position to the limit, that is in so far as it is becoming non-different. Because there is no limit to the growing proximity, because there is an inexhaustible potency of getting closer to the limit, we say that the movement tends "toward" the negation of distance, "toward" the negation of difference, of inequality of of otherness. The movement cannot get beyond the state of "being toward", although the "being toward" increases as the movement proceeds, and to this, again, there is no limit.

Because of this we may say that the variable, open and getting ever closer to the limit, tends to enclose the limit as its own ultimate value: the open series tends to close itself by reaching beyond itself. It tends toward homogeneity with its limit. The matter of the variable, the movement of the approaching values, tends somehow to disrupt, to break through, the definition of the variable and to assume the form of the limit.

2. The kind of distinction we make between the form of the variable and its matter was understood to be proper to the variable as such (supra n.1). It has this distinction whether we consider it as ordered to a limit or not. The limit itself is not as such a variable, and hence does not, in this respect allow this distinction. Nevertheless, the limit regards both the matter and the form

of the variable. We might say, speaking elliptically, that that which in the variable is the common proximate form and that which is a proper proximate form of one of its values are, in the corresponding limit, identical. It is like a class with one member only, or a universal which can be predicated of only one individual.

It is when we consider the limit formally, that is, in the light of the tendency of a series toward the limit, that we must somehow distinguish form and matter in the limit itself. For limit is primarily the limit of the values of the variable as if limit were the ultimate value of the variable, that is, the common limit of the series. In other words, if, per impossibile, the series could reach its limit, the limit would be the common limit of the series, hence part of the series and homogeneous with the members of the series as having a proper form(the form on the part of the matter) of the order of the variable; it would belong to the matter of the variable. The limit of polygon would be both circle and a species of polygon, that is both circle and polygon, a one-many-sided plane figure, an unbroken broken line, etc. On the other hand, no limit would be limit if it did not have and retain its proper definition a other than that of the variable. The circle which we define: "one-sided plane figure whose.....", and the cir-

cle which we define: "limit of a regular inscribed polygons whose....", must be the same circle. Hence, to attain the limit would be identical with destroying it.

13. All this brings out more clearly what we have said of the peculiarity of the movement toward a limit. If it were to be defined as a movement which can attain the limit, the movement itself would be as contradictory as the attained limit. When we use such expressions as "the variable tends to attain its limit", we should consider the "tendency to attain" as one indivisible form, and not as a "tendency to attain", on the one hand, and the possibility of attaining, on the other. Similarly, when we say that the movement toward a limit is a movement toward contradiction, we must understand this hypothetically: to attain the limit toward which it tends would be contradictory, just as a relation of reason would be contradictory if it were in the thing toward which it is.

Limit as a State of Becoming

11. If it is already difficult to conceive what movement is (for movement is neither determinately act nor determinately potency), it should be even more difficult to conceive what limit is; for, besides being movement, it is also rest, and it seems that it should be both determinately so. If it is held that the Aristotelian analysis of movement renders obscure what is so clear ——clear indeed if we intended to

"save the name" only—, we dread to think what should then be said of the present attempt toward an analysis of what limit is.

So far we have considered the movement toward a limit as a state of becoming on the part of the variable. The state of becoming which is on the part of the variable is its state of becoming the limit. By running through its values, or rather by acquiring ever other and new values, the variable tends to become the limit in its very otherness. We should note, however, that the variable is not said to tend toward otherness merely in so far as it is acquiring ever new and other values, for this alone does not imply a tendency toward what is other than the variable itself. This tendency must be viewed in the light of the limit toward which it converges, something which differs from the variable by definition.

In other words, the variable ordered to a limit *is* the state of becoming of the limit itself, but this obviously means that both the variable and the limit each have a double state. For the variable is always other than its limit and absolutely identical with itself; but it is also becoming the limit, and in becoming the limit it is becoming identical with the limit. To become the limit must mean to become identical with it, as is obvious from the fact that if the term of the becoming could be reached, the variable would have to be wholly and determinately identical with the determinate otherness that is the limit in its absolute state. We might add that in acquiring its values, that is, in accomplishing itself, the variable is at the same time undoing itself and tending to vanish in otherness.

The same must be said of the limit. It has a state of absolute identity with itself, in which it is absolutely other than the variable; but it must also have a state of becoming, a state of "coming from" the variable. In other words, the limit must be coming from the otherness that is the variable as if it were precontained in that otherness, and the variable must be moving away from itself and becoming identical with what is otherness to it, viz. the limit.

In turn, the absolute identity of the limit with itself must be considered in two ways: absolutely, and with respect to the becoming. When we take 2 as the limit of 1, $1\frac{1}{2}$, $1\frac{3}{4}$...., we must consider 2 as absolutely identical with itself; but if 2 is a limit, we must consider this absolute identity with itself as that which the series itself is tending to be. This absolute identity, *taken* relatively, must be distinguished from what we have called "becoming identical"; it is that term because of which the becoming is denominated "identical". The "becoming identical" is predi-

sated of that which would be identical with the absolute identity, if, *per impossibile*, it could be reached. If, *per impossibile*, it were reached then the variable's state of absolute identity and the limit's state of absolute identity would be identical.

1.. The limit in its state of becoming is the becoming identical of the limit with its state of absolute identity. Furthermore, since the variable in its state of becoming identical with the limit is the state of becoming of the limit itself, the state of becoming of the variable as the state of becoming of the limit are the same state of becoming.

This shows clearly enough that the limit cannot be identified with either one of its states, and that the variable as ordered to a limit cannot be identified with either one of its own states. For if limit were identical with its state of becoming on the one hand, and on the other hand with its state of absolute identity, its state of becoming would be identical with its state of absolute identity.

This would raise a false [illegible] tion, if the two states of the limit were [illegible] "[illegible]" state of rest [illegible] state of [illegible]. But they are not comparable, for the contrary states of the limit must be [illegible] as contraries

in the same subject at the same time. Yet this is contradictory.

What we have so far called the "notion of limit" involves contraries which can hardly be reconciled with the unity of a notion. It would not do to say that limit is a multiplicity of notions (such as "circular polygon"), that is, two mutually exclusive notions which have only the unity of grammatical expression. By limit we do not mean that which, reaching the limit, would have to be, i.e. the identity of the variable with the limit, for that, plainly, is a contradiction.

The notion of limit envelops contraries. It does not represent them as being simultaneously in the same subject, nor as being successively in the same subject. It represents them as contraries <u>tending</u> to be simultaneously in the same subject. To suppress the tendency would destroy the notion of limit. It is the "tendency" which gives to the notion of limit its peculiar unity. It envelops the contraries only as principles of the tendency toward identity, an identity which is no more represented than reached. The tendency is the nexus which establishes the unity of the notion of limit, embracing all the states of both variable and its limit. That it does embrace them all is clear from the fact that the whole structure breaks down as soon as we drop out any state of either term.

The absolute generation of a limit

13. The becoming of the limit may be called the generation of the limit, but we must be careful to distinguish what we here call generation from natural generation.[14] The novelty of the limit coming to be is not like the novelty of some concrete nature, say Socrates, but of the very abstract nature itself, say man as such, which is to say, that of which there is strictly no generation. The 2 that is becoming in the series 1, $1+\frac{1}{2}$, $1+\frac{1}{2}+\frac{1}{4}$, $1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}$, ..., is not just "some" 2, but "the" 2. Twoness itself is becoming. The novelty, then, is not merely on the part of the "what it is" of the abstract nature itself; its very knowability "secundum se" is becoming. It is as if 2 could become rational only by such a generation. The givenness of 2 in its absolute state is, with respect to its becoming, as a state of irrationality becoming rational in the movement of the variable. In other words the givenness of the absolute state of limit is as a barrier to its complete rationalization.

The tendency toward a limit is always toward absolute generation, but, as we have seen, the limit must have an absolute state in which it is unattainable and irreducible otherness. However, when the absolute state of a limit lies itself within an order, such as 2 within the order of integers 1, 2, 3, ..., then the tendency toward the fullness of that limit cannot be accomplished from one direction alone. The absolute

state of 2 is not fully seen when considered only as the upper limit of the series 1, $1\frac{1}{2}$, $1\frac{1}{2}\frac{1}{4}$... , for the same absolute state of 2 is the lower limit of the series 3, $3-\frac{1}{2}$, $3-\frac{1}{2}-\frac{1}{4}$, $3-\frac{1}{2}-\frac{1}{4}-$... .

When we move in one direction alone, 2 lies not only beyond the order of the series; it lies beyond order. It escapes pursuit as if it were forever receding into an open background. Its becoming rational lies forever before it, the rear remaining safe. As the higher limit of one series it escapes circumvention, and the generation is unilateral; but the direction of the withdrawal is somehow arrested and overcome when, at the same time, we move toward 2 from the opposite direction.

14. This consideration introduces an entirely new idea. When a limit is both upper and lower, the upper limit of the lower series may be viewed as the limit coming from the lower series. This is what we call the absolute generation of that limit.

To consider the limit from the viewpoint of its absolute generation implies a certain advantage but also entails a difficulty. When we say that the limit we are moving toward in the lower series is, not the absolute state of that limit in its otherness lying wholly beyond that order, but the limit that is coming from the upper series, we seem to rule out one condition which we have so far asserted to be essential, viz. that any limit must have an absolute state in which it is

absolutely determined otherness, unattainable and irreducible. Now the only otherness we choose to consider is the one coming from the other direction.

This difficulty, though, is only apparent. We still suppose that otherness in two ways. We first suppose as already established that a given limit is the upper limit of one series, and the lower limit of another, the one independently of the other. We then bring the two together, and henceforth define absolute limit as the limit toward which one series is moving, not in its absoluteness alone (although we do suppose that) but in its absoluteness as coming from the other series. Therefore, we have not eliminated anything. We have merely added a new respect based on a comparison between the two series cum converging toward one another. Although in this greater complexity we have become more independent of the absoluteness of the limit, we still presuppose it. It is only the independence of the mode of approaching it that has increased.

We might now add as a rule that any limit should be so approached if it is at all possible. This is possible whenever a limit may be considered as lying between two open series.

18. In an earlier paragraph (n. 6) we have spoken of the "more" and the "less" as belonging to the very notion of limit. We meant thereby the dynamic correlatives, related as concave and convex and to be found in any single series converging toward a limit, such

as the "increasing" value of the series 1, 1 + ½, 1 + ½ + ⅓....., and the directly corresponding "decreasing" value of the difference from the limit. The increase alone would have no reference to limit. We must bear in mind, furthermore, that both this "more" and this "less" imply infinity and movement, for there is no end to the more, nor to the less, and apart from a state of movement they have no bearing here. We should also understand them as being on the part of the matter of the variable and as tending to acquire another form, viz. the form of the limit. The more is many and one or many-one, and so is the less. In the attained limit, the more and the less would be identical and one only. The more and the less would appear again when we move from one limit to the next, as from 1 to 3.

Now the absolute generation of a limit introduces a more and a less of another order: the more and the less that are the two series converging from "above" and from "below" to an identical limit, each series having in itself its own more and less. Now the more and the less that are the two series may be viewed in two ways, since each series may be called both the more and the less with respect to its limit. Thus on the one hand 2 may be considered as coming from that which is always more than 2 however small, that is, from 3 or from any value of the series $3-\frac{1}{1}$, $3-\frac{1}{2}$-...; the common form of this series is more than that of its limit. On the other hand, but with

respect to the same series, we may also say that 2 is coming from the less, that is, from the greater qua getting smaller. The limit is here the limit of a decrease, of a degradation; at the limit, if it could be reached, the ultimate value would be infinitely small with respect to the series. It is from the "getting smaller" of the greater that the limit is becoming. It is qua getting smaller that the upper series has infinity and movement. "Servatis servandis", the same distinction is found on the part of the lower series. The upper limit is becoming from the "getting greater" of the smaller.

16. These are the more and the less that go into the absolute generation of a limit. The limit is as the form toward which they tend. If the lower series could become infinitely great, and the upper infinitely small, the same limit would be their proper proximate form, that is to say, they would be identical. In this perspective, the unity of the limit-form comes from the duality of the great and small. If the limit could be reached, it would be the form of the infinitely small and the infinitely great, which would there be identical.

Since the "infinitely small" and the "infinitely great" may be understood in two ways, it might be good to specify the meaning here employed. "Infinitely small" may be said of the absolute state of a limit considered relatively. Thus we may say that 1 is infinitely small with respect to its upper series $2-\frac{1}{2}, 2-\frac{1}{4}-\ldots$, or that 2 is infinitely small with respect to its upper series, $3-$, etc. The expression "infinitely small" is also applied to what is becoming ever smaller without end.[15] The one and the other would be identical only if the limit could be reached. The same should be said of the "infinitely great". We have here taken these expressions in their first meaning. We might point out again, however, that even in their first meaning they have significance only with reference to becoming. That the two meanings have been frequently identified may be explained from the fact that the getting smaller, or greater, of the variable is identical with the limit's state of becoming.

Abstract and Concrete Identity

16. The variable tends toward concrete identity with the limit. The very meaning of this tendency would be destroyed if we confused it wit what we shall call the abstract identity of the variable and the limit. This abstract identity is as a prerequisite of their tendency toward concrete identity. It is at the first principle upon which hinges the tendential unity of limit itself. The abstract identity at the same time determines the confines within which the difference tends to be-

cuse concretely identical with the other.

Polygon and circle are distinct species having "plane figure" as their common proximate genus. (For the sake of simplicity we will prescind from the intermediary division into "regular" and "irregular"). Now we must say that the plane figure predicated of polygon and of circle is the same plane figure, that is it is the same for both. If it were not, polygon and circle would differ as plane figure. This is not what we mean by the abstract identity of differences. Although the plane figure that is predicated of polygon and of circle is the same plane figure, the same plane figure cannot be predicated of those species with identity. We cannot say that polygon and circle are the same plane figure, for that would mean that the different species are the same species, that polygon is circle and vice versa. The reason is that the proximate genus is divided by its species. To say that polygon and circle are the same plane figure would be identifying the identity of plane figure with that which divides it.

