248; see 226: "How can one achieve mathematical rigor in the transition from the discontinuous to the continuous, with respect to the modalities of action?" The proposed rigor is the infinitesimal calculus.
[72] Newton, *Principia*, 19.

and impressed forces through successive points of space and time.[73] However, the inertial states of the body can provide no unity to the motion occurring at these successive parts of space. There is no intrinsic difference between these moments of motion, nor intrinsic *per se* order among them, nor intrinsic relation to the moments succeeding or preceding any given moment. There is no intrinsic difference to these moments because the moments are only differentiated through the extrinsic denomination of order provided by the individuated parts of time and space. Inertia is only a principle of the motion at these points in space and time, not through a unified whole of them. Neither is there a *per se* order among the moments of a motion, for the order of the parts of space grants this order. Inertia merely stipulates an endless, extrinsic directionality *in rectum* to the body, not a goal or *terminus ad quem* that draws the places in between it and the *terminus a quo* of the body into an *intrinsically* determined order (an order determined precisely because the body has the goal of the motion *within its potency*). Neither is there an intrinsic relation to the moments succeeding or preceding the present moment, for, as clear from the preceding two arguments, the differentiation and order of the places along a motion-path are extrinsically denominated. Hence any given moment is merely another point along the line at which inertia acts against an impressed force.[74]

So, generally, the force of inertia does not yeild a continuous motion "for it is not exercised except in the presentation of an obstacle."[75] However, obstacles are only present on a (possible) inertial motion-path given in space and time. Now, a body (whether a point or 'whole' body) as the 'form' of a traversed space derives its individuality from that part of space. A single motion-path provides only an infinite series of spots where the mobile is found in accord with its law (function of time with respect to distance). However, each part of space is individuated through its mutual order

[73] See Hassing, "Animals Versus the Law of Inertia," *The Review of Metaphysics* 46 (1992): 53-56. I say "founded on,"not "completely determined by" since a force law (how a force varies with respect to distance or time) is needed to let the consequences of the Second Law (force proportioned to acceleration or change in quantity of motion) play out.
[74] The substance of the last three arguments drawn from Nieto, "Continuity and the Reality of Movement," 112-13.
[75] Ibid., 114.

to all other parts of space and time, a principle already argued against. Therefore, the subject of motion has no real identity or continuity, and consequently neither does its motion. Newton has not, then, provided an account of the oneness of a mobile subject. What is missing is the ability of a mathematical solid to be conceived with a perduring, instrinsic subject. This is the final answer as to whether a mathematical solid can be considered in motion.

Now, the tool of the infinitesimal calculus gives the Newtonian a backdoor to see unity in motion—a certain mode of understanding using limits. The fundamental differences in species between the continuous and discrete, the straight and the curved are circumvented by a methodic process in thought.[76] Since a full investigation of Newton's calculus and its given mode of understanding is not possible here, this point must be assumed. However, granting the method's functionality, it provides a way around the infinite series of points in space and moments of time which determine the motion of a body and the infinite parts which compose a body. These quantities as known through calculus are provided a unity built up from their parts. However, nothing is explicitly noted by this mode of knowledge that provides a unity to the whole as such. Rather, the motion of the whole is such because all of its parts coincide in a relative rest with respect to the whole. Thus Newton holds: "It is a property of motion, that the parts, which retain given positions to their wholes, do partake of the motions of those wholes."[77] His method explains this participation in the whole's motion from the character of its parts. This integration of parts moving in accord with a law of motion determined through laws relating indivisible points of space and time

[76] See Charles de Koninck, "Concept, Process, and Reality," *Philosophy and Phenomenological Research* 9 (1949): 445-46: "Defining a nature as the limit of an unending process we at least approach noetic identity; to use Hermann Weyl's expression: our mind 'is open toward the infinite,' toward that absolute 'which is not of this world, and of which the eye of our consciousness perceives but reflected gleams.' However, in consequence of the empirical nature of our mind, we can never attain to it by the method we are bound to employ—i.e., the tentative reduction toward identity of formally distinct objects—since at 'infinity' both object and concept would be destroyed. For this reason the tendency toward a limit can be no more than the symbolic expression of the identity found in that perfect concept which we have termed a universal in power." See also Charles de Koninck, "The Dialectic of Limits as Critique of Reason," "Notes on Marxism," and "Concept, Process, and Reality," reprinted in *The Writings of Charles de Koninck*, v.2, ed. and trans. R. McInerny (Notre Dame: University of Notre Dame Press, 2009) 365-89, 405-15.

