DISSERTATION

PRESENTED

TO THE FACULTY OF PHILOSOPHY

OF LAVAL UNIVERSITY

TO OBTAIN

THE DEGREE OF DOCTOR OF PHILOSOPHY

BY

BERNARD I. MULLAHY, C.S.C.

LICENTIATUS IN PHILOSOPHY

GREGORIAN UNIVERSITY, ROME

Thomism and Mathematical Physics.

JULY 1946

THOMISM AND MATHEMATICAL PHYSICS

TABLE OF CONTENTS.

Section One.

THOMISM AND MATHEMATICAL PHYSICS

TABLE OF CONTENTS.

Section One.

Section Two.

Appendix

CHAPTER ONE

THE PROBLEM OF MATHEMATICAL PHYSICS.

1. A Symbol of Progress.

"On the second floor of the Hall of Science at the Century of Progress Exposition, held at Chicago in the summers of 1933 and 1934, reaching up into the great tower of the building was a smaller tower designed to symbolize the interrelations and interdependence of the physical sciences. The huge base on which the remaining sciences were supported and uplifted was assigned to mathematics. Astronomy, physics, chemistry, the medical sciences, geology, geography, engineering, architecture, the industrial arts — all had their roots in the science by whose methods and attainments they have learned and continue to learn to express themselves." (1)

The milling throngs that crowded the pavillions of Chicago's Exposition found a great many things to make their visit rewarding. For there, under a great variety of forms, were the concrete and tangible results of a century of amazing scientific and technological progress which had gone to almost incredible lengths in penetrating into the inner secrets of Nature and in controlling its hidden forces. But for those who were interested not merely in things, but in their meanings, the tower of the sciences resting upon the base of mathematics was the most significant object in the whole Exposition. For it was the symbol of a human triumph that was the source from which had come all the other remarkable achievements on display — a source so fruitful that it reached beyond the limitations of these particular achievements, and would ever continue to reach beyond the even more remarkable accomplishments that would come from it in the future. More than that, it was the symbol of something that was far too great to be put on display: the amazing theoretical attainments of Einstein, Planck, Bohr, Heisenberg, Schrodinger, Dirac, and De Broglie — to mention only a few of the names which have made modern physics great.

But there were even more far-reaching implications in this symbolism. For it was a revelation of what has happened to the human intellect in modern times. And here we have in mind, not merely a question of scientific methodology, but something far deeper. In this symbolism could be found an indication of the precise direction in which the mind of man has progressed in the modern era. For in so far as the speculative intellect is concerned, modern progress has not been a progress in wisdom, but in sciences; and not in science in the full and perfect sense of the term in which it was understood by the Greeks and the Medievalists — the sense in which it signifies an intellectual triumph over the obscurity of matter to the extent of laying hold of

the objective <u>logos</u> of nature with clarity and certitude — but in that dialectical type of knowledge into which science necessarily issues as it pursues its development in the direction of increasing concretion in matter. And in so far as the practical intellect is concerned, modern progress has not been a progress in prudence, but in art; and, once again, not in the higher form of art, the art of imitation or fine art, in which the darkness of matter is transfused by the light of the mind, but in technological art, in which the intellect is bent upon the exploitation of matter, and at best achieves only a kind of compromise with it. And as this development has gone on, not only has dialectical science tended to dispute the hegemony of wisdom in the speculative order, and technological art that of prudence in the practical order, but science and art have been drawn closer and closer, and united in a new and strange intimacy.

Obviously, the matrix of this distinctive intellectual growth, so characteristic of our times, is something highly complex, and it would be a naive oversimplification to attribute it to any one factor. Nevertheless, we feel that the source which has contributed most to it, and given it its strongest impetus, and dictated its precise direction has been the erection of the tower of the sciences upon the base of mathematics: the interpretation of the physical world in the light of the world of mathematics.

For the moment we shall not attempt to establish this point. It has been suggested here merely to orientate properly the problem we are undertaking to discuss, and further development of it now would take us too far afield and make it necessary to anticipate much of what is to follow. But perhaps it would not be irrelevant to quote a passage from one of the greatest contemporary mathematical physicists, in which what we have been saying finds at least a general confirmation. In the introduction to his Electrons, Protons, Neutrons, and Cosmic Rays, Professor Millikan points out that it is only through the application of mathematics to the physical world that the secrets of nature can be effectively laid bare, and the road thrown open to man's control over nature through technological art:

> For it usually happens that when nature's inner workings have once been laid bare, man sooner or later finds a way to put his brains inside the machine and to drive it whither he wills. Every increase in man's knowledge of the way in which nature works must, in the long run, increase by just so much man's ability to control nature and to turn her hidden forces to his own account. . .
> In this presentation I shall not shun the discussion of exact quantitative experiments, for it is only upon such a basis, as Pythagoras asserted more than two thousand years ago, that any real scientific treatment of physical phenomena is possible. Indeed, from the point of view of that ancient philosopher, the problem of all natural philosophy is to drive out qualitative conceptions and to replace them by quantitative relations. And this point of view has been emphasized by the farseeing throughout all the history of physics clear down to the present. One of the greatest of modern physicists, Lord Kelvin, writes: "When you can measure what you are speaking about and express it in numbers, you know something about it, when you cannot express it in numbers, your knowledge is of a meagre and unsatisfactory kind. It may be the beginning of knowledge, but you have scarcely in your thought advanced to the stage of a science." (2)

Perhaps enough has been said to suggest that there is hardly a more important or more pressing task confronting contemporary philosophy, nor one which promises greater intellectual fruitfulness, than the analysis of the significance of the symbolism of the scientific tower resting upon the base of mathematics, the attempt to unfold one by one its manifold implications in their proper focus. Such is the purpose of this study. We shall not attempt to unravel completely the whole complicated maze of epistemological problems that have arisen out of mathematical physics, and particularly out of its more recent development. The state of this development is still too fluid perhaps to make any attempt of that kind feasible. We shall content ourselves with an analysis of the basic significance of the interpretation of nature in terms of mathematics.