A common genus could be predicated with identity of differences only if it were not divided by those differences; but that is exactly the case of the remote genus with respect to the species of

of the proximate genus, and of the proximate genus with respect to the individuals of its species. Here are a few classical texts on the subject:

> "Things are said to be 'one' whose genus is one and differs in its opposite differentiae. All these things too are said to be 'one' because the genus, which is the substrate of the differentiae, is one (e.g. 'horse', 'man', and 'dog' are in a sense one, because they are all animals); and that in a way very similar to that in which the matter is one. Sometimes these things are said to be 'one' in this sense, and sometimes their higher genus is said to be one and the same (if they are final species of their genus) -- the genus, that is, which is above the genera of which their proximate genus is one: e.g. the isosceles and equilateral triangles are one and the same **figure** (because they are both triangles), but not the same triangles." (Aristotle-- V Metaphysics, ch.6; 1016a 25--33)

> "It is said rightly, too, that the number of the sheep and of the dogs is the same **number** if the two numbers are equal, but not the same **decad** or the same **ten**; just as the equilateral and the scalene are not the same **triangle**, yet they are the same **figure**, because they are both triangles. For things are called the same so-and-so if they do not differ by a differentia of that thing, but not if they do; e.g. triangle differs from triangle by a differentia of triangle, therefore they are different triangles; but they do not differ by a differentia of figure, but are in one and the same division of it." (Aristotle--IV Physics, ch.14; 224 a 1-15)

> "...Trianguli enim aequilateri non sunt idem triangulus: eo quod dividuntur et plurificantur per propria et essentiales differentiam trianguli: est enim divisio essentialis et immediata trianguli in isopleurum qui est laterum aequalium, et isoscelon qui est habet latera aequalia; tamen hi duo sunt eadem figura: eo quod non exeunt a figura per immediatam et essentialem divisionem figurae: quia figura dividi-

tur in triangulum, et quadratum, et alias differentias figurae. Idem enim dicitur, a quo non differt differentia propria essentiali. Non autem idem dicitur, a quo differt differentia propria et essentiali." (St.Albert the Great-- Liber IV Physicorum, tr.III, cap.17)

"....Genus potest cum additione unitatis vel identitatis praedicari de pluribus individuis existentibus in una specie, et similiter genus remotum de pluribus speciebus existentibus sub uno genere propinquo; neque tamen species de individuis, neque genus propinquum de speciebus diversis potest praedicari cum additione unitatis vel identitatis.

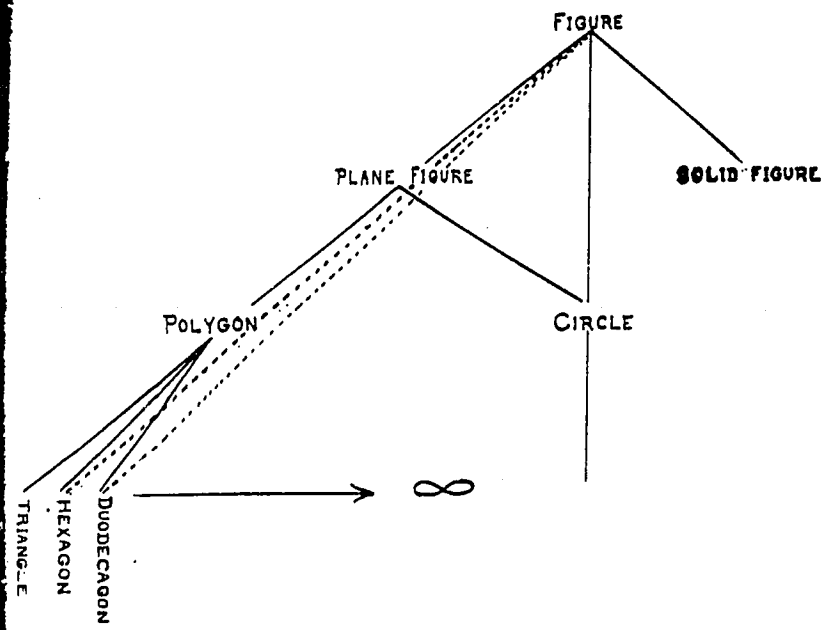
"Et huius consequenter ponit exemplum. Sunt enim duae species trianguli, scilicet aequilaterus, idest habens tria latera aequalia, et gradatus, idest habens tria latera inaequalia; figura autem est genus trianguli. Non ergo possumus dicere quod aequilaterus et gradatus sint idem triangulus; sed possumus dicere quod sunt eadem figura, quia utrumque continetur sub triangulo, qui est una species figurae. Et huius assignat rationem: quia cum idem et diversum seu differens opponantur, ibi possumus identitatem dicere, ubi differentia non invenitur; sed non possumus dicere identitatem, ubi invenitur differentia. Manifestum est autem quod aequilaterus et gradatus differunt ad invicem differentia trianguli, idest quae est propria trianguli divisiva; et hoc ideo quia sunt diversae species trianguli. Sed aequilaterus et gradatus non differunt secundum differentiam figurae, sed sub una et eadem differentia divisiva figurae, continentur. Et hoc si patet. Si enim dividamus figuram in suas species, quae per differentias constituuntur, invenietur quod alia erit circulus, et alia triangulus, et sic de aliis speciebus figurae; sed si dividamus triangulum, invenimus quod alia species eius est aequilaterus, et alia gradatus. Manifestum est igitur quod aequilaterus et gradatus sunt una figura, quia continentur sub una specie figurae, quae est triangulus; sed non sunt unus triangulus, quia sunt diversae trianguli species." (St.Thomas-- IV Physics, lect.13, n.13)

"Sed sciendum est, quod unum ratione generis dicitur dupliciter. Quandoque enim aliqua dicuntur ita unum in genere sicut dictum est, quia scilicet eorum unum est genus qualitercumque. Quandoque vero non dicuntur aliqua esse unum in genere, nisi in genere superiori, quod cum adjunctione unitatis vel identitatis praedicetur de ultimis speciebus generis inferioris, quando sunt aliquae aliae superiores species supremi generis, in quarum una infimae species conveniunt. Sicut figura est unum genus supremum continens sub se multas species, scilicet circulum, triangulum, quadratum et hujusmodi. Et triangulus etiam continet diversas species, scilicet aequilaterum, qui dicitur isopleurus, et triangulum duorum aequalium laterum, qui dicitur aequitibiarum vel isosceles. Isti igitur duo trianguli dicuntur una figura, quod est genus remotum, sed non unus triangulus, quod est genus proximum. Cujus ratio est, quia hi duo trianguli non differunt per differentiam quibus dividitur figura. Differunt autem per differentias quibus dividitur triangulus. Idem autem dicitur a quo aliquid non differt differentia." (St.Thomas-- V Metaphys. lect.7, n.863)

This is what we mean by the abstract identity of differences. Polygon and circle are the same figure. Hence we move from polygon to circle, we tend from their abstract identity toward concrete identity where polygon and circle would be the same plane figure, that is, where polygon would be circle, and vice versa. The movement toward a limit is, therefore, as a tendency to deduce the concrete identity of the differences from their abstract identity, to generate the one from the other; or, more appropriately, to deduce the irreducible differences from their abstract identity.

to say "more appropriately", for, as we shall soon point out, the aim of the tendency toward a limit is not to reduce the differences to identity (for this would defeat our purpose), but rather to deduce the differences, to generate them from non-difference. However, if we could actually generate them, they would then constitute nothing more than a sterile identity.

The following illustration shows how the tendency toward a limit hinges on the abstract identity of the variable and the limit:



When we view the system in its downward direction and prescind from the movement, we observe a tendency toward pure plurality (each species of polygon might be divided indefinitely into perfectly homogeneous individuals). However, when we view the system horizontally, moving from left to right, we see that the plurality has a unity due to the movement and to the identity toward which it is tending. We can also see, on the other hand, that if the limit were actually reached, the whole system would be reduced to sterile identity and contradiction. Polygon would be circle; these would in turn be identical with their proximate genus, plane figure; the proximate genus would have the properties of the remote genus; in other words, plane figure and solid would be the same figure. Consequently, all these terms could differ only as the words of different languages signifying exactly the same thing, as "horse", "equus" and "cheval".

17. Both from what has been said in n.4 and in the preceding paragraph, it is clear that the whole procedure is purely logical. The genus used throughout is a pure form regarding its inferiors as the matter. The differences appear only on the part of the matter. The procedure consists in trying to deduce the formal differences from the matter, to somehow generate these differences, instead of taking them in their mere givenness. For instance, the formal difference of polygon and circle is given. From the matter of polygon, that is, from its various species, we try to account for that which is formally different from polygon, viz. circle, as if the difference of circle came from the matter of another difference; or, as if one difference came from another by means of its matter, the matter being as the expression of the fecundity of the generating difference.

Provided we establish the kind of logical community required for abstract identity, a system may be expanded indefinitely and made to hinge upon one single term expressing its fecundity in differences by movement.

What should never be forgotten is that the whole procedure always remains within the bounds of "as if".

Limit and Dialectical Movement

16. There are many reasons why we should call the movement toward a limit, "dialectical" movement.

The form and the matter of the variable, from which the movement starts and within which it proceeds, must be taken logically. The movement itself never gets beyond these logical terms. The abstract identity, within which the whole system is contained as in a form, hinges upon a genus which is to be taken logically.

The movement itself is a movement produced by reason, a movement of reason as projected into the object. It remains within the bounds of what can come from reason alone. Although it has a "nature" as a term, it has it only as a term "toward which", but forever unattainable.

This movement which is on the part of an object considered in its "esse cognitum" (that is, as to what it has because of its being known, or according to the "modus intelligendi rem ipsam" as distinguished from the "modus rei intellectae in suo esse")- this movement is comparable to that relation of reason which is a second intention. It does not terminat "in" that "toward" which it is moving, and in one respect it is more an "intention" of reason than the second intention which is the object of logic, for it is a "dynamically tending toward", namely toward that which lies

beyond what is of reason alone.

Again, it may be called dialectical in the more restricted sense of "dialectical", because it is a tendency of reason to overcome a contrary, without itself (of itself) ever reaching "what is", as is the case with the dialectical syllogism. Moreover, just as no dialectical reasoning could, of its own accord, reach the truth without contradiction, so this movement could not reach its term (as to what it is in itself), without contradiction.

Furthermore, the peculiar unity of the system produced by this movement, its unity in becoming, is a unity due merely to our way of looking at the system, and not to what things are in themselves.

19. Viewed in this light, all things are, as it were, afloat in their very immobility. While this mode is like the mode of universal mobilism, nevertheless its universality here is purely relative, for the mobility would have no meaning if the things that are viewed as mobile were not absolutely immobile. The mobility is but a means, the better with which to reach the immobility.

By means of this dialectical movement, reason constructs a floating imitation of some "one" and "immovable". What is the original of this imitation?

The Purpose of the Method of Limits

20. The purpose of the method of limits needs no justification here when we confine our attention to its application in the calculus and in experimental science. What we want to establish at this point is, first of all, its general purpose simply as a mode of knowing when used merely to view something as a limit and, then, the purpose of its application to a domain from which it has been enthusiastically banished by so many philosophisers.

Obviously, the purpose of this method is not to reduce all differences to void identity and sterile contradiction. "...Sous peine de détruire le terme même de cette réduction, il faut se rendre compte qu'elle est purement dialectique, que le mouvement imprimé aux choses n'est qu'un mouvement de la raison projeté dans les objets, et que cette réduction demeure à l'état de tendance. Ce mouvement n'a pas pour but la réduction

des natures connues: cette réduction se fait dans la connaissance strictement scientifique où une nature est connue comme la raison de l'autre, l'une et l'autre demeurant radicalement distinctes; il a pour but la réduction des moyens de connaître. Mais cette réduction ne peut être que tentative; si on la faisait aboutir, elle serait frustrée par la destruction des natures que nous voulons atteindre dans leur différence."[16]

Yet, it might be asked here, if the purpose of this mode of knowing is the better to arrive at a knowledge of different things in their differences, why do we attempt to reduce these things, as it were, to a unity which would exclude all multiplicity and variety from itself? The answer is that we are not really trying to reduce the differences to unity just for the purpose of reducing them to unity or to absolute "oneness" exclusive of all multiplicity; what we are really trying to reduce, without any hope of ever getting beyond the stage of "trying", is the multiplicity of our means of knowing.

For the proper knowledge of any nature we need a distinct means of knowing. Polygon and circle, for example, may be known in a common way

as "plane figure", or in a still more common way merely as "figure". Such common knowledge, however, does not attain those specific figures in their differentiated nature. It is a proper knowledge that is required if polygon and circle are both to be known as to what they are, not in their common, but in their different nature. This shows that the means by which we know are themselves divided one from the other. The cause of this is on the part of the knower, not on the part of the known absolutely.

For us, to know something by a more universal means is to know it only in a confused manner; for, as St. Thomas says:

"...[C]ognoscere aliquid in universali, dicitur dupliciter. Uno modo ex parte rei cognitae, ut scilicet cognoscatur solum universalis natura rei; et sic cognoscere aliquid in universali est imperfectius; imperfecte enim cognosceret hominem qui cognosceret de eo solum quod est animal. Alio modo ex parte modi cognoscendi; et sic perfectius est cognoscere aliquid in universali: perfectior enim est intellectus qui per unum universale medium potest singula propria cognoscere, quam qui non potest." [17]

"....Cognoscere aliquid in universali, potest intelligi dupliciter. Uno modo ut referatur ad cognitionem ex parte cogniti; et sic cognoscere aliquid in universali est cognoscere naturam universalem cogniti .....; ...quando cognoscitur de aliquo natura universalis tantum, imperfectius cognoscitur quam si cognoscantur eius hee propria ipsius. Alio modo ut referatur ad cognitionem ex parte ejus quo cognoscitur; et sic cognoscere aliquid in universali, idest per medium universale, est perfectius, dummodo cognitio usque ad propria reducatur."[18]

Now, according to this principle may be determined the degree of perfection of an intellect.

"...In hoc sunt in rebus aliqua superiora, quod sunt uni prime, quod est Deus, propinquiora et similiora. In Deo autem tota plenitudo intellectualis cognitionis continetur in uno; scilicet in essentia divina, per quam Deus omnia cognoscit. Quae quidem intelligibilis plenitudo in intelligibilibus creaturis inferiori modo et minus simpliciter invenitur. Unde oportet quod ea quae Deus cognoscit per unum, inferiores intellectus cognoscant per multa; et tanto amplius per plura, quanto amplius intellectus inferior fuerit. Sic igitur quanto angelus fuerit superior, tanto per pauciores species universalitatem intelligibilium apprehendere poterit: et ideo oportet quod ejus formae sint universaliores, quasi ad plura se extendentes unaquaeque earum. Et hoc per exemplum aliqualiter in nobis perspici potest. Sunt enim quidam qui veritatem intelligibilem capere non possunt, nisi sis particulatim per singula explicetur; et hoc ex debilitate intellectus eorum contingit. Alii vero, qui sunt fortioris intellectus, ex paucis multa capere possunt."[19]

That is why we could say: "Il est très vrai que la réduction dialectique du volume à la surface, de la surface à la ligne, et de la ligne au point, rend notre connaissance plus parfaite et plus semblable à la connaissance divine qui dans une espèce unique, dans un universel moyen de connaître, atteint toutes choses dans ce qui leur est le plus propre. Nous connaissons mieux l'intelligence humaine quand nous pouvons la voir comme la limite d'une dégradation dans la raison même d'intelligence."[20]

Hence, if we cannot actually attain to a definite unity for knowing things in their proper distinction, we can move toward such a unity and reach it in a tentative way.

If, then, the end of our intellectual life is to imitate as much as possible the mode of knowing of an ever more perfect intellect, the mode of knowing proper to the method of limits is not only one human means of imitating superhuman knowledge, but indeed one of the most excellent- and no wise man could look upon it with indifference.

"...La tentative de voir le cosmos tout entier comme une grande coulée, comme un immense torrent débordant continûment d'un logos unique, d'une raison première, et où les natures sont comme des tourbillons du flux, est très louable, voire essentielle à une vue sapientiale, pourvu qu'on se rende compte des limites de cette méthode et de ses conditions."[81]

Hence, if it is in any way possible, this method should be extended to the whole realm of human knowledge, and most particularly, then, to that science which is wisdom proper.

81. Let us consider how man may be viewed as a

limit, and even as a limit of two converging series, which would give us the absolute generation of man.