[77] Newton, *Principia*, 15.

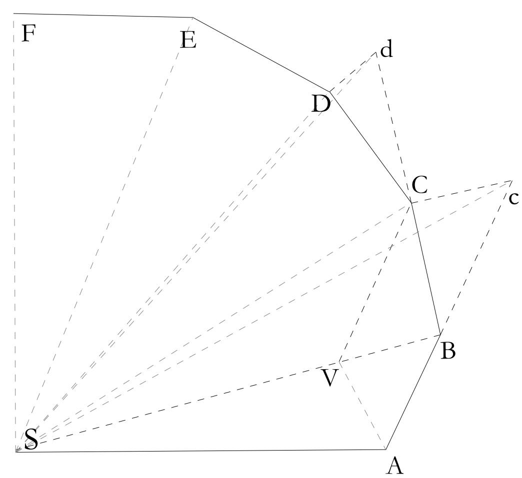
is the way that Newton can know that what moves is divisible.

§3: Newton's Science of Bodies and Motion

It now remains to consider the character of Newton's science of bodies and motion. First, I will examine Prop 2 of Book 1, and then discuss a proof from Roger Cotes' "Editor's Preface" to the *Principia*. Upon this basis, I will consider whether the *Principia* can be considered a middle science.

The second proposition of the first book is as follows:

> *Every body that moves in some curved line described in a plane, and by a radius, drawn to a point either immovable, or moving forward with an uniform rectilinear motion, describes about that point areas proportional to the times, is urged by a centripetal force toward that same point.*
>
> For every body that moves in a curved line is (by Law 1) turned aside from its rectilinear course by some force acting on it. And that force by which the body is turned off from its rectilinear course, and is made to describe the equal least triangles SAB, SBC, SCD, &c., about the immovable point S in equal times, acts in the place B, according to the line parallel to cC (by Elem. I.40 and Law 2), that is, along the line BS, and in the place C, according to the line parallel to dD, that is, in along the line CS, &c. Therefore it acts always along lines tending to the immovable point S. Q.E.D.[78]



First, consider the enunciation. Newton holds that "every body [*corpus omne*]" is considered by this proposition. The definitions and laws were framed in similar terms: *vis insita* is defined to belong to every body, and the first law applies to every body, etc. In the definition of inertia, the mind has been led "to recognize that, conceived as possessed of such a force, bodies cannot but persist in any definite state," and the examples used to support the law of inertia—projectiles, a spinning hoop, planets, and comets—are propaedeutic, "these instances allow us to satisfy and to strengthen the imagination in the consideration of the relation of the force of inertia to the motions of bodies."[79]

[78] Newton, *PNPM*, 41; simplified diagram. The second case proves the same granting that the entire system moves with a uniform rectilinear motion, which is easily resolved through Corollary 5 to the laws, see ibid., 24-25.
[79] Nieto, "Continuity and the Reality of Movement," 106.

Newton is preparing the reader for universal gravitation—the heavenly and corporeal bodies are subject to the same laws of motion. Thinking of "every body" is not to consider a common nature,[80] but the same specific nature present in multiple apparent kinds—a Baconian tactic.[81] Newton need not worry about corruptible or incorruptible bodies when applying his mathematical principles.

The argument itself is as follows. *Every body that moves in any curved line in given parameters of deflection* (deflecting from its rectilinear course such that the body is on a radius drawn to a point—Newton does not say "center" to include open curves—and describing areas around that point proportional to the times of its motion) *is acted upon by some force producing those parameters of deflection*. However, *a force deflecting with such parameters is a force that acts along lines tending towards that point*. Therefore, *every body with such parameters is acted upon by a force towards that point*.