"Ubicunque enim est diversitas graduum, oportet quod gradus considerentur per ordinem ad aliquod unum principium. In substantiis igitur materialibus attenduntur diversi gradus specierum diversificantes in ordine ad primum principium, quod est materia; et inde est quod primae species sunt imperfectissimae; posteriores vero perfectiores, et per additionem se habentes ad primas; sicut mixta corpora habent speciem perfectiorem quam sint species elementorum, utpote habentes in se quidquid habent elementa, et adhuc amplius; unde similis est comparatio plantarum ad corpora mineralia, et animalium ad plantas. In substantiis vero immaterialibus ordo graduum diversarum superiorum attenditur, non quidem secundum comparationem ad materiam, quam non habent, sed secundum comparationem ad primum agens, quod oportet esse perfectissimum; et ideo prima species in eis est perfectior secunda, utpote similior primo agenti; et secunda diminuitur a perfectione primae, et sic deinceps usque ad ultimam earum. Summa autem perfectio primi agentis in hoc consistit, quod in uno simplici habet omnimodam bonitatem et perfectionem. Unde quanto aliqua substantia immaterialis fuerit primo agenti propinquior, tanto in sua natura simplici perfectiorem habet bonitatem suam, et minus indiget inhaerentibus formis ad sui completionem; et hoc quidem gradatim procedit usque ad animam humanam, quae in eis tenet ultimum gradum, sicut materia prima in genere rerum sensibilium; unde in sui natura non habet perfectiones intelligibiles, sed est in potentia ad intelligibilia, sicut materia prima ad formas sensibiles; unde ad propriam operationem indiget ut fiat in actu formarum intelligibilium, acquirendo eas per sensitivas potentias a rebus exteriori bus; et cum operatio sensus sit per organum corporeum, ex ipsa conditione suae naturae competit ei quod corpori uniatur, et quod sit pars speciei humanae, non habens in se speciem completam."[82]

This text gives us the framework for a dialectical consideration. Man may be considered as the

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joint limit of the two orders whose principles are the most remote in being: matter and God. When looking toward man, starting from those material things which are most remote from him, we may view him as the limit of the order of those material things; just as, within that order, we may consider plant as the limit of the order inferior to it, and brute as the limit of the different and ever more perfect species of plant. Furthermore, this same order may be viewed in the opposite direction.

In like manner, man may be considered as the inferior limit of the order of separated substances. This may be done from the viewpoint of substance, as is done when we consider the composite substance of man as the lower limit of the decreasing simplicity of the separated substances. From the viewpoint of intellect, the human intellect may be considered as the lower limit of the decreasing pure intellectuality of the separated intelligences:

"Envisagée dans sa condition de nature, l'intelligence des substances séparées est toujours en acte. Elle juge sans composition ni division; elle connaît les raisons des choses les

unes dans les autres sans discours; elle saisit intuitivement dans un mouvement quasi circulaire l'essence d'où elle émane et à la lumière de laquelle elle voit. Parce que l'ange est trop parfait pour subir les autres créatures dans la connaissance, Dieu lui a infusé depuis le matin de son existence des espèces intelligibles représentatives de l'univers qu'il avait choisi de former, espèces antérieures aux choses elles-mêmes. Imitant Dieu qui connaît toutes choses dans une espèce universelle, les esprits purs, à proportion qu'ils sont plus rapprochés de Lui, connaissent cet univers au moyen d'un nombre d'espèces toujours plus petit. Mais quand nous regardons la hiérarchie angélique dans le sens de son éloignement de l'intelligence première, l'intuition de l'essence s'appauvrit selon l'imperfection même de cette essence et de l'intelligence qui en émane. Pour connaître les autres choses, cette intelligence a besoin d'idées de plus en plus nombreuses, son activité est de plus en plus morcelée; le temps discret constitué par la suite toujours croissante de pensées et de vouloirs est de plus en plus atomisé; le présent se diffuse, s'éparpille en passé et avenir toujours plus distants. L'intel-

ligence est de plus en plus éloignée d'elle-même et des autres choses qu'elle connaît. A la limite de cette dégradation surgit une intelligence vouée hors d'elle-même, en pure puissance, semblable à la matière-première, tabula rasa, intelligence non-intuitive qui ne pourra s'éveiller à son acte propre qu'au moyen du singulier sensible, intelligible en puissance seulement. 'Ratio oritur in umbra intelligentiae: La raison humaine surgit dans l'ombre de l'intelligence.' Elle ne peut se connaître qu'en dépendance d'une espèce représentative d'autre chose que soi. Pour connaître des choses dans leur nature propre, il lui faut un nombre d'espèces intelligibles égal au nombre des natures qu'elle connaît; elle se met sous la dépendance des sens auxquels il faut autant d'espèces qu'il existe de formes singulières connues. La connaissance requiert, à ce niveau, non seulement un grand nombre de facultés sensibles internes et externes, mais aussi un dédoublement de la puissance intellectuelle en un intellect qui dévance la connaissance en pénétrant dans la pénombre du monde sensible pour éclairer les objets afin de les rendre assimilables, et un intellect qui connaît proprement les choses et qui se les dit. Notre intelligence ne peut vivre que dans la pénombre. La nécessité des ténèbres du monde sensible prend origine dans la faiblesse de notre intelligence. Par nature, notre vie raisonnable est la vie intellectuelle la moins parfaite qui se puisse concevoir."[23]

This same order may be looked at in the opposite direction toward the "intelligere subsistens", for instance.

23. It should be noted that in all these cases we presuppose distinct knowledge, just as in moving from polygon to circle, we presuppose distinct knowledge of the one and the other. The dialectical mode is added to this and tends to improve the mode of knowing. Now, if the mode of knowing is better, then the thing known is better known.

When applied to the domain of metaphysics, for instance, this mode of knowing presupposes the mode of knowing exemplified in the Metaphysics of Aristotle, and its fecundity will depend upon the greatness of the distinction. The tendency toward reduction must be toward the reduction of what is known to be distinct and in itself irreducible. Furthermore, and this may sound paradoxical enough, the purpose of the tendency toward reduction is not

the reduction of the natures known, but their very distinction, that is, a more distinct knowledge of their very distinction by the improvement of the mode of knowing. If we did not tend toward more distinct knowledge, we would be striving after the most imperfect intellectual knowledge possible.

> "Circa differentiam universalitatis et particularitatis specierum intelligibilium, hoc primo considerandum est quod, sicut hic dicitur et in libro Praecli, superiores habent formas magis universales; inferiores vero minus universales. Et hoc etiam Dionysius dicit, XII cap. Coelestis hierarchiae, ubi dicit quod Cherubin ordo participat sapientia et cognitione universaliori; sed inferiores substantiae participant sapientia et scientia particulariori. Quae quidem universalitas et particularitas non est referenda ad res cognitas, sicut aliqui male intellexerunt, existimantes quod Deus non cognosceret nisi universalem naturam entis. Cui consequens esset quod in inferioribus intellectibus tanto uniuscujusque magis in universali sisteret, quanto esset altior; puta quod unus intellectus cognosceret solum naturam substantiae, inferior vero naturam corporis. Et sic usque ad individuas species. Quae quidem existimatio aperte continet falsitatem. Cognitio enim qua cognoscitur aliquod solum in universali, est cognitio imperfecta: cognitio vero qua cognoscitur aliquid in propria specie, est cognitio perfecta. Cognitio enim speciei includit cognitionem generis; sed non e converso. Sequeretur igitur quod quanto intellectus esset superior, tanto esset cognitio ejus imperfectior. Est ergo haec differentia universalitatis et particularitatis attendenda solum secundum id quod per intellectus intelligit. Quanto enim aliquis intellectus est superior, tanto id quo intelligit est universalius. Ita tamen quod in illo universali ejus cognitio extendatur etiam ad propria cognita, multo magis, quam cognitio inferioris intellectus qui per aliquid magis particulare cognoscit."[84]

23. It might be objected that this mode of knowing which is here presented as an improvement on the kind of knowledge presupposed, is in reality most imperfect since it does not actually get beyond the logical order.

To this, we answer that this mode of knowing should not be considered in itself absolutely, but rather together with the kind of knowledge we initiate in obliquo when using it; for, a fortiori, the same objection might be raised against the via negationis of theology. Nevertheless, "quanto plures negationes de ipsis (substantiis immaterialibus) cognoscimus, tanto minus est confusa earum cognitio in nobis, eo quod per negationes sequentes prior negatio contrahitur et determinatur, sicut genus remotum per differentias".[25] And we might even compare it to the way in which probable knowledge may be far better than certain knowledge

> "Sed hoc est in quibusdam scientiis diversum: quia aliquae sunt magis certae aliis, et tamen sunt de rebus minus honorabilibus: aliae vero sunt de rebus magis honorabilibus et nobilioribus, et tamen sunt minus certae. Nihilominus tamen illa est melior quae de rebus melioribus et honorabilioribus est. Cujus ratio est, quia sicut dicit Philosophus in lib. undecimo de Animalibus, magis concupiscimus scire modicum de rebus honorabilibus et altissimis, etiam si topice et probabiliter illud scimus, quam scire multum, et per certitudinem de rebus minus nobilibus. Hoc enim habet nobilitatem ex se et ex sua substantia, illud vero ex modo et ex qualitate.[26]

Objections

24. When the mathematician views the area under a curve as the limit of the sum of the areas of the inscribed rectangles, he does not do this for the purpose assigned in the previous chapter. He uses this method in order to know the value of the area under a curve.

To this we answer that the mathematician does indeed use the method of limits for that purpose. In so doing, he may define the value of that area as precisely as he wishes, without however reaching infinite precision. It is this use of the method of limits about which there is perfect agreement on the part of all in mathematics as in the experimental sciences. Nevertheless this alone, far from exhausting the utility of the method of limits, is but one aspect of it and may be considered as purely subservient to the higher purpose we have described, which higher purpose has not been ignored by the greater mathematicians. Indeed, it receives its most universal application in the effort to reduce geometry to arithmetic,

for instance, and in the effort to reduce all mathematics to what the mathematicians call "formal logic". As Bertrand Russell has said:

> "Mathematics and logic, historically speaking, have been entirely distinct studies. Mathematics has been connected with science, logic with Greek. But both have developed in modern times: logic has become more mathematical and mathematics has become more logical. The consequence is that it has now become wholly impossible to draw a line between the two; in fact, the two are one. They differ as boy and man: logic is the youth of mathematics and mathematics is the manhood of logic..... So much of modern mathematical work is obviously on the borderline of logic, so much of modern logic is symbolic and formal, that the very close relationship of logic and mathematics has become obvious to every instructed student. The proof of their identity is, of course, a matter of details: starting with premisses which would be universally admitted to belong to logic, and arriving by deduction at results which as obviously belong to mathematics, we find that there is no point at which a sharp line can be drawn, with logic to the left and mathematics to the right."[27]

25. We have said that the limit in its absolute state is absolutely determined, and that it is determinately distinct from the variable, having its own definition. Does not this imply that the area under a curve is something quite definite, or that the square root of 2 is an entirely determined value? Now, is it not precisely because these values are unknown that we employ the method of limits?

The answer is that the area under the curve has a value to which the sum of the inscribed rectangles is approaching. The approaching would have no meaning if we did not suppose that we are approaching some definite value. We know that there always remains an irreducible difference between the proper area under the curve and any approaching sum of the rectangles. How do we know this? We would not if we did not know that the former must have some definite value which, however, we cannot define in terms of what is specifically different.

Furthermore, when we say that the area under a curve, or the square root of 2, has some definite value different from any we can express in the terms we employ when tending toward it or toward a limit, this does not imply that we can express their value in terms of their own species, such as the area under one curve in terms of the area under another curve. When we say that we know that there is a definite difference, we are not saying that we know what that definite difference is.

This applies equally to other domains where we have only a negative knowledge of the difference toward which we are moving. We do not know "what" God is, but we do know that He is "not this", "nor

that", etc. Such kind of knowledge of a difference is sufficient for our purpose.

26. This whole dialectical process seems to be contrary to what is firmly established in the Metaphysics. For, as has been stated here above, n. 13 by applying the method of limits to the very "what it is" of things, we try to derive one "propter quid" from another as if we were searching for the "propter quid" of a "propter quid", in the sense wholly rejected by Aristotle.

> "Now when we ask why a thing is, it is always in the sense 'why does A belong to B?' To ask why the cultured man is a cultured man is to ask either, as we have said, why the man is cultured, or something else. Now to ask why a thing is itself is no question; because when we ask the reason of a thing the fact must first be evident; e.g., that the moon suffers eclipse; and 'because it is itself' is the one explanation and reason which applies to all questions such as 'why is man man?' or 'why is the cultured person cultured?' (unless one were to say that each thing is indivisible from itself, and that this is what 'being one' really means), but this, besides being a general answer, is a summary one. We may, however, ask why a man is an animal of such-and-such a kind. It is clear, then, that we are not asking why he who is a man is a man; therefore we are asking why A, which is predicated of B, belongs to B. (The fact that A does belong to B must be evident, for if this is not so, the question is pointless.) E.g., 'why does it thunder?' means 'why is a noise produced in the clouds?' for the true form of the question is one thing predicated in this way of another. Or again, 'why are these things, e.g. bricks and stones, a house?' Clearly then we are inquiring for the cause (i.e., to speak abstractly, the essence); which is the case of some things, e.g. house or bed, the end, and in others the prime mover-- for this also is a cause. We look for the latter kind of cause in the case of generation and destruction, but for the former also in the case of existence.
>
> What we are now looking for is most obscure when one term is not predicated of another; e.g. when we inquire what man is; because the expression is a simple one not analysed into subject and attributes. We must make the question articulate before we ask it; otherwise we get something which shares the nature of a pointless and of a definite question. Now since we must know that the fact actually exists, it is surely clear that the question is 'why is the matter so-and-so?' e.g. 'why are these materials a house?' Because the essence of house is present in them. And this matter, or the body containing this particular form, is man. Thus what we are seeking is the cause (i.e. the form) in virtue of which the matter is a definite thing; and this is the substance of the thing."[26]

This objection wholly misunderstands the purpose of the present method. It is true that we are proceeding as if we were searching for such a reason. But that is not what we are actually searching for, as has been sufficiently shown. We are trying to reduce the means of knowing, and not what we are apparently tending to reduce. If what Aristotle shows to be true were not true, then we could not even start applying the method of limits; and if by applying this method we intended such a reduction, we would be defeating our very purpose.

27. It is difficult to see how this method may be

legitimately extended beyond the field of mathematics. During the whole discussion, emphasis was laid on the expression "values", which expression is quite unobjectionable when understood mathematically, that is in terms of quantity or of something reducible to quantity. But how can we consider a genus as a variable?

Any genus may be considered as a variable, and a species as one of its values, since the "more" and the "less" may be said of a species in exactly the way required by this method, as St.Thomas shows:

> ".. Magis et minus est dupliciter. Uno modo secundum quod materia eiusdem formam diversimode participat, ut lignum albedinem; et secundum hoc magis et minus non diversificat speciem. Alio modo secundum diversum gradum perfectionis formarum; et hoc diversificat speciem. Diversi enim colores specie sunt secundum quod magis et minus propinque se habent ad lucem; et sic magis et minus in diversis Angelis invenitur."[29]

This shows sufficiently that the method is not properly an extension of the mathematical theory of limits, but rather that it is based on common notions of which the case of mathematics is but an instance.

28. The method of limits always introduces infinity and movement on the part of what is under consideration, which would suggest that only mobile things are susceptible to this method. Then, mathematics, which does not admit of motion, would be thereby excluded. Furthermore, how could we introduce movement into the order of separate substances?

Even when the things considered furnish a direct basis for infinity, such as the series of integers, the species of brute, or even the species of separate substances, the infinity formally used in the method of limits is but an artifice of reason to which we give the status of a means of knowing on the part of the knower. As for the case of movement, we even use the notion of movement when speaking dialectically of God, as may be seen from the de Divinis Nominibus, ch.9, lesson 4. Furthermore, our notion (negative) of actual infinity is based on our notion of potential infinity: we use it when conceiving actual infinity.

29. Does not this method of limits, in which we consider the differences as coming from the subject, make us fall into the error of the Platonists? For, St. Thomas says: "quod ponere diversitatem rerum propter diversitatem susceptivi tantum, est opinio

platonica, quae posuit unum ex parte formae, et dualitatem ex parte materiae; ut tota diversitatis ratio ex materiali principio proveniret. Unde et unum et ens posuit univoce dici, ut unam significare naturam sed secundum diversitatem susceptivorum, rerum species diversificari."[30]

This "opinio platonica" is false when understood in the natural sense, not when it is understood in the dialectical sense. What St. Thomas calls a "platonist error" is a confusion of logical inquisition with natural inquisition. As he had said in a preceding paragraph: "Est autem considerandum, quod multa quidem secundum abstractam considerationem vel logici vel mathematici non sunt aequivoca, quae tamen secundum determinatam rationem naturalis ad materiam applicantis, aequivoce quodammodo dicuntur, quia non secundum eandem rationem in qualibet materia recipiuntur: sicut quantitatem et unitatem, quae est principium numeri, non secundum eandem rationem contingit invenire in corporibus caelestibus et in igne et in aere et aqua."[31]

Analogy and the Method of Limits

30. When we say that the method of limits may be extended to all being, we mean that it may be applied to the whole scale of being in such a manner that each degree of the scale may be considered as an upper limit of the order below it, or the lower limit of what lies above, and that the whole scale of limits may be considered with respect to an absolutely upper limit, God, and with respect to an absolutely lower limit, absolute non-being. According as we look upward or downward, we then may that anything that lies within the scale tends either toward God or toward non-being. This might be compared with the series of integers, the upper limit of which would be actually infinite multiplicity (if such is possible), and the lower limit, zero.

Although one might concede the procedure as a fact, it remains difficult, nevertheless, to see how its legitimacy could be established in terms of the foregoing analysis. So long as we may conceive different things with respect to a common logical genus, we can easily see how the differences may have abstract identity. But how can the method of limits be applied to the whole scale of being when being is not univocal? The differences

of being do not come from the subject, nor does being have the unity which would make it predicable with identity of differences.

That we do require a term more universal than real being and that, at the same time, we must presuppose analogy, is clear enough from the fact that the application of the method of limits here means a tendency to reach beyond analogy. Since the tendency toward identity presupposes some abstract identity, and since in this method we do tend toward identity, therefore in this method we tend to deny analogy. In fact, we must consider this effort to get away from analogy as being essential to our purposes in using the method of limits.

But why should one try to reach beyond analogy, and, in so doing, tend to destroy it? The simple reason is, as we are well aware and cannot forget, that knowing by analogy ever remains an extremely imperfect mode of knowing. When we apply the method of limits to the whole ~~realm~~ scale of being, we tend toward a divine mode of knowing, from which knowledge according to analogy must be excluded. Tending toward this mode, we tend to negate the merely proportional unity of our concept of being. In other words, we tend to break through the ratio entis

and to know God sub ratione Deitatis. We tend to know Him as to that which is not known of Him by analogy. The limit, then, is to know positively that which we know of Him only by negation, that is to know that which lies beyond the reach of a knowledge sub ratione entis entis. (We must be prompt to add, however, that this tendency is purely dialectical and that an arrival at the term of the tendency would result in contradiction.) The expression "only by negation" does not refer merely to the negation implied in all analogical knowledge of God. The negation formally considered in this application of the method of limits turns us away from the previous negation, so that it is as a negation of negation. The former (i.e. the negation implied in analogical knowledge of God) is static, fixed. By the latter, however, we keep moving away from that which is denied of God toward God as to that which surpasses any conception we can have. For instance when we say that God is good, we must understand that His goodness is other than the goodness of the creature as creature, and therefore, with respect to this goodness, we must say that He is not good. The goodness that is predicated of Him, consequently, must in some way suppose the otherness of His goodness expressible only by negation. Now the method of limits adds to this

by gradually bringing us, in an oblique way, closer to that divine goodness in its otherness. The negation now takes on a dynamic form and approaches a term. For instance, we consider the goodness of some lower creature and then negate thus: Divine Goodness is not like this. Then we move on to a higher creature and again negate, and so on. The first general negation (i.e. God is not good with the goodness of any creature) left us in a static generality, in which we always remained remote (and definitely remote, so to speak) from the Goodness of God. The second dynamic negation, however, while never getting beyond the boundary of that general negation, does imply a certain "getting ever closer" and, in this respect, tends to break through the barrier of the general negation. It is this tendency to break through the first negation that makes the dynamic negation differ fundamentally from the former.

Whereas the emphasis of analogical knowledge is on affirmation, that of the present dialectical knowledge is on negation. Yet, just as the movement toward the otherness of the limit remains within the variable, so the movement toward "what God is" remains within the boundary of analogical knowledge and is never actually severed from the negation which it is to deny. The decreasing difference is infinite and is forever "not yet" null.

31. We can now narrow down the difficulty and express it in the form of a simple question: what can be common to "what is knowable by analogy" and "what is known by negation only", in the manner required for the application of the method of limits? In other words, what is the univocal term involved? The negation in question is in the mind only; it is nothing more than a tentative, dialectical negation of the negation already involved in the knowledge by analogy. Its term is never reached and never actually expressed.

Now, when we consider the scale of created beings as tending toward uncreated being as toward a limit, we must suppose that both the variable and the limit convene in some "ratio univoca". Where can we find this "ratio"? From what we have said in the previous chapters, it is clear that the method of limits is confined to things considered in their "esse objecti" as distinct from the "esse rei". Again, when we speak of the "knowable by analogy" and the "knowable by negation", it is not a question of the knowable in the absolute knowability that it has "in esse rei", for we are not speaking in esse rei we are not

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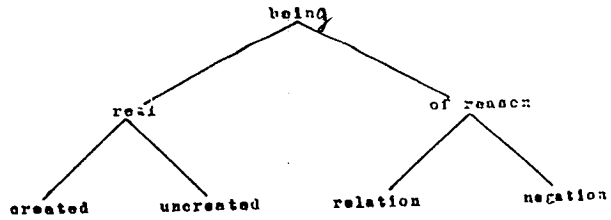
of the "objectum in esse rei", but of the "objectum in esse objecti".

So long as we consider being, not entitatively, but in a purely objective manner (that is, being formally taken in what it has as object), it is a "ratio univoca" in which convene both real being and being of reason, created being and uncreated being, substance and accident, etc. "...Licet entitative ens reale et ens rationis analogentur, tamen objective, cum unum ad instar alterius representetur, possunt in ratione univoca objecti convenire etiam quae entitative univoca non sunt, ut Deus et creatura, substantia et accidens in ratione scibilis metaphysici vel intelligibilis ab intellectu."[31]

Provided that we confine ourselves strictly to this viewpoint of "esse objecti" and that we regard the "esse rei" in a purely dialectical manner, we may then construct the following diagram, wherein each term is predicated univocally of its inferiors:



Now, since being, so taken, is a "ratio univoca", it does not actually include the differences; it is divided by "real being" and "being of reason" and, therefore, cannot be predicated with identity of these terms. It is important to notice, however, that it is not divided by "created" and "uncreated". Because of this, we may say that, considered "in esse objecti", created being and uncreated being are the same being "in esse objecti".

This is enough as a starting point for the whole system, which is then "one in becoming" by the approach from created toward uncreated. If, per impossibile, the term of this movement were achieved, created and uncreated would be identical; real being would be identical with either; real being with being as divided by real and of reason; real being and being of reason would be identical; to consider being entitatively and to consider it objectively would be identical.

31. In the foregoing paragraph we have merely applied what Saint Thomas says in a text we have already quoted (supra n.29): "... quod multa quidem secundum abstractam considerationem vel logicae vel mathematici non sunt aequivoca, quae tamen secundum concretam rationem naturalis ad materiam applicantis, aequivoce quodammodo dicuntur, quia non secundum eandem rationem in materia recipiuntur".

There is perhaps no clearer instance of how reason can put together and contain in one wholly indistinct unity, things that absolutely cannot be together, than the instance of infinite names. For example "not-man" may be predicated of horse, of being of reason, and of the impossible.[33] The signified of "not man" has a unity which is absolutely impossible in reality. "...Nomen infinitum quodam modo significat unum. Non enim significat simpliciter unum, sicut nomen finitum, quod significat unam formam generis vel speciei aut etiam individui, sed inquantum significat negationem formae alicujus, in qua negatione multa conveniunt, sicut in quodam uno secundum rationem. Unum enim eodem modo dicitur aliquid, sicut et ens; unde sicut ipsum non ens dicitur ens, non quidem simpliciter, sed secundum quid, idest secundum rationem, ut patet in IV Metaphysicae, ita etiam negatio est unum secundum quid, scilicet secundum rationem. Introducit autem hoc (Aristoteles), ne aliquis dicat quod affirmatio, in qua subiicitur nomen infinitum non significet unum."[34]

That being may be predicated of non-being is clear from the fact that being may be predicated of being of reason, which is divided into relation and negation. Non-being, in this sense, has a kind

of being . "Unde dicimus quod non-ens est non-ens. Quod non diceretur nisi negationi aliquo modo esse competeret." [33] "...Non ens dicitur multipliciter sicut et ens. Uno modo dicitur quod est secundum compositionem et divisionem propositionis. Et hoc, cum non sit in rebus, sed in mente, ..." [34]

33. In the present application of the method of limits we are tending toward a mode of knowing whose absolute limit is the properly divine mode of knowing. Paradoxically, we tend toward this mode of knowing by overtly exploiting that which is most expressive of the imperfection proper to human knowledge: negation. By negation we tend to wrestle away from the mode of knowing by negation. Both negations are from reason, but one is turned against the other, in so far as that one tends to break through the other. In the use of this method, the order of reason is other than the real order, turns against itself. If we call this pure reason, we might say that reason places itself in contradiction with itself and tends to negate itself by moving from within toward the otherness of that is most remote from its Qui Est.

Reason and Intellectuality

34. The human intellect differs from all other created intellects in that it can never actually know all that it is capable of knowing.

> "Hoc autem ad naturam angelicam pertinet, ut actu habeant notitiam omnium quae naturaliter scire possunt; sicut nos naturaliter actu habemus notitiam primorum principiorum, ex quibus procedimus ratiocinando ad acquirendam cognitionem conclusionum; quod in angelis non contingit, quia in ipsis principiis intuentur omnes conclusiones quae ad naturalem eorum cognitionem pertinent. Et ideo sicut immobiliter nos habemus in cognitione primorum principiorum; ita intellectus eorum immobiliter se habet circa omnia quae naturaliter cognoscit." [35]

Strictly speaking, the human intellect is intellectual only by participation.

> "Anima autem humana intellectiva dicitur per participationem intellectualis virtutis. Cujus signum est quod non tota est intellective, sed secundum aliquam sui partem. Pertingit etiam ad intelligentiam veritatis cum quodam discursu et motu arguendo. Habet etiam imperfectam intelligentiam; tum quia non omnia intelligit, tum quia in his quae intelligit, de potentia procedit ad actum." [36]

Human science, then, is imperfect in so far as it involves discourse.

> "...quia illud quod in Deo est absque omni imperfectione, in creaturis cum aliquo defectu invenitur; propter hoc oportet, ut si aliquid in creaturis inventum Deo attribuatur, separemus totum quod ad imperfectionem pertinet, ut solum maneat hoc quod perfectionis est; quic secundum hoc tantum creatura Deum imitatur. Dico igitur quod scientia quae in nobis invenitur, habet aliquid perfectionis, et aliquid imperfectionis. Ad perfectionem ejus pertinet certitudo ipsius; quia quod scitur, certitudinaliter cognoscitur; sed ad imperfectionem pertinet discursus intellectus cognoscens principia, concludit in poten-

tia tantum conclusiones; si enim motu cognosceret, non esset ibi discursus; cum motus non sit nisi exitus de potentia ad actum."[39]

That which the angelic intellect possesses from the very beginning is as the limit toward which the human intellect tends by means of what is properly human.

> "...Inferiores intellectus, scilicet hominum, per quendam motum et discursum intellectualis operationis perfectionem in cognitione veritatis adipiscuntur; dum scilicet ex uno cognito in aliud cognitum procedunt. Si autem statim in ipsa cognitione principii not inspicerent, quasi notas, omnes conclusiones consequentes, in eis discursus locum non haberet. Et hoc est in angelis; quia statim in illis quae primo primo naturaliter cognoscunt, inspiciunt omnia quaecunque in eis cognosci possunt. Et ideo dicuntur intellectuales; quia etiam apud nos ea quae statim naturaliter apprehenduntur, intelligi dicuntur, unde intellectus dicitur habitus primorum principiorum. Animae vero humanae, quae veritatis notitiam per quendam discursum acquirunt, rationales vocantur. Quod quidem contingit ex debilitate intellectualis luminis in eis. Si enim haberent plenitudinem intellectualis luminis, sicut angeli, statim in primo aspectu principiorum totam virtutem eorum comprehenderent, intuendo quidquid ex eis syllogizari posset."[40]

Now, when we consider the movement of human reason in the light of the method of limits, it is paradoxical that it should be using precisely that which makes human science imperfect, in order to imitate more closely pure intellectuality. The potentiality of the human intellect is "somehow" overcome by the act of its dialectical movement specified by the term, pure intellectuality. It is as if the intellect, by acquiring an act in its potency as it is in potency (that is, in the potency that always lies beyond science definitely in act), acquired, in this "always reaching beyond", an act more definite, in its dynamism, than any definite act of science.

Dialectical Movement and Finality

39. It might be said that to call this movement something more definite and ultimate than any definite act we can adequately reach is wholly ridiculous, since movement cannot be an end. "Motus enim, ex ipsa sui ratione repugnat ne possit poni finis, et quod motus est in aliud tendens; unde non habet rationem finis, sed magis ejus quod est ad finem. Cui etiam attestatur, quod est actus imperfectus...."[41]

To this we reply that our position should not be understood to imply that the movement is an end in itself as movement. The end we propose is "the getting closer" to the term of the movement. For, to get ever closer to the term can be an end if by end we mean that which can be actually reached. That this involves movement does not imply that we pursue

the imperfection of movement for its own sake. Or again, if we remain in a state of movement, it is not because we choose the movement for its own sake, but because it is the only means of getting closer to the term. The necessity of remaining in the state of movement is but a consequence. It is therefore only by accident that the movement has "ratio finis".