The minor premise is an instance of the first law of motion. There must be a force acting to produce such parameters, otherwise the body in its motion would behave inertially, against the hypothesis. The second premise has a more complicated justification.[82]

First, note that Newton is proving the converse of the previous theorem of the *Principia*: areas that bodies moving in orbits describe by radii drawn to an immobile center of force lie in the same immobile planes and are proportional to the times—his version of Kepler's law of areas. The first proposition assumes the center of force and deduces the parameters, while the second assumes the parameters and deduces the center of force. Further, the first proposition, using the same diagram, starts with triangular and parallelogramic representations of force (e.g., VbcC) and then, through the process of limits, obtains the curved orbit as a limit of the bases of the evanescent triangles (ΔSAB, ΔSBC, etc.) and retains the direction of force in the evanescent line VB. However,

[80] See above, ch. 2, p. 51.
[81] See Francis Bacon, *The New Organon*, II.2-4, ed. and trans. by L. Jardine and M. Silverthorne (Cambridge: Cambridge University Press, 2000) 102-104; and Hassing, "Physical Continuum," 111, and "The Exemplary Career of Newton's Mathematics," *The St. John's Review* 44 (1997): 91, n. 11.
[82] See also Dana Densmore, *Newton's Principia: The Central Argument* (Santa Fe, NM: Green Lion Press, 2003) 142-46.

the second proposition starts with the curved orbit and then "reads into" this curve the limit of the series of triangles and parallelograms used before. This "reading" recovers VB, the line to be proven parallel to cC, that Newton uses as a pointer to the center of force.

Line VB is a force-pointer because of Corollary 1 to the laws of motion, Newton's force analyzer: "a body by two forces conjoined will describe the diagonal of a parallelogram, in the same time that it would describe the sides, by those forces apart."[83] The side Bc is the path that the body would describe in a given unit of time through its inertial force. However, upon reaching B, a force is supposed instantly impressed on the body in the direction of S, i.e., along the line VB. Now, in the non-ultimate case, C is the point which the body arrives at in the next given time unit, and hence the line cC is a measure of the instantaneous force applied at B. However, by Euclid I.40 (equal triangles which are on equal bases, and on the same side, are also between the same parallels) shows that cC is parallel to VB.[84] This same situation holds at the points D, E, F, etc.

Now, in the ultimate or limit case, the lines such as Cc, Dd (Ee, Ff, etc.) approach coincidence with each other, and also coincidence with BS and AS. By the second law (the second part only: an alteration of quantity of motion by an impressed force is made in the direction of the right line in which the force is impressed), in the limit case of the curved line, the force pointers coincide with the radii of the curve at each of its points. Therefore, a force with given parameters acts along a line to the point S. The major premise is established.

Note that the minor premise, although established by the law of inertia, is minimally physical. The "body" considered needs only a point to represent it. The law of inertia is not applied here to any recognizable physical body, but is merely used to reveal the "path" of this point—line Ac—that is the basis for constructing the force parallelogram VbcC through Corollary 1. These

[83] Newton, *Principia*, 20.

[84] ΔSAB = ΔSBc (by Euclid I.38, triangles on equal bases and in the same parallels are equal) and ΔSAB = ΔSBC by hypothesis (equal areas described in equal times), thus ΔSBC = ΔSBc, and Euclid I.40 applies.

mathematical quantities, whether or not they are essentially different from physical quantities, are named with physical names (a body with motion, suffering a force) but all that matters is the quantity involved as revealed through the first law and postulated by the parameters in the enunciation. At this point, a mathematical physicist sees quantity only as a *possible* measure of physical bodies.[85]

The major premise can be interpreted as minimally physical in a similar way. The force involved is revealed only by the change of direction "read into" the curve from the limiting process borrowed from Proposition 1, and thus only as to its direction, a purely quantitative relationship. The orbit is seen as a locus of force-pointers. Furthermore, one is able to predicate "a force acting along such and such a radius" of the "force producing such and such parameters of deflection" only because one sees through the infinitesimal calculus that the limit involved "is" the radius of a curve.

Thus, the middle term in this argument, a force acting to produce such and such parameters, is elicited from the minor term via Law 1, and identified with a force having a certain direction (centripetal) by a calculus of the force parallelograms given through Law 1, Corollary 1, and Law 2. Is this force considered only mathematically? In order for this to be true (and for Newton to be true to his word) then it must be the case that only quantity is being considered. This is certainly the case. What gives one pause, however, is that the various quantities that represent the presence of force in the diagram (VB, Cc) were obtained through definitions and axioms of motion that seem to use physical language (motion, velocity, force). The enunciation is speaking with physical words while intending only an implied mathematical meaning. This will be reconsidered later.