We may therefore compare this dialectical movement to the movement of the celestial bodies, according to the first hypothesis stated by St.Thomas in the very article from which the above objection is drawn: "Oportet ergo finem motus coeli ponere aliquid quod coelum per motum consequi possit, quod sit aliud a motu, et eo nobilius. Hoc autem dupliciter potest poni. Uno modo ut ponatur finis motus coeli; quod quidem fit ipso motu durante; unde secundum hoc non convenit quod motus coeli deficiat; quia deficiente motu, finis ex motu provenions cessaret. Alio modo, etc.".

Now it is true that St. Thomas excludes this opinion as less probable. But we should note the reason: "Licet autem utraque positionum praedictorum possit rationabiliter sustineri; tamen secundum, quae fidei est, videtur esse probabilior propter tres rationes....". Now all the reasons he gives[43] are based on the fact that the term of the movement of the heavenly bodies is something which can actually be achieved. If such is the case, then that must be the end. When he says that "causare non potest esse finis, cum sit operatio habens operatum, et tendens in aliud; hujusmodi enim operationibus meliora sunt operata,...", he does suppose that that "aliud" is something which can be achieved, the "operatum" as distinct from the operation.

This, however, does not hold in the case of dialectical movement, where the term of the movement cannot be attained. It is better then, to tend toward it, and get ever closer, even without the possibility of ever reaching it, than to remain in a state of rest- and especially because, in an oblique way, this movement is a better imitation of "active immobility" than is the "rest" of the state of potentiality prior to movement.

Again, we should not forget that St. Thomas is speaking of a movement in reality, whereas we have confined the dialectical movement to reason.

However, when we consider the actual order of things, namely the elevation of the human in-

tellect to the supernatural order, the case of the dialectical movement we are studying becomes similar to that of the celestial bodies. This movement is no longer something ultimate, for, due to grace and not to the dialectical movement, the term of this ever closer approach can actually be attained; for this term is the divine mode of knowing on the part of the Blessed.

36. According as the separated substances are more perfect, they know the many in an ever smaller number of ideas. God knows all things down to their ultimate diversity in one single means, identical with His essence. We have already shown how the application of the method of limits enables us to imitate (remotely, of course, but truly nevertheless) this mode of knowing.

There is, however, another aspect of the knowledge of separated substances which this method allows us to imitate, namely the priority of their knowledge to the things which they know by means of intelligible species. The angels receive these species, not from the things they know, but directly from the First Cause, so that the being things have in the angelic intellect is prior to the being they have in themselves. In fact, we may say that they have three kinds of being, according to St. Augustin's interpretation of Genesis I.

"...Secundum Augustinum per illa tria significatur triplex esse rerum. Prime quidem esse rerum in Verbo; nam res prius habent esse in arte divina, quod est Verbum, quam in seipsis; et hoc significatur cum dicitur: Dixit autem Deus, fiat; id est, Verbum gessit, in que res erat ut fieret. Secundo esse rerum in mente angelica; quia nihil Deus produxit in rerum naturo cujus naturam non impresserit in mente angelica; et hoc significatur cum dicitur "factum est; scilicet per influentiam Verbi in mente angelica. Tertio esse rerum in propria natura; et hoc significatur per hoc quod dicitur "fecit". Sicut enim ratio qua creatura conditur prius est in Verbo quam in ipsa creatura quae conditur, ita et cognito ejusdem rationis prius est ordine naturae in mente angelica quam sit ipsa creaturae productio; et sic angelus triplicem cognitionem habet de rebus, videlicet prout sunt in Verbo, prout sunt in mente ejus, et prout sunt in propria natura; prima vocatur cognitio matutina, aliae duae sub vespertina cognitione comprehenduntur. Et ad ostendendum hunc duplicem modum cognitionis rerum in creatura spirituali, dicitur: Factum est vespere et mane dies unus."[45]

The variable ordered to a limit is as a means of knowing prior to the limit in its absolute state. It is as if we drew the knowledge of all that we know, from within ourselves.

37. This mode of knowing, in which human reason tends toward a superhuman mode of knowing, is at the same time most proportioned to the human intellect, since it is essentially discursive. As we have seen, the human intellect is rational properly but is intellectual only by participation. Whenever anything can be known by us in the discursive mode, it is

better known than when known only in an intuitive, intellectual manner. That is why we surround even our first notions and first principles, whose scientific inference would be contradictory, by dialectical discourses. By so doing, we somehow make them more proportioned to our proper mode of knowing. It is to be noted, for instance, that even the principle of contradiction, the first of all principles, is nevertheless the limit toward which converge all the sapiential discourses in defense of its primacy.

The Method of Limits and Will

58. Any complete definition of a limit will comprise the expression: "to which we can approach as much as we wish", or, "autant que l'on veut". This should be understood to mean "as much as we choose". The tendency toward a limit, therefore, implies not only will in its very definition; it actually implies free will.

One might object that this is not al all characteristic of the method of limits, for, as St. Thomas says: "ipse actus speculativae rationis, secundum quod est voluntarius, cadit sub electione et consilio quantum ad suum exercitium..."[44] But this very text contains the answer to the objection, viz. this act depends on the will- as to the exercise, however, and not as to the specification. Will does not enter into the very definition of our speculative science. For instance, there is a definite conclusion which is true whether we draw it or not. Speculative science is measured by the object. Yet, in the case of a variable ordered to a limit, there can be no definite amount of steps to be taken. Although we must make the steps according to definite rules, the steps we make are not definitely there until we do actually make them. All that is there is the infinity which, within the bounds of the rule, we may cut up as, and as much as, we wish. We can get as close to the limit (without ever reaching it) as we wish. If this were not a matter of choice, it would mean that there is a closest step.

One might suggest that the method of limits is an art, and that art implies free will. But this is far too simple. Speculative art, such as logic, has nothing to do with free will. Sculpture, however, as an instance of practical art, does suppose freedom- but only in regard to this statue willed as an end to be achieved, rather than that statue. But there is no freedom concerning the means, once the end is well chosen. Although the

choice of the degree of approximation in the method of limits is somehow comparable to the choice of an end in sculpture, the degree of approximation that is chosen is essentially provisional. The choice must go on and on. Furthermore, that toward which we tend when applying the method of limits is not something whose conception is made by us as to the "what it is", as in the case of the quasi-universal that it "this statue".

In the freely constructed approximation, we tend, nevertheless, to conceive immutable essences as ideas to be formed, as if they were suspended to freedom, even as to what they are. If, per impossibile, the limit could be achieved, abstract natures would be the fruits of creative will. This brings out clearly enough the absurdities involved in a misunderstanding of the meaning of tendency toward a limit.

The method of limits is indeed an art, but its exercise requires an intervention of will (and of free will) different from that required for the exercise of the other habitus of the intellect. It is important to note this, for the misinterpretation of this method may very reasonably account for much that is perverted in modern philosophy.

The "Thing in Itself" as a Limit

39. We have already seen in what sense the knowledge of God as He is in Himself is the limit of the universal application of the method of limits. The same holds for the most humble of creatures, the natural beings which are the object of our experience, for the knowledge of these things in their ultimate and specific concretion is itself but a limit, which can ever be approached without ever being attained. Thus, what Eddington call the "absolute world condition" is but a limit of experimental science.

The case of the experimental sciences is indeed infinitely more complex than that of mathematics, for in the former we never have but general and confused knowledge of that toward which we are tending, such as the proper matter of man or of elephant, or the ultimate physical constituent of things. The reason why the natural things in themselves can never be but a limit, so soon as we wish to attain them in their proper and complete natural concretion, will be sufficiently accounted for here, when we consider the method to which we are confined in the experimental sciences. If we define the experimental sciences as those branches of natural

discourse which formally start from purely experimental propositions (that is propositions whose terms are united not because we see, either immediately or through demonstration, the "why" of their connection, but because we encounter them together in sense experience), then no knowledge derived from them can be definite.

The Evolutionary Method

40. The evolutionary method is but one phase of the method of limits, namely the tendency toward an upper limit. When we apply this phase of the method to natural species, we try to envisage the higher differences as arising from the matter of the lower. This upward movement takes on a realistic aspect because of its association with time. The realistic appearance of the method is even greater when we realize that there is evolution in nature. However, the error of most evolutionary philosophies is due to the fact that the natural evolution is believed to be sufficiently accounted for by the evolutionary method. But this is wholly absurd, unless it were maintained that the differences of things are merely apparent. In such a case, however, we could not speak of evolution

except in a very weak sense; neither would the method then be the method of limits.

A strictly natural account of evolution must be made in terms of the four natural causes. Man then, for instance, is not merely the limit of what precedes him in time but is actually the final cause. If nature had to make all the possible steps between some given species of plant and some given species of brute, the latter would never be reached. Nature does proceed according to a certain order, but not strictly according to the order of the method of limits. The latter may be used as a dialectical frame-work to be used for the discovery of the order of nature, but the two cannot be identified without contradiction.

However, the reaction against evolutionary conceptions has been, in general, most inept, since no account was taken of these distinctions. There was a notable failure to recognize and acknowledge this method for what it really is, viz. an incomparably useful instrument and, more profoundly, an attempt to see the dynamic unity of nature- but to see it, as far as possible, by a single means.

It remains true that there is a profound resemblance between the "becoming", essential to the method of limits, and the becoming of natural things, just as there is a profound similitude between the human intellect and matter because of their potentiality. Just as matter accounts for the becoming of natural things, so does the potentiality of the human intellect account for its ratiocinative nature. We may safely say that that which is most "really" similar to the dialectic of the method of limits is the universe of mobile being. Because of this similarity, modern philosophers have been prone to confuse them and to see dialectic in nature itself.

Let us consider a concrete example in which sense experience might be used to elucidate the confusion. We might construct a polyhedron and increase the number of its surfaces to the point where the senses could no longer distinguish it from a sphere. One might then say that the polyhedron has become a sphere, or, more grossly, that it is both a polyhedron and a sphere. It is just this sort of naïve procedure that accounts for current misinterpretations of the calculus when applied to physics.

PART III

[illegible] MODERN PHILOSOPHICAL
TEXTS RELEV. [illegible] METHOD OF LIMITS

Some Ancient and Modern Philosophical Texts Relevant to The Method of Limits

The objections that are raised against the application of the method of limits to the realm of what is called philosophy proper are usually expressed somewhat in the following manner:- "You are talking like Anaximander"; "Now you fall back into the error of the Pythagoreans"; "Do you take Parmenides seriously?"; "Was not that the error of Zeno?"; "Did not Aristotle refute all these platonic positions?"; "How is this to be distinguished from Neoplatonism?". The objections may go on up to more recent philosophers, such as Bergson.

For the sake of convenience, we shall henceforth call the application of the method of limits the dialectical mode, as opposed to the natural mode, the latter being characterized by its consideration of "ens naturae" in terms of "ens naturae".

When viewed according to the dialectical mode, many of the most fundamental statements and theories made by non-aristotelian philosophers, both ancient and modern, are not only entirely correct, but they do express something which cannot be accounted for

in any mode. Indeed, if the method of limits has the depth and importance for wisdom that we claimed for it, it follows that much of the criticism that has gone under the label of "aristotelian" or "thomist" has not only been indiscriminate, but actually destructive of that without which there can be no profound imitation of divine wisdom.

It is true that when a dialectical statement is put forth as a strictly realistic one, it should then be vigorously criticised, precisely as was done by Aristotle. The criticism, however, should not be intended to reflect upon the dialectical mode, but upon the false identification of this mode with the natural one. We must point out in what very definite sense these statements may be true.

Some Texts from the Greeks

41. In early Greek philosophy, the dialectical mode and the scientific mode were never seen in that clear distinction which Aristotle's philosophy allows us to make. The situation was all the more more perplexing, because that part of the dialectical mode and that use of it which should come only after the natural mode, were given precedence over the latter, as if the latter were being dispensed with. This dialectical mode was somehow given the status of the dialectic of the Topics, which is but a preparation to the natural mode. We find many statements, indeed fundamental positions, in Heraclitus and Parmenides, which, when understood in the dialectical mode as we here take it, are entirely true; but they can only be understood in this mode when we presuppose the natural mode. In this respect, the early Greek philosophers, and perhaps even Plato, "brulaient les etapes".

Let us consider a few examples.

42. The universal application of the method of limits requires that we see infinity everywhere, both separated from everything, and as that from which everything comes, and in everything, as that

from which all things come. It is also that into which all things are resolved. This aspect of Anaximander's apeiron must be retained. It is not enough, then, to convert this infinite into actual infinity, and to see in Anaximander's first principal of all things but an inept conception of divinity. On the contrary, from an absolutely universal point of view, Anaximander's infinity calls for a separation into actual infinity (which we would identify with God) and a potential infinity (not the one which is to be identified wholly with prime matter, but an infinity which still has universal extension, namely the infinity we conceive as an artifice indispensable for the dialectical mode).

As is clear from what Aristotle says in the Physics, Anaximander's apeiron is an impossible combination of these two infinities:

> "So the 'unlimited' cannot be derived from any other principle, but is itself regarded as the principle of the other things, 'embracing and governing all', as it is said to do by such as accept it, such as 'Intelligence' or 'Amity'. This unlimited, then, would be the divinity itself, being 'immortal and indestructible', as Anaximander and most of the physicists declare it to be."[45]

It would therefore be foolish to say, for instance, that according to Anaximander, prime matter was the wholly first principle of all things, not only because the apeiron is infinitely more potential than matter, but because it is also what Aristotle says of it in the text just quoted. Neither could we object to an intimate union between what we shall call the two infinites, for the dialectical mode posits its own for the purpose of reaching toward the other in its very otherness. To tend toward the actually infinite is conditioned by the potentially infinite.

In other words, the universal extension of the method of limits requires that we bring again into play one aspect of Anaximander's apeiron.

45. The ideas that are fundamental to Pythagoras' philosophy are at the same time essential to the dialectical mode:- limit (peras), infinity (apeiron), the orderly "binding" (harmonia -- in its active and in its passive sense[46]), and the One. An examination of Pythagoras' conception would be far too involved for our present purpose. We must note, however, that peras is not just a common limit, for it actually bounds infinity; and the orderly binding is as a dynamic function of the "one" which is other than limit and infinity. The "one" may be compared to what we have called the absolute limit, and the One par excellence, with

the upper limit of the universal system. The "one" offers precisely the paradox which we have seen in the action of limit: it is both separated and mixed somehow with the infinite, as Professor Hack has shown: "It is the One (viewed as a combining agent) that creates order in each particular thing and group of things, from the Psyche to the city-state and the universe, by combining with itself some definite amount of the apeiron,.. "[47]

The mathematical bent of the pythagorean school is, again, in conformity with the dialectical mode, for as we have already seen, even when treating of non-mathematical entities, we always lean back on a mathematical instance. The reason of this is the homogeneity required for any consideration of limit. We might cite as an example a speculation on the separated substances in the platonic mode:

> "...Non videtur esse universaliter verum, quod imperfectior differentia generis in plures species multiplicetur. Corpus enim dividitur per animatum et inanimatum; plures tamen videntur esse species animatorum corporum quam inanimatorum, praecipue si corpora coelestia sint animata, et omnes stellae ad invicem specie differant. Sed et in plantis et in animalibus est maxima diversitas ~~specierum~~ specierum. Ut tamen hujus rei veritas investigetur, considerandum est quod Dionysius Platonicis contrarian sententiam proferre videtur. Dicunt enim Platonici quod substantiae quo sunt primo unai propinquiores, eo sunt minoris numeri. Dionysius vero dicit in cap.