The above was an instance of a proposition that Newton considers to be mathematical. The structure of the *Principia* is such that the first two books present mathematically demonstrated propositions, and the procedure of the third book fits certain observed phenomena to these

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[85] One can argue against this reading if one interprets Newton with Koyré, see "The Significance of the Newtonian Synthesis," in *Newtonian Studies*, 9-10. Then, the minor premise here is physical because the physical world is mathematical—an ontological claim. Certainly this view has weight, although De Gandt's cannot be dismissed out of hand (fn.17).

mathematical principles.[86] Newton states that "these principles are the laws and conditions of certain motions, and powers or forces, which chiefly have respect to philosophy [*ad philosophiam maxime spectant*]."[87] This is Newton's intention to reach natural science. Roger Cotes provides an example of one of these arguments that utilize mathematical principles to explain a phenomenon:

> Now, it is reasonable to accept something that can be found by mathematics and proved with the greatest certainty: namely, that [M:] [B] *all bodies* moving in some curved line described in a plane, which by a radius drawn to a point (either at rest or moving in any way) describe areas about that point proportional to the times, *are* [C] *urged by forces that tend toward that same point*. Therefore, [m:] since it is agreed among astronomers that [A] *the primary planets* [B] *describe areas around the sun proportional to the times*, as do [a] *the secondary planets* around their own primary planets, it follows that the forces by which [A/a] they are continually pulled away from rectilinear tangents and are compelled to revolve in curvilinear orbits [C] are directed toward the bodies that are situated in the centers of the orbits. Therefore this force can, appropriately, be called centripetal with respect to the revolving body, and attractive with respect to the central body, *from whatever cause it may in the end be imagined to arise*.[88]

I have labelled the major premise [M:], minor premise [m:], major term [C], minor terms [A] & [a], and middle term [B], in Cotes' argument. Note that the "neutrality" towards physical causes remains, to Cotes' mind, even in the conclusion. The major premise will be recognized as the conclusion of Proposition 2, discussed above. Since it serves in this argument as the major premise, it plays a more causal role insofar as it presents the predicate which the middle term connects the subject to. The minor premise is an induction from observations taken from astronomy concerning the five known primary planets and the known moons of Earth, Jupiter, and Saturn.[89] Here is the required measurement and comparison Newton speaks of to "illustrate" the general case mathematically demonstrated in the phenomena of nature.[90] The mathematical principle is inserted into the physical

[86] Newton describes this relationship in several key places; see *Principia*, "Preface to the Reader," 4; the "Scholium" to Book 1, Section 11, 153-54; and the "Preface" to Book 3, 319.
[87] Ibid., 319.
[88] See Newton, *The Principia*, Cohen ed., 387-88. My emphases. Roger Cotes was the editor of the *Principia*, and wrote the "Editor's Preface" to the second edition of 1713. Here he is synthesizing Props 1 and 2 of the *Principia*'s third book. See Cohen, *Guide*, 22. See also Wallace, *Modeling of Nature*, 359-63.
[89] See Newton, *Principia*, Phenomena 1, 2, 5, and 6, 322-25. Mars' moons had not yet been discovered; the Moon follows the law of areas with noticeable irregularities, but which are (ibid.), "minute errors that are negligible."
[90] See ibid., 154-54.

realm, and the conclusion follows: the planets are urged by forces that tend towards the central point. Yet Cotes surreptitiously substitutes the central point for the central body. To assert that the physical central body (the sun) is acting on the planets requires another (physical) argument or postulate.[91] The mathematical quantity alone cannot distinguish the center of the sun from the center of the planets' orbits (only a rational distinction). Now, one could reformulate the argument using Proposition 75 of Book 1 and its third corollary,[92] but for all the mathematician can tell from Cotes's argument as stated, the center of the orbits (a point in space) and the center of the sun (a point in a body) happen to always coincide. The mathematical physicist must stop short and say: "from whatever [physical] cause it may in the end be imagined to arise."

So is this argument part of a middle science? A likely case can be made for either an affirmative or negative answer. On the negative side, an issue can be raised with the mathematics: the force of the arguments is derived from the calculus. However, is the calculus a rigorous mathematics? By ancient or medieval standards, "rigor" minimally includes scientific discourse about species of quantity as such.[93] However, as noted above, calculus seeks to methodically meld species of quantity for its own ends. By this standard, calculus is not a rigorous mathematics.

Difficulties with the physical aspects can also be raised. One difficulty with the physical assumptions is the mathematical species-neutrality used to construct the variegated and heterogenous world of nature. This is the case whether on the side of a body's actual structure,[94] or the measurement of its motion.[95] Furthermore, the specifically physical aspect of the proof is assumed from outside the mathematical genus: one allows talk of rays of light, heavy triangles, and

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[91] See Wallace, *Modeling of Nature*, 362. Wallace gives a version of this argument that includes such physical suppositions.
[92] See above, fn. 20.
[93] For example, having proven the possibility of the alternation of a proportion of magnitudes, one must use a different proof for the alternation of a proportion of numbers; see Euclid's *Elements*, 5.16, and 7.13.
[94] See, for example, Newton, *Principia*, Book 3, Proposition 9, 335-36.
[95] See, for example, ibid., Book 2, Proposition 48, 301.

musical numbers. This is a recognized trait of middle science.[96] Newton, for his part, must of necessity include such physical aspects, lest his propositions illuminate nothing about the physical world. Thus, inertia as a principle and law is stated so as to reveal specifically mathematical characteristics about the motion of physical bodies. However, inertia was unable to unite the moments of a motion into a unity because it acts in essentially individuated points of space and time.

The affirmative answer sees the first two books of the *Principia* as a mathematical shell. Odd uses of words (body, force, motion) are present because Newton is looking towards physics. He remains neutral to physical interpretation because the mathematical middle terms he provides do not reveal a physical account of things. Furthermore, the imprecision concerning the mathematical middle term due to the irregularity of corruptible physical quantity the Newtonian can ignore: the error of imprecision arises not from the art, but from the artificer. Now, Newton's assumption is that there is no essential difference between physical and mathematical body as such that will interfere with the outcome of his physics.[97] This assumption essentially proposes a program of perfecting the formal object of mathematical physics by providing empirical measurements of physical bodies that reveal the *uniformity* and *regularity* of their motions. However, the difficulty, as always, is with the calculus. Calculus is the most formal element in the science. This was seen in Cotes demonstration: it relies upon Proposition 2 as a major term, which, as a conclusion, relies upon a major term that is supported by Euclid I.40, the Second Law, and the procession to the limit case of an infinite series of force parallelograms (VBcC, above). It is the calculus that transforms the "static" quantities used at every point along the orbit to a "dynamic" functionality. Yet as we have seen, this functionality is still that of an infinite series of terms—this is a division on the side of form. The real individuality and subjectivity of the mobile have not been accounted for, and this is the keenest difference

[96] See Aristotle, *Posterior Analytics*, 1.7, 75b10-20, 1.13, 78b37-79a16; Weisheipl, *Commentary*, 23.
[97] I owe this observation and its phrasing to Dr. Richard Hassing.

between Newton and Aristotle's approaches.

As for the affirmative aspect of physics, it was already noted that Newton of necessity must make certain physical assumptions from outside the genus of mathematics. These hypotheses (in Aristotle's sense) are the real sticking points—philosophers and physicists argue the most about them. What it manifests, however, is that Newton's physics intends knowledge of the physical world as its intellectual terminus or rest. This is the same reason St. Thomas assigns for why a mixed science is more physical, and it is the more general but surer probable sign that Newton's attempts can be characterized as a middle science.