XIV Angelicae hierarchiae (P.G.III,312), quod Angeli omnes materialem multitudinem transcendunt. Utrumque autem verum esse aliquis potest ex rebus corporalibus percipere; in quibus quarto corpus aliquod invenitur superius, tanto minus habet de materia, sed in majorem quantitatem extenditur. Unde cum numerus quodammodo sit causa quantitatis continuae, secundum quod punctus constituit unitas, et punctus linean, ut more Platonicorum loquamur: ita est etiam in rebus rerum universitate, quod quanto aliqua sunt superiora in ordine, tanto plus habent de formali multitudine, quae attenditur secundum distinctionem specierum; et in hoc salvatur dictus Dionysii: minus autem de multitudine materiali quae attenditur secundum distinctionem individuorum in eadem specie; in quo salvatur dictus Platonicorum. Quod autem est una sola species animalis rationalis, multis existentibus speciebus irrationalium, ex hoc provenit, quia animal rationale constituitur ex hoc quod natura corporea attingit in sui supremo naturam substantiarum spiritualium in sui infimo. Supremus autem gradus alicujus naturae, vel etiam infimus, est unus tantum; quamvis possit dici, plures esse species rationalium animalium, si aliquis ponaret corpora coelestia animata."[48]

44. Parmenides' distinction between the way of truth, i.e. the path of "It is", and the way of belief or path of that which is not", is often cited as an extreme example of hopeless philosophical folly.

Again we must be careful not to throw out the baby with the bath. When we consider the Being that is the upper limit of all things, then indeed all that is outside it, is but an appearance of being. Just as the variable becoming the absolute limit is but an appearance of this limit, just as the limit in its state of becoming is but an appearance of the absolute state. The Being of Parmenides may be con-

pared to the absolute state of the universal upper limit formally considered in its unattainability, but toward which and from which all things become. Being is "immovable within the Limits of mighty bonds, without beginning and without end, because coming-to-be and passing away have wandered far away, and truth has cast them away." It is the same, and it rests in the self-same place, abiding in itself. And thus it remaineth constant in its place; for mighty Necessity holds it in the bonds of Limit, which fences it in round about. Therefore it is not in accordance with divine law that What Is be incomplete, for it is not in need of anything, but if it were in need of anything, it would need everything.... Since there is an extreme (punaton, uttermost or final) Limit, Being is complete on every side, like the form of a well-rounded sphere, equally poised from the center in every direction; for it cannot be greater or smaller in one place than in another. For there is nothing that could keep it from reaching out equally, nor can aught that is be more here and less there than What is; ...and it is all Inviolable, for since there is equality throughout Being, Being lies uniformly within limits. (v.26-30)".[49]

Again, is it not true that so long as we do not know things that are caused, in their cause, our knowledge is only knowledge of appearance and, compared with the knowledge of them in their cause, merely apparent knowledge? For a thing that is caused, when known in itself as distinct from "known in its cause", is not fully known as to what it is, since its very being is "being from". There is therefore a sense in which the whole created universe is but an appearance of "the Is". And the tendency to know all things in the light of "the Is", is itself but knowledge in becoming.

The way of belief is not just pure veil. It is distinguished from the way of truth because it contains "two forms, one of which must not be named". As Professor Mack says in his chapter on Parmenides: "This phrase admirably expresses the reluctance that Parmenides felt when the force of his own argument compelled him to retain, though attenuated and filed down to the verge of breaking, the causal bond of form (morphe). Here in the world of phenomena there must be one more form, which will correspond with the One divine form as the effect corresponds with its cause. At the boundaries or limits of this second form, its coincidence with the divine causal

form will be perfect, and the form of non-being will therefore be a sphere; Parmenides accordingly says that all is 'full at the same time of light and of night devoid of understanding'. But though the two forms coincide at their limits, in every other respect they are opposites, since non-being is the opposite of Being; and Parmenides works out the opposite attributes of Being and of non-being as they appear in the world of phenoma, and goes into considerable detail which is irrelevant to our subject. It should be noted, however, that Parmenides has placed himself in a position where he must attribute to the form of Being many qualities which it did not possess when it abode alone in the Way of Truth; it is now, so to speak, keeping bad company, which is full of negative attributes such as darkness and lack of understanding and solidity and heavyness, and therefore the supreme god, who when alone was pure Being and Thought and Form, promptly acquires the additional attributes of Aither and Flame and Fire and Light, and is spoken of as being 'gentle' and 'weightless'."[50]

"...The Limit, that bounds (the) sphere of pure Being, coincides with and is the divine cause of the limit that bounds the sphere of non-being, and the supreme god of Parmenides is still causally

connected with the world of phenomena, and is still present in that world as its total and connective Form." (p.89)[51] Hack p.51

"And in this world of phenomena and among the forms that are discoverable in it, the same scale of values that has now been established will endure; the forms that change least will be most nearly like the supreme god, and those that change most will be most imperfect. No science that deals with phenomena can be a real science, since real science, so Parmenides and his followers believe, deals exclusively with the immutable perfection of Thought and Being and Form, and that is the way of Truth. Nevertheless phenomena and 'non-being' do not utterly cease to exist, even for Parmenides; and therefore Parmenides, through the goddess who spoke for him, was compelled to add 'the beliefs of mortals' to his 'trustworthy speech and thought about Truth.' The beliefs of mortals are of course the pseudo-science that deals with phenomena; and this section of the poem of Parmenides finds its exact parallel in the 'myths' of Plato, or as Plato himself puts it in the Timaeus (29 C): 'What Being is to becoming, Truth is to belief'."[52]

45. The most fundamental ideas of Anaxagoras, when confined to the dialectical mode, are again extremely suggestive. There is no doubt that Anaxagoras did not determinately take them in that way, as can be seen from the application he made of them. But, as we have already stated, we are here not primarily interested in what the authors of these texts meant, but in what sense their texts can be understood as true. Let us consider the more abstract fragments.[53]

> (1) All things were together, infinite both in number and in smallness; for the small too was infinite. And, when all things were together, non of them could be distinguished for their smallness. For air and aether prevailed over all things, being both of them infinite; for amongst all things these are the greatest both in quantity and size.
>
> (3) Nor is there a least of what is small, but there is always a smaller; for it cannot be that what is should cease to be by being cut. But there is also always something greater than what is great, and it is equal to the small in amount, and, compared with itself, each thing is both great and small.
>
> (4) And since these things are so, we must suppose that there are contained many things and of all sorts in the things that are uniting, seeds of all things, with all sorts of shapes and colors and savors, and that men have been formed in them, and the other animals that have life, and that these men have inhabited cities and cultivated fields as with us; and that they have a sun and a moon and the rest as with us; and that their earth brings forth for them many things of all kinds of which they gather the best together into their dwellings, and use them. Thus much have I said with regard to separating off, to show that it will not be only with us that things are separated off, but elsewhere too.

But before they were separated off, when all things are together, not even was any color distinguishable; for the mixture of all things prevented it— of the moist and the dry, and the warm and the cold, and the light and the dark, and of much earth that was in it, and of a multitude of innumerable seeds in no way like each other. For none of the other things either is like any other. And these things being so, we must hold that all things are in the whole.

(5) And those things having been thus decided, we must know that all of them are neither more nor less; for it is not possible for them to be more than all, and all are always equal.

(6) And since the portions of the great and of the small are equal in amount, for this reason, too, all things will be in everything; nor is it possible for them to be apart, but all things have a portion of everything. Since it is impossible for there to be a least thing, they cannot be separated, nor come to be by themselves; but they must be now, just as they were in the beginning, all together. And in all things many things are contained, and an equal number both in the greater and in the smaller of the things that are separated off.

(7) ...So that we cannot know the number of the things that are separated off, either in word or deed.

(8) The things that are in one world are not divided nor cut off from one another with a hatchet, neither the warm from the cold nor the cold from the warm.

(9) ...as these things revolve and are separated off by the force and swiftness. And the swiftness makes the force. Their swiftness is not like the swiftness of any of the things that are now among men, but in every way many times as swift.

(10) How can hair come from what is not hair, or flesh from what is not flesh?

(11) In everything there is a portion of everything except Nous, and there are some things in which there is Nous also.

(12) All other things partake in a portion of everything, while Nous is infinite and self-ruled, and is mixed with nothing, but is alone, itself by itself. For if it were not by itself, but were mixed with anything else, it would partake in all things if it were mixed with any; for in everything there is a portion of everything, as has been said by me in what goes before, and the things mixed with it would hinder it, so that it would have power over nothing in the same way that it has now being alone by itself. For it is the thinnest of all things and the purest, and it has all knowledge about everything and the greatest strength; and Nous has power over all things, both greater and smaller, that have life. And Nous had power over the whole revolution, so that it began to revolve in the beginning. And it began to revolve first from a small beginning; but the revolution now extends over a larger space, and will extend over a larger still. And all the things that are mingled together and separated off and distinguished are all known by Nous. And Nous set in order all things that were to be, and all things that were and are not now and that are, and this revolution in which now revolve the stars and the sun and the moon, and the air and the aether that are separated off. And this revolution caused the separating off, and the rare is separated off from the dense, the warm from the cold, the light from the dark, and the dry from the moist. And there are many portions in many things. But no thing is altogether separated off nor distinguished from anything else except Nous. And all Nous is alike, both the greater and the smaller; while nothing else is like anything else, but each single thing is and was most manifestly those things of which it has most in it.

(13) And when Nous began to move things, separating off took place from all that was moved, and so much as Nous set in motion was all separated. And as things were set in motion and separated, the revolution caused them to be separated much more.

(14) And Nous, which ever is, is certainly there, where everything else is, in the surrounding mass, and in what has been united with it and separated off from it.

(15) The dense and the moist and the cold and the dark came together where the earth is now, while the rare and the warm and the dry (and the bright) went out towards the further part of the aether.

(16) From these as they are separated off earth is solidified; for from mists water is separated off, and from water earth. From the earth stones are solidified by the cold, and these rush outwards more than water.

(17) The Mellenes follow a wrong usage in speaking of coming into being and passing away; for nothing comes into being or passes away, but there is mingling and separation of things that are.
(21) From the weakness of our sense we are not able to judge the truth.
(21a)What appears is a vision of the unseen.

The dialectical meaning of all the fragments down to 11 excl., is clear enough. But the fragments 11 to 14 inc. seem to bring in a new idea which calls for an explanation. With what are we to identify this Nous? What is the meaning of the movement described here? The most obvious meaning would be the mind and its movement which sets in motion and operates the tendency toward the limit, thereby causing (i.e. tending to cause) the separation and order of things. But Nous is two-fold: there is the Nous which is entirely separated. "and there are some things in which there is Nous also". But even the Nous which is in things is somehow separated and mixed with nothing, for to be mixed with anything would hinder it.

We may identify the Nous which is entirely separated with the strictly divine intellect, that is the intellect in which all things are known and ordered, and which we imitate. The nous that is in us is also separated in a way, and it is this that allows us to imitate Nous.

Again, when we speak of Nous, we cannot avoid describing its way in terms of the way of our nous.

Fragment 17 too offers a special difficulty. When understood in the natural mode, the statement is false, and the problem it supposes is to be solved by the distinction between act and potency. But, even when we understand it in the dialectical mode, it seems contrary to what we have established in our earlier analysis, namely that "coming to be" is essential to the notion of limit. Anaxagoras excludes here all coming-to-be and passing away:"but there is mingling and separation of things that are". On the other hand, he seems at this very point, to contradict openly what he had asserted about infinity, smallness and greatness, and being together without distinction, as if he here reduced everything to pure discontinuity. Yet, it must be said that we too should use this language when we formally consider the limit in its absolute state. With respect to this state, becoming is only appearance. Again, in this state, the limit is an actual whole, with its parts actually within it.

Meno's arguments have definitely demonstrated the absurdities which follow from the confusion of the dialectical mode with the natural one. There is no solution to his problems unless we distinguish the two modes.[54] If natural movement were identical with what we have called "dialectical movement" (and the latter is obviously what one is talking about), then natural movement would be impossible, for to reach any term by such a movement, we would have to go through an infinite amount of steps. Again we could say that in order that things be different, they would have to be identical, and so on.

Even modern authors have solved Meno's problems only be denying movement.

In a fairly recent book,[55] intended to serve as a "basic vulgarization" of the science of mathematics, the authors attempt to demonstrate the impossibility of motion and the changelessness of our universe. After a brief, but extremely clear and interesting discussion of the instantaneous rate of change of a function, they announce that "all the vagaries, the mysteries, and uncertainties indissolubly linked with the idea of motion, are thus swept away, or more appropriately, transformed into a few precise and definable aspects of the idea of function".[56] Their conclusion, as well as their manner of arriving at it, is the same as in a considerable number of other modern books of the same "genre". Their mistake is to imagine that the world of nature is exactly the same as their mathematical picture of it, a mistake that Eddington warns against in books that these very authors describe as "worth reading". By such a mistake, they forget that the method of limits is only a method. Furthermore, it would seem that such a conclusion could be valid only if it is a question of measuring indivisibles and not intervals of time or space; but regardless of the manner of looking at Δx, it is always an interval. To believe it otherwise, is to believe in the attainment of the limit, which we have shown to be contradictory.

The present paper was practically finished when we read A.E.Taylor's extremely suggestive study on "Forms and Numbers".[57] According to Professor Taylor, "the fundamental novelty about the Platonic theory is that it represents the first discovery, in an incomplete form, of the real numbers, as the ultimate determinants of

geometrical structure, and so mediately of the physical characters of things." Such difficulties as the ratio of the diagonal to the side (the Pythagorean "scandal") and the length of the edge of a cube whose volume is double that of a given cube (the "Delian problem") were realized by Plato to be reappearances in plane and solid geometry of the same difficulties in pure arithmetic. Familiar with a method of forming an endless series of increasingly close numerical approximations to $\sqrt{2}$, Plato in some way envisaged a mathematical program that would provide arithmetical treatment of all the geometric incommensurables. In other words, he was introducing quadratic surds (and implicitly cubic surds) into the number system, for evidently the integers themselves were incapable of doing what Plato felt some numbers should do. These real numbers he regarded as the limits of double converging series of unending fractions. It was because of this that he was influenced in substituting for the Pythagorean "apeiron" the "duality" of "the great and the small." After illustrating successive convergents, Taylor continues:

> "The general character of the procedure is thus that in the expression of $\sqrt{a}$ as an "unending continued fraction", by forming the series of "convergents" we pin down a

> between two values, one of which is a little too small and the other a little too large, but the difference between the too small and the too large is decreasing at every step and can be made less than any fraction we like to assign, though we never quite get rid of it, because we cannot actually arrive at a last convergent. To put it another way, in approximating to $\sqrt{2}$ by this method, we are not merely approximating to a 'limit,' we are approximating to it from both sides at once; 2 is at once the upper limit to which the series of the values which are too small, 1, 7/5,... are tending, and the lower limit to which the values values which are too large, 3/2, 17/12,... are tending. This, as it seems to me, is manifestly the original reason why Plato requires us to substitute for the apeiron as one thing, a 'duality' of the great and small. $\sqrt{2}$ is an apeiron, because you may go on endlessly making closer and closer approximations to it without ever reaching it; it never quite turns into a rational number, though it seems to be on the way to do so. But also, it is a 'great-and-small' because it is the limit to which one series of values, all too large, tends to decrease, and also the limit to which another series, all too small, tends to increase.
>
> "The meaning of what is said in our passage of the Epinomis about plane geometry will thus be that the real problem of the study is to evaluate all quadratic surds ($\sqrt{3}$, $\sqrt{5}$, etc), by the same method which has proved successful in the case of the 'double;' they are all, in modern phraseology, to be expressed as unending 'continued fractions,' and our conception of numbers is to be enlarged to include these 'irrationals', which by the proposed method can be made rational to within whatever 'standard' we like to adopt. It is the indispensability of providing a means of checking the interval within which the 'error' of an approximation falls which is the real reason for replacing the simple 'apeiron' by a 'duality'."[58]

This notion can be applied to all limits, which are "irrational" in one way or another and located in some infinite "field". It is this double approach by way of the great and of the

small that localizes and hems in, as it were, the object of the dialectical hunt; it is the still more complete determination of that which, ultimately, will never admit of perfect determination. The circle, then, is the limit of two converging polygons; the straight line, the limit of two converging curves; the area under the curve, the limit of the sums of both the inscribed and the circumscribed rectangles; etc.