Conclusion

Wonder and Wisdom About Mobile Being

Motion is indeed difficult to penetrate scientifically. We have followed Aristotle's method and arrived at some determinations. He is able to alleviate many difficulties, unlike his doppelgänger in Plato's *Parmenides*, because he has the true definition of motion. This definition leads to a resolution of one of the problems of the mobile whole: it must of necessity have parts. The physicist sees the quantitative divisibility of the mobile subject through the nature of motion. This entails, as St. Thomas points out, that the subject of physics is mobile being, not mobile body, for the divisible nature of body is proven through a prior nature (the movable) and hence that nature must be prior in the given scientific discipline. Further, since the the existence of this nature is assumed by the scientist in his speech, it constitutes the subject of his endeavors. The physicist begins general physics in wonder about mobile being (if indeed every science ought to begin in the expectant ignorance of wonder and it behooves a scientist to wonder about his proper subject). This wonder begins with an indeterminate grasp of its object,[1] so that it may discover the principles of knowing the subject and key definitions (nature, form, matter, agency, finality, motion, place, time, etc.), the being that is the principle of the subject (the First Mover), and the concrete details of the subject (as St. Thomas explains at the end of his *Prooemium* to the *Physics* commentary). Relative to the genus of mobile being, general physics is a qualified form of wisdom.[2]

That this is the case is obvious from the fact that it is this part of physics that discovers the first principle of the entire mobile order, for one calls "wisdom" what considers the first principle of a genus.[3] This also follows since physics considers the *communia* (common things, aspects, properties)

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[1] See Aristotle, *Physics*, 1.1, in particular 184a21ff: "But the things which are first obvious and certain to us are rather confused, and from these, the elements and principles become known later by dividing them." The mobile is known indistinctly or confusedly as a whole, and its property, motion, leads to a distinct grasp of its elemental parts.

[2] See Aristotle, *Metaphysics*, 4.3, 1005b1. Aristotle says physics is a kind of wisdom, but see *prooemium* [8] p. 6, above.

[3] See St. Thomas, *In Meta*., "Prooemium," 1.

of the subject genus of mobile being. Just as first philosophy treats of the *communia* of being as being, so also "first physics"[4] for mobile being as mobile. Further, just as the former's subject is analogical, so also is the latter's.[5] The principles of motion (form and matter) are named in the definition of motion by "act" and "potency." However, since motion is found in several categories, and act and potency are found in every category, but analogically,[6] insofar as they define the motion proper to that category, they are found analogically in the general definition of motion. That is, the names "act" and "potency" in the general definition have an analogical relation when their use in naming one species is compared to another. By contrast, they are univocal within the same species, e.g., "act" and "potency" said of the principles of this or that local motion.[7] General physics, since it considers such *communia* for the species of mobiles with analogous breadth, has a sapiential role.

Besides the first sapiential function of metaphysics (reflecting upon itself), one can see that

> from the very fact that it has being as its object, [metaphysics] also covers somehow all the inferior sciences which treat of particular beings or particular aspects of beings, and may judge them, defend these sciences and use them, just as theology uses philosophy in general. Reflecting on mathematics, metaphysics becomes philosophy of mathematics, which is only materially mathematical, even though the data used be formally mathematical. And just as there is a metaphysics of mathematics, there is a metaphysics of philosophy of nature.
>
> Philosophy of nature participates in this second sapiential function of metaphysics in which it goes beyond its limits as a science . . .[8]

So the "first physicist" will judge, defend, and use the particular parts of the study of mobile being, yet does not argue demonstratively *to* them.[9] Rather, in scientific subsidiarity, there is a propriety to the parts of physics. Experience and dialectic brought in at the proper levels are not only judged for consistency against and defended from error by the more general and more certain determinations

[4] I borrow this expression (and invent appropriate denominatives) from James Chastek; see above, p. 5, fn. 1.
[5] See Cajetan, *DSNP*, 211a45-58; John of St. Thomas, *Nat.Phil.*, 15-16.
[6] See Aristotle, *Metaphysics*, 12.4, 1070b16-19. See also Cajetan, *DSNP*, 209b11-210a5; John of St. Thomas, *Nat.Phil.*, 12-13. I will assume without argument that these principles are analogous in being and predication.
[7] See John R. Mortensen, *Understanding St. Thomas on Analogy* (Lander, WY: The Aquinas Institute for the Study of Sacred Doctrine, 2010) 58.
[8] Charles de Koninck, "Thomism and Scientific Indeterminism," *Proceedings of the American Catholic Philosophical Association* 13 (1937) 75. By "philosophy of nature" De Koninck means general or first physics. See also De Koninck, "Reflections on the Problem of Indeterminism," in *Writings*, vol. 1, 437-38.
[9] It acts "beyond its limits," i.e., to species of mobiles, as a wisdom, not a science: See ibid., and above, ch. 1, fn. 12.

of first physics, but are also used to detail the initial grasp of the first principles.