The relevance of all these notions concerning limit to metaphysics or the philosophy of nature is hinted at in the concluding words of Taylor's study:

> "The identification of the forms (eide) with numbers means that the 'manifold' of nature is only accessible to scientific knowledge in so far as we can correlate its variety with definite numerical functions of 'arguments.' The 'arguments' have then themselves to be correlated with numerical functions of 'arguments' of a higher degree. If this process could be carried through without remainder, the sensible world would be finally resolved into combinations of numbers, and so into the transparently intelligible. This would be the complete 'rationalization' of nature. The process cannot in fact be completed, because nature is always a 'becoming,' always unfinished; in other words, because there is real contingency. But our business in science is always to carry the process one step further. We can never completely arithmetize nature, but it is our duty to continue steadily arithmetizing her... [59]

We could give no better example of the dialectical mode in Plato. It would be interesting to analyse the Sophistes in this light. But that would carry us too far from our present purpose. But we shall come back to this fundamental dialogue on another occasion.

Some Texts from Modern Philosophers

48. Let us now consider a few examples taken from modern philosophies.

The whole of Hegel's philosophy would be gone over in the light of the distinction we have made between the natural mode and the dialectical mode of considering things. Such an examination would reveal much that is legitimate in his works, and at the same time permit a more pertinent criticism.

To some extent, we may identify what he calls Understanding with our natural mode.

"Thought, as Understanding, sticks to fixity of characters and their distinctness from one another: every such limited abstract it treats as having a subsistence of its own."[60]

Provided we apply what Hegel here says to the proper "proper quid" of a thing, as we understand it in a previous paragraph (p.13), the statement is true. For the "why" man is one has no "why".

He then goes on to say:

"...The action of understanding may be in general described as investing its subject-matter with the form of universality. But this universal is an abstract universal: that is to say, its opposition to the particular is so rigorously maintained, that it is at the same time also reduced to the character of a particular again...."[61]

And Hegel recognises the importance of this understanding:

"...It must be added however, that the merit and rights of the mere Understanding should unhesitatingly be admitted. And the merit lies in the fact that apart from Understanding there is no fixity or accuracy in the region either of theory or of practice.

"Thus, in theory, knowledge begins by apprehending existing objects in their specific differences. In the study of nature, for example, we distinguish matters, forces, genera and the like, and stereotype each in its isolation. Thought is here acting in its analytic capacity, where its common is identity, a simple reference of each attribute to itself. It is under the guidance of the same identity that the process in knowledge is effected from one scientific truth to another. Thus, for example, in mathematics magnitude is the feature which, to the neglect of any other, determines our advance..."[62]

"...Understanding, too, is always an element in thorough training. The trained intellect is not satisfied with cloud and indefinite impressions but grasps the objects in their fixed character..."[63]

There is one point to be noticed particularly in the foregoing text. Hegel refers only to the unity of a thing with itself, and not to what we have termed abstract identity of differences. His confusion about "identity" will be responsible for many confusions following logically from it.

He next proceeds to the Dialectical stage, or that of negative reason:

"In the Dialectical stage these fixite characterisations or formulae supersede themselves, and pass into their opposites".[64]

It is true that anything we know according to the natural mode, is known in a finite mode, as when we know God by analogy. But this is due to our natural mode and reflects the limitation of our light.

"...By dialectic is meant the indwelling tendency outwards by which the one-sidedness and limitation of the predicates of understanding is seen in its true light, and shown to be the negation of them. For anything to be finite is just to suppress itself and put itself aside. Thus understood the dialectical principle constitutes the life and soul of scientific progress, the dynamic which alone gives immanent connexion and necessity to the body of science; and, in a word, is seen to constitute the real and true, as opposed to the external exaltation above the finite."[55]

The only trouble with this statement is that it attributes the dialectic to the things in themselves, and not merely to the purely objective being they have in our mind. While it is true that the dialectical mode allows us to know things better as to what they are, this is not due to what they are; it is not because they have a dialectical aspect which the natural mode does not account for, but merely because our natural mode is imperfect, as we have shown before.

Let us read on:

"It is of the highest importance to ascertain and understand rightly the nature of Dialectic. Wherever anything is carried into effect in the actual world, there Dialectic is at work. It is also the soul of all knowledge which is truly scientific. In the popular way of looking at things, the refusal to be bound by the abstract deliverances of understanding appears as fairness, which, according to the proverb 'Live and let live', demands that each should have its turn; we admit the one, but admit the other also. But when we look more closely, we find that the limitations of the finite do not merely come from without; that its own nature is the cause of its abrogation, and that by its own act it passes into its counterpart. We say, for instance, that man is mortal, and seem to think that the ground of his death is in external circumstances only; so that if this way of looking were correct, man would have two special properties, vitality and--also-- mortality. But the true view of the matter is that life, as life, involves the germ of death, and that the finite, being radically self-contradictory, involves its own self-suppression."[56]

This text carries through the initial confusion. The truly interesting point, however, is that Hegel is using as an illustration a case taken from natural becoming and destruction, which indeed has much in common with the becoming and vanishing of dialectic. We have already called attention to this resemblance, and warned against the confusions that follow from their identification.

When Hegel says: "its purpose is to study things in their own being and movement and thus to demonstrate the finitude of the partial categories of understanding", we would say: "its purpose is to overcome in a purely tendential manner, the finitude of our means of knowing employed in the natural mode".

Let us now consider the third stage of thought:

"The Speculative stage, or stage of Positive Reason apprehends the unity of terms (propositions) in their opposition,--the affirmative, which is involved in their disintegration and in their transition."[67]

"...The speculative is in its true signification, neither preliminarily nor even definitively, something merely subjective: that, on the contrary, it expressly rises above such oppositions as that between subjective and objective, which the understanding cannot get over, and absorbing them in itself, evinces its own concrete and all-embracing nature. A one-sided proposition therefore can never give expression to a speculative truth. If we say, for example, that the absolute is the unity of subjective and objective, we are undoubtedly in the right, but so far one-sided, as we enunciate the unity only and lay the accent upon it, forgetting that in reality the subjective and objective are not merely identical but also distinct."

"Speculative truth, it may also be noted, means very much the same as what, in special connexion with religious experience and doctrines, used to be called Mysticism. The term Mysticism is at present used, as a rule, to designate what is mysterious and incomprehensible: and in proportion as their general culture and way of thinking vary, the epithet is applied by one class to denote the real and the true, by another to name everything connected with superstition and deception. On which we first of all remark that there is mystery in the mystical, only however for the understanding which is ruled by the principle of abstract identity; whereas the mystical, as synonymous with the speculative, is the concrete unity of those propositions, which understanding only accepts in their separation and opposition. And if those who recognise Mysticism as the highest truth are content to leave it in its original utter mystery, their conduct only proves that for them too, as well as for their antagonists, thinking means abstract identification, and that in their opinion, therefore, truth can only be won by renouncing thought, or as it is frequently

expressed, by leading the reason captive. But, as we have seen, the abstract thinking of understanding is so far from being either ultimate or stable, that it shows a perpetual tendency to work its own dissolution and swing round into its opposite. Reasonableness, on the contrary, just consists in embracing within itself these opposites as unsubstantial elements. Thus the reason-world may be equally styled mystical, -- not however because thought cannot both reach and comprehend it, but merely because it lies beyond the compass of understanding."[68]

Speculative thought, then, would be, in our terms, thought reaching a concrete identity of what were first known to be only abstractly identical. That Hegel does not hesitate before open contradiction, is clear enough from the following statement:

"...A notion, which possesses neither or both of two mutually contradictory marks, e.g. a quadrangular circle, is held to be logically false. Now though a multiangular circle and a rectilineal are no less contradict this maxim, geometers never hesitate to treat the circle as a polygon with rectilineal sides."[69]

But we might also understand our equivalent for Hegel's speculative thought in another way. As we have seen, the limit toward which we really tend when we apply universally the method of limits, is none other than the knowledge of all things in one single means only; we tend to know the many in the One. And this is nowhere better realized, as a pure tendency of course, than in the dynamic via negationis, which is the subject of Dionysius' Mystica Theologia. Now, if this limit were actually reached, we would

know God sub ratione Deitatis. But to talk of reaching this by the dialectical mode is to talk blasphemy and contradiction.

47. The dialectical mode of knowing is, again, essential to an even elementary understanding of Marxism, which openly professes contradiction as a simple fact. Engels has given a concrete example which would indeed mean the destruction of metaphysics, as he himself points out:

"Straight and curved in the differential calculus are in the last resort put as equal: in the differential triangle, the hypotenuse of which forms the differential of the arc (in the tangent method), this hypotenuse can be regarded "comme une petite ligne toute droite qui est tout à la fois l'élément de l'arc et celui de la tangente"-- if now the curve is regarded as composed of an infinite number of straight lines, or also, however, "lorsqu'on la considère comme rigoureuse; puisque le détour à chaque point M étant infiniment petit, la raison dernière de l'élément de la courbe à celui de la tangente est évidemment une raison d'égalité." Here, therefore, although the ratio continually approaches equality, but asymptotically in accordance with the nature of the curve, yet, since the contact is limited to a single point which has no length, it is finally assumed that equality of straight and curved has been reached. Bossut, Calcul. diff. et integr. (Differential and Integral Calculus), Paris, An.VI, I, p.149. In polar curves the differential imaginary abscissae are even taken as parallel to the real abscissae and operations based on this, although both meet at the pole; indeed, from it is deduced the equality of two triangles, one of which has an angle precisely at the point of intersection of the two lines, the parallelism of which is the whole basis of the equality!

"When the mathematics of straight and curved lines has thus pretty well reached exhaustion, a new almost infinite field is opened up by the mathematics that conceives curved as straight (the differential triangle) and straight as curved (curve of the first order with infinitely small curvature). O metaphysics!"[70]

This example allows us to put our finger on the confusion. Unless by 'straight as curve' we mean straight as the limit of curve, the expression is a contradiction in terms.

For Plekhanoff, too, real movement is an obvious instance of real contradiction which throws out the principle of contradiction as having value only for things considered in their fixity:

> "La base de tous les phénomènes de la nature est constituée par le mouvement de la matière. Mais qu'est-ce que le mouvement? Il fait une contradiction évidente. Si l'on vous demande si un corps en mouvement se trouve en un moment donné à tel endroit, vous ne pourrez, malgré votre bonne volonté, répondre selon la règle d'Aberuag, c'est-à-dire selon la formule: 'oui est oui, et non est non'. Un corps en mouvement se trouve à un endroit donné, et en même temps il ne s'y trouve pas. On ne peut pas juger de lui autrement que d'après la formule: 'oui est non et non est oui'. Ce corps se présente donc comme une preuve irréfutable en faveur de la 'logique de la contradiction', et quiconque ne veut pas prendre son parti de cette logique doit proclamer avec Zénon que le mouvement n'est rien d'autre qu'une illusion des sens..."[71]

The trouble with this position is that it tries to answer a question which has no meaning when applied to a body in motion as it is in motion. Ironically, to concede such a question about movement supposes the very negation of movement.

The relevance of a rudimentary understanding of the dialectical mode might be shown from the following text of Joseph Stalin:

> "Contrary to metaphysics, dialectics does not regard nature as an accidental agglomeration of things, of phenomena, unconnected with, isolated from, and independent of, each other, but as a connected and integral whole, in which things, phenomena, are organically connected with, dependent on, and determined by, each other.
>
> "The dialectical method therefore holds that no phenomenon in nature can be understood if taken by itself, isolated from surrounding phenomena, inasmuch as any phenomenon in any realm of nature may become meaningless to us if it is not considered in connection with the surrounding conditions, but divorced from them; and that, vice versa, any phenomenon can be understood and explained if considered in its inseparable connection with surrounding phenomena."[72]

This text brings out clearly enough the absurd consequences of identifying dialectic with nature. If things in themselves were dialectical, what Stalin here maintains would be true. But even from this absurd hypothesis it would also follow that we could never know anything.

> "Contrary to metaphysics, dialectics holds that nature is not a state of rest and immobility, stagnation and immutability, but a state of continuous movement and change, of continuous renewal and development, where something is always arising and developing, and something always disintegrating and dying away.
>
> "The dialectical method therefore requires that phenomena should be considered not only from the standpoint of their interconnection and interdependence, but also from the standpoint of their movement, their change, their development, their coming into being and going out of being."[73]

"Contrary to metaphysics, dialectics does not regard the process of development as a simple process of growth, where quantitative changes do not lead to qualitative changes, but as a development which passes from insignificant and imperceptible quantitative changes to open, fundamental changes, to qualitative changes; a development in which the qualitative changes occur not gradually, but rapidly and abruptly, taking the form of a leap from one state to another; they occur not accidently but as the natural result of an accumulation of imperceptible and gradual quantitative changes.

"The dialectical method therefore holds that the process of development should be understood not as movement in a circle, not as a simple repetition of what has already occurred, but as an onward and upward movement, as a transition from an old qualitative state to a new qualitative state, as a development from the simple to the higher....."[74]

"Contrary to metaphysics, dialectics holds that internal contradictions are inherent in all things and phenomena of nature, for they all have their negative and positive sides, a past and a future, something dying away and something developing; and that the struggle between these opposites, the struggle between the old and the new, between that which is dying away and that which is being born, between that which is disappearing and that which is developing, constitutes the internal content of the transformation of quantitative changes into qualitative changes.

"The dialectical method therefore holds that the process of development from the lower to the higher takes place not as a harmonious unfolding of phenomena, but as a disclosure of the contradictions inherent in things and phenomena, as a 'struggle' of opposite tendencies which operate on the basis of these contradictions.