First physics exercises this sapiential office with respect to any one of its subjective parts (those portions of scientific speech related as species to the analogical genus of *ens mobile*). That is, insofar as any scientific effort pursues a conclusion about mobile being (an object within physics), first physics may inspect its workmanship. A mixed science, sharing a material object (conclusion) with physics, and speaking (materially) about the same subject, is judged and also defended formally by first physics. This is especially true regarding the aspects of physical things which must be assumed by mathematics as extrinsic to the mathematical genus (the assumptions that give a physical-scientific *telos* to the use of mathematics) so that the mixed science can progress. Since the character of the mobile as divisible, and the nature of divisibility itself, are established by *Physics* 6, it follows that this part of physics will especially judge and defend a mathematically-principled study of physical things. That is, as was argued above, in *Physics* 6 Aristotle studies quantity not as a mathematician or middle scientist, but properly as a physicist. The assumption of mobile quantity by mathematics for the sake of *scientia media* is overseen by that part of first physics.

Newton's science of mobile bodies, therefore, insofar as it falls under the account of a middle science, is subject to the judgment and defense of first physics. For example, Newton's science cannot defend the unity of the mobile subject. The first physicist can judge this as falling short of actual conditions of physical quantity and its motion. It is a different problem to address more fundamental aspects of Newton's philosophy: absolute space, time, and motion, or inertia. Here, first physics could indicate problems whose solutions belong to first philosophy.[10]

The first physicist can judge a Newtonian treatment of bodies because he can see further than the mathematical physicist. He possesses a wisdom about mobile being the other lacks. This is

[10] See Aristotle, *Physics* 1.2, 185a1-20. The physicist does not dispute with those denying his principles, but with those drawing false conclusions from his principles. See also St. Thomas, *In Phys*., lib. 1, lect. 2, n. 7 (Leon.2.9-10).

not a lack of what the other should have by nature, but what is simply beyond the reach of his scientific discipline. It is, then, a sort of foolishness—gainsaying the wise—for the Newtonian to reduce the study of physical things to a mathematical mechanism. While this can be argued *ad hominem,*[11] this route establishes an independent vantage point. When the divisibility of a mobile is shown through the nature of motion, one connects the principles of motion to "the many" that are present to the moving thing (the quantitative parts of the mobile, the parts of the motion, the parts of time). These are unified in one subject through the material principle (what underlies and unifies the motion) and diversified by the formal principle (what allows for the specific terms of motion). This is shown at a very general level, where one still names the subject mobile being. The mathematical physicist, even at the level of his own domain, quantity, is forced to yield when speaking of mobile being—he must listen before he speaks.

Indeed, he is not forced to quit the field. The first physicist, because of the generality of his treatment, attains the most certainty *with respect to us*, but the least depth as to *things in themselves*:

> With respect to the later treatises [in natural philosophy], the first, the *Physics* and *De Anima*, give us only a superficial knowledge of things, and that is why they are more certain for us. So it is right to reproach those who are content with this kind of consideration as if they had attained the ultimate causes, their air of false profundity, unless one calls the confused and undetermined profound.[12]

The first physicist needs the lower parts of physics to more fully determine his grasp of the physical world. As a lower part, mathematical physics "can be useful to the philosopher only insofar as it has established itself in its own right."[13] Beginning in confused wonder about mobile being, knowing the principles and details about this subject is the *telos* of the physicist. His sapiential use of first physics needs mathematical physics to achieve his speculative end—scientifically adequate speech and thought about the natural world.

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[11] See Hassing, "Wholes, Parts, and Laws of Motion," *Nature and System* 6 (1984): 195-215; and "Animals versus the Laws of Inertia."
[12] De Koninck, "Experimental Sciences Distinct?" 452.
[13] De Koninck, "Thomism and Scientific Indeterminism," 76.

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