" 'In its proper meaning,' Lenin says, 'dialectics is the study of the contradiction within the very essence of things'." [75]

All these positions follow logically enough from a confusion of the dialectical with the real. It is rather ironical that, in dialectical materialism, it is the dialectical that carries off the real.

These few examples should show convincingly the necessity of knowing and of developing the method of limits.

NOTES

1. An extremely interesting and instructive treatment of this problem with all its philosophical implications and consequences as found in the doctrine of Plato and Aristotle has been given by Professor A. E. Taylor in an article entitled "Forms and Numbers". Cf. A. E. Taylor-- Philosophical Studies (London: MacMillan & Co.; 1934), pp.91-150.

2. Mullins and Smith-- Freshman Mathematics (New York: Ginn & Co.; 1927), p.34.

3. J. G. Leatham-- The Mathematical Theory of Limits (London: G. Bell & Sons; 1925), p.9. To this excellent work we are completely indebted.

4. J. G. Leatham--op.cit., p.9.

5. Smith, Salkover, Justice-- Calculus (New York: Wiley & Sons; 1938), p.2.

6. Alfred N. Whitehead-- An Introduction to Mathematics (London: Thornton Butterworth Ltd.; 1911), p.339.

7. In using these terms, "infinitesimal" and "infinite" we shall adhere to the definitions given in the text, for we do not believe, of course, that there is any actual quantity (whether continuous or discrete) that is actually infinitely small or infinitely large.

8. Leatham-- op.cit., p.10.

9. Whitehead says (op.cit., p.238), "Now, according to the Weirstrassian explanation the whole idea of h tending to the value 0, though it gives a sort of metaphorical picture of what we are driving at, is really off the point entirely. Indeed it is fairly obvious that, as long as we retain anything like 'h tending to 0', as a fundamental idea, we are really in the clutches of the infinitely small; for we imply the notion of h being infinitely near to 0. This is just what we want to get rid of." By all means, we should get out of the clutches of the infinitely small, but if we always bear in mind that this tendency is only a "dialectical" tendency, we

shall be out of danger. This point will be developed in our second section. Later on (p.234), Whitehead concludes after an excellent exposition: "Now have we, by this definition of a limit, really managed to avoid the notion of 'infinitely small numbers' which so worried our mathematical forefathers? For them the difficulty arose because on the one hand they had to use an interval x to x + h over which to calculate the average increase, and on the other hand, they finally wanted to put h = 0. The result was they seemed to be landed into the notion of an existent interval of zero size. Now how do we avoid this difficulty? In this way-- we use the notion that corresponding to any standard of approximation, some interval with such and such properties can be found.' This shows that an interval always remains, which is true. Those who use the calculus to disprove the existence of movement in the real world, would seem to forget this very point, for their application of the mathematical results directly to the real world would depend on the reduction of any interval, x, to zero or to an indivisible of the same order (a relative zero).

10. See Editor's Preface in Taylor's Philosophical Studies.

11. A. E. Taylor-- op.cit., p.133. At least by 1783 the mathematicians were clear about this point. Cf. L. N. M. Carnot-- Réflexions sur la métaphysique du calcul infinitésimal (2nd.ed.; Paris, 1813), p.11, n.16. The text is: "On voit par ce qui précède, que les quantités appelées infiniment petites en Mathématiques, ne sont point des quantités actuellement nulles, ni même des quantités actuellement moindres que telles ou telles grandeurs déterminées, mais seulement des quantités auxquelles les conditions de la question proposée et les hypothèses sur lesquelles le calcul est établi, permettent de demeurer variables, jusqu'à ce que le calcul soit entièrement achevé, en décroissant continuellement, jusqu'à devenir aussi petites qu'on le veut, sans que l'on soit obligé de changer en même temps les valeurs de celles dont on veut obtenir la relation. C'est en cela uniquement que réside le véritable caractère des quantités auxquelles on a donné le nom d'infiniment petites, et non dans la

tomiti dont leur dénomination semble supposer qu'elles sont effectivement données, ni dans la multitude absolue qu'on pourrait leur attribuer; et la notion, comme on le voit, en est parfaitement simple, et dégagée de toute idée vague ou contentieuse."

11. We are using the terms "open class" and "closed class", not in a strictly technical sense, but in our own and a popular sense. For instance, by an "open class" we merely mean a class of numbers to which more may be added; and by "closed", one to which no more may be added, for one reason or another.

13. Cf. De Spiritualibus Creaturis, a.8,c.— "Manifestum est autem quod in omnibus individuis unius speciei non est ordo nisi secundum accidens: convenient enim in natura, et differunt secundum principia individuantia, et diversa accidentia, quae per accidens se habent ad naturam speciei. Quae autem specie differunt ordinem habent per se, et secundum essentialia principia. Invenitur enim in speciebus rerum una abundare super aliam, sicut et in speciebus numerorum, ut dicitur in VIII Metaph." When the author's name is not mentioned, as in the present case, it is understood to be St.Thomas Aquinas.

14. The natural generation here used as a term of comparison is taken in the broad sense described in IV Metaphy., lect.7, nn. 1233,1234:- "...Omne moveri praecedit motum esse propter divisionem motus. Oportet enim quod quacumque parte motus data, cum divisibilis sit, aliquam partem ejus accipi, quae jam peracta est, dum pars motus data peragitur. Et ideo quidquid movetur, jam quantum ad aliquid motum est. Et eadem ratione quidquid fit, jam quantum ad aliquid factum est. Licet enim factio in substantia quantum ad introductionem formae substantialis sit indivisibilis, tamen si accipiatur alteratio praecedens cujus terminus est generatio divisibilis est, et totum potest dici factio."

15. Cf. footnote 7.

16. Charles DeKoninck— De La Primauté Du Bien Commun contre les Personnalistes (Québec: Editions de l'Université Laval; 1943), pp.109-116.

17. I, 88, 3, ad 2.

18. De Veritate, VIII, 10, ad 1.

19. I, 88, 3,c. Cf. also De Causis, lect.10.

20. DeKoninck— op.cit.,p.109.

21. DeKoninck— op.cit.,p.168.

22. Quaest. De Anima, a.7, c.

23. Charles DeKoninck— Ego Sapientia- La Sagesse Qui Est Marie (Québec: Editions de l'Université Laval; 1948), pp. 77-79.

24. De Causis, lect. 10.

25. De Trinitate, VI, 3, c.

26. X De Anima, lect.1, n.5.

27. Bertrand Russell— Introduction to Mathematical Philosophy (London: 2nd ed., Allen & Unwin; 1920), p.194.

28. Aristotle— VII Metaphysics, ch.17;1041a10-b10.

29. In De Anima, a.7, ad 6. Cf. also De Spirit. Creat., 8, ad 6.

30. VII Physics, lect.7, n.11.

31. VII Physics, lect.7, n.9.

32. John of St. Thomas— Cursus Philosophicus (Reiser edition), t.I; p.693, a10-b7. The text immediately preceding is equally important:- "Signum naturale et ad placitum convenient univoce in ratione signi. Ergo non potest alterum esse reale, alterum rationis, quia ad relationem realem et rationis nihil est univocum, nec utrumque est aliquid reale, cum signum ad placitum constet esse aliquid rationis; ergo utrumque aliquid rationis est.

"Antecedens probatur: Ratio objecti seu cognoscibilis est univoca in ente reali et rationis, quia ad univocam scientiam et potentiam eandem pertinet. Logica enim, quae agit de ente rationis, et Metaphysica, quae de ente reali, univoce sunt scientiae. Ergo et objecta earum univoce sunt objecta et scibilia. Ergo similiter univoce sunt signa; siquidem ratio signi et significabilis est de genere objecti et cognoscibilis, pro quo substituit.....

"...Ratio cognoscibilis et objecti ex ente reali et rationis potest esse universa; aliae enim sunt divisiones entis in esse rei, aliae in genere scibilis, ut bene Cajetanus docet 1.p.q.1.art.3. Et sic ratio cognoscibilis non est ratio entis formaliter, sed praecognoscitive solum est ens et ex secundum ad ens; verum enim est passio entis, et sic formaliter non est ens, sed consecutum ad ens et praecognoscitive ens; idem est autem verum quod cognoscibile. Unde bene stat, quod aliquod ens incapax existentiae sit capax veritatis, non ut subjectum, quatenus non habet in se entitatem, quae tamquam subjectum fundet veritatem et cognoscibilitatem, sed habet, quod tamquam objectum possit cognosci ad instar entis realis et sic objective esse in intellectu tamquam verum. (idem.pp.682, 683)

33. "Omne enim nomen significat aliquem naturam determinatam, ut hoc; aut personam determinatam, ut praesens; aut utrumque determinatum, ut Socrates. Sed hoc quod dico non-homo, neque determinatam naturam neque determinatam personam significat. Impositio enim a negatione hominis, quae aequaliter dicitur de ente, et non-ente. Unde non-homo potest dici indifferenter, et de eo quod non est in rerum natura; ut si dicamus, chimaera est non-homo, et de eo quod est in rerum natura; sicut cum dicitur, equus est non-homo. Si autem imponeretur a privatione, requireret subjectum ad minus existens: sed quia imponitur a negatione, potest dici de ente et de non-ente, ut Boethius et Avicenna dicunt. Quia tamen significat per modum nominis, quod potest subjici et praedicari, requiritur ad minus suppositum in apprehensione. Non autem erat nomen positum tempore Aristotelis sub quo hujusmodi dictiones concluderentur. Non enim est oratio, quia pars ejus non significat aliquid separata, sicut nec in nominibus compositis; similiter autem non est negatio, id est oratio negativa, quia hujusmodi oratio superaddit negationem affirmationi, quod non contingit hic. Et ideo novum nomen imponit hujusmodi dictionibus, vocans eam nomen infinitum propter indeterminationem significationis, ut dictum est. (In Perihermeneias, I, lect.4, n.13)

34. In Perihermeneias, II, lect.1, n.3.

35. IV Metaph., lect.1, n.539.

36. XI Metaph., lect.11, n.2368.

37. De Malo. XVI, 5,c.

38. I, 79, 4, c.

39. De Veritate, II, 1, ad 4.

40. I, 58, 3, c.

41. De Potentia, V, 5, c.

42. "Primo quidem, quia nihil differt dicere finem alicujus esse assimilationem ad Deum secundum aliquid, et illud secundum quod assimilatio attenditur, sicut supra dictum est, quod finis rerum posset dici vel ipsa assimilatio divinae bonitatis; vel esse rerum, secundum quod res Deo assimilantur. Idem ergo est dictu finem motus caeli esse assimilari Deo in causando et causare. Causare autem non potest esse finis, cum sit operatio habens operatum, et tendens in aliud; hujusmodi enim operationibus meliora sunt operata, ut dicitur in principio Ethic.; unde hujusmodi factiones non possunt esse fines agentium, sum non sint perfectiones facientium, sed magis factorum; unde et ipsa facta sunt magis fines, et patet XI Metaph.... et in I Ethic., cap. I; ipsa autem operata non sunt fines, cum sint viliora caelo, ut supra dictum est. Unde relinquitur non convenienter dici, quod finis motus caeli sit assimilari ad Deum in causando. Secundo vero, quia cum caelum moveatur in Deo existente sola aptitudine ad motum, principio vero active activo existente extra, ut dictum est, movetur et agit sicut instrumentum; haec est enim dispositio instrumenti, ut patet in artificialibus; nam in securi est sola aptitudo ad talem motum; principium autem motus in artifice est. Unde et secundum philosophos, quod movet motum, movet ut instrumentum. In actione autem quae est per instrumentum, non potest esse finis aliquis in ipso instrumento nisi per accidens, inquantum instrumentum accipitur ut artificiatum et non ut instrumentum; unde non est probabile quod finis motus caeli sit aliqua perfectio ipsius, sed magis aliquid extra; ipsum. Tertio, quia si similitudo ad Deum in causando est finis motus caeli, praecipue attenditur haec similitudo secundum causalitatem ejus quod a Deo immediate causatur, scilicet animae rationalis, ad cujus causalitatem concurrit caelum per motum suum materiam disponendo. Et ideo probabilius est quod finis motus caeli sit numerus electorum quam assimilatio ad Deum in causalitate generationis et corruptionis, secundum quod philosophi ponunt. Et ideo concedimus quod motus

eculi impleto numero electorum finietur. (De Potentia, V, 5, c.). Cf. also Contra Gentiles, III, 82 together with the Comm. of Torrerlensis.

43. De Potentia, IV, 2, ad 8.

44. II-II, 47, 2, ad 2.

45. Aristotle-- III Physics, ch.4; 203b10-15.

46. "Since it is the One that creates order in each particular thing and group of things, from the Psyche to the city-state and the universe, by combining with itself some definite amount of the apeiron, we shall search for a term to designate the function of the One as a combining agent, and we shall choose the word harmonia. This word means a "binding": Homer has used it of the clamps or joints that Odysseus used in building his raft (Od., V, 248). Since every binding made by the One is orderly, we shall use harmonia either in its active sense as "that which binds and produces order" or in its passive sense as "that which is bound into order". We are acquainted with such musical intervals as the octave, the fifth, and the fourth, and with the mathematical proportions which correspond to these intervals; but the word Harmonia will always mean a binding, whether or not it refers to a concordant sound or a relief (Roy Kenneth Mack-- God in Greek Philosophy, Princeton University Press, 1931; pp.54,55.)

47. Mack-- op.cit., p.54

48. De Spirit. Creat., a.8, ad 10.

49. Cf. Mack (op.cit.,p.53) and John Burnet-- Early Greek Philosophers (London: 4th ed., A.& C. Black Ltd.; 1930) pp. 175,176.

50. Mack, pp.90, 91.

51. Mack, p. 89.

52. Mack, pp.89, 90.

53. Burnet-- op.cit., pp.257-261.

54. Whether Zeno himself considered his proofs of the impossibility of the multiple, of movement, etc. as solutions or not, is another question. We refer to Plato's Parmenides, 128c.

55. Kasner and Newman-- Mathematics And The Imagination (New York: Simon and Schuster; 1940).

56. Kasner and Newman-- op.cit., p.361

57. A. E. Taylor-- "Forms and Numbers" in Philosophical Studies (London: MacMillan & Co.;1934).

58. op.cit.,pp.102, 109.

59. op.cit.,pp.149, 150.

60. William Wallace-- The Logic of Hegel: transl. from The Encyclopaedia of the Philosophical Sciences (Oxford Univer. Press: 2nd ed.; 1892)n.80, p. 143.

61. op.cit., n.80, p. 143.

62. op.cit., n.80, p. 144.

63. op.cit., n.80, p. 145.

64. op.cit., n.81, p. 147.

65. op.cit., n.81, pp.147, 148.

66. op.cit., n.81, p. 148.

67. op.cit., n.82, p. 152.

68. op.cit., n.82, pp.153, 154.

69. op.cit., n.119, p.221.

70. Frederick Engels-- Dialectics of Nature (New York: International Publishers; 1940),pp.200,201.

71. G. V. Plekhanov-- Les questions fondamentales du Marxisme (Editions Sociales Internationales, Paris), p.98.

72. Joseph Stalin-- Dialectical And Historical Materialism (New York: International Publishers; 1940), p.7.

73. op.cit., pp. 7, 8.

74. op.cit., pp. 8, 9.

75. op.cit., p. 11.