It would be interesting to know how many of the hundreds of thousands of visitors at the Chicago Exposition found the tower within the tower worthy of special interest, and how many grasped the profound meaning of its symbolism. *Prima facie*, it would undoubtedly seem preposterous to suggest that no one among those who had reaped the fruits of modern progress, or even among those whose genius had been immediately responsible for its great achievements, could understand this symbolism quite so well as some who lived centuries before the Century of Progress began. Yet it does not seem necessary, or even possible to rule out such a supposition in *a priori* fashion. And if this supposition could be proved to be true, it would provide striking evidence that not everything that has happened in the century of progress has been progress. In any case, it is important to understand that modern progress has not been ambiogenetic. The mathematical interpretation of nature is indeed characteristic of the modern mind, but not in the sense that it was first discovered or created in recent times. Like most modern things it has its roots deep in the past. This has already been suggested in the passage just quoted from Millikan, and it will be one of the main purposes of this essay to show how important these roots are. But for the present it is necessary to examine its historical background only in a summary way, so that our problem will be thrown into proper focus.

2. Historical Perspective.

Not a few historians have considered the Renaissance as the origin of the physico-mathematical method in science and have generally accorded to Galileo or to Descartes the honor of being its creator. But history is there to contradict the historians, and Pierre Duhem, among others, has shown with what remarkable clarity of outline the so-called modern scientific method had already been conceived in ancient times. We shall have occasion later to show that this is true of all the major elements in this scientific method, but for the moment we are interested only in the application of mathematics to physics. It is true, of course, that only in modern times have the far-reaching possibilities and remarkable fruitfulness of this application been fully realized — realized both conceptually and practically. That is why Duhem himself could write: "Créée au XVII siècle, la physique mathématique a prouvé qu'elle était la saine méthode physique par les progrès prodigieux et incessants qu'elle a faits dans l'étude de la nature." (3) It is also true that the modern developments of mathematical physics have brought to light, or thrown into sharper outline, certain new epistemological aspects of the general physico-mathematical method. And it is probably these new aspects that have led Sir James Jeans to declare: "The fact that the mathematical picture fits nature must, I think, be conceded to be a new discovery of science, embodying new knowledge of nature such as could not have been predicted by any sort of general argument." (4) But these new aspects do not change the essence of the method. And it is this essence which has its roots in the past. It is, moreover, this essence which has the deepest and most interesting philosophical implications. That is why we must, if we would see things in their proper perspective, try to situate our problem in its historical context.

Already among the ancient Greeks the physico-mathematical method was clearly conceived, and actually put to considerable use. In this connection the name of Archimedes comes readily to mind, for it was through him that this method achieved its fullest fruitfulness in ancient times, and actually led to the definite and clear cut formulation of the sciences of mechanics and hydrostatics. But Archimedes was not the inventor of the method. Long before his time, the Greek astronomers, such as Eudoxus of Cnidos, had united mathematics and physics by attempting to "save the phenomena" through deduction drawn from geometrical hypotheses. (5) In the same way mathematics had been applied successfully in other sciences, such as optics. But since the purpose of this historical sketch is to orientate a philosophical problem, we are interested less in those who actually

applied mathematics to nature, than in those who in some reflective way attempted to bring to light the philosophical significance of this application. And in this connection it has become customary to designate two Greek philosophers as the ones who in ancient times grasped more clearly than any others the meaning of the mathematical interpretation of nature and the reach of its possibilities. They are Pythagoras and Plato.

The basic doctrine of the Pythagoreans is well known. The ultimate reality of things was for them essentially mathematical; the structure of the universe was based on numbers and their relations. Aristotle characterizes their position in the following terms:

> Contemporaneously with these philosophers and before them, the so-called Pythagoreans, who were the first to take up mathematics, not only advanced this study, but also having been brought up in it they thought its principles were the principles of all things. Since of these principles numbers are by nature the first, and in numbers they seemed to see many resemblances to the things that exist and come into being — more than in fire and earth and water (such and such a modification of numbers being justice, another being soul and reason, another being opportunity — and similarly almost all other things being numerically expressible; since, again, they saw that the modifications and the ratios of the musical scales were expressible) in numbers; — since, then all other things seemed in their whole nature to be modelled on numbers, and numbers seemed to be the first things in the whole of nature, they supposed the elements of numbers to be the elements of all things, and the whole heaven to be a musical scale and a number. And all the properties of numbers and scales which they could show to agree with the attributes and parts and the whole arrangement of the heavens, they collected and fitted into their scheme; and if there was a gap anywhere, they readily made additions so as to make their whole theory coherent. (6)

For the Pythagoreans the divine One was a mathematical god; he was the supreme number, and the source and cause of all the numbers that constituted the universe. (7) All this seems to be a distant anticipation of the conclusions that one of the greatest contemporary mathematical physicists has arrived at as the result of his many years of work in the field and of his philosophical reflections upon its meaning. "Our contention," writes Sir James Jeans, "is that the universe now appears to be mathematical in a sense different from any which Kant contemplated or possibly could have contemplated — in brief, the mathematics enters the universe from above rather than from below." (8) "From the intrinsic evidence of his creation, the Great Architect of the universe now begins to appear as a pure mathematician." (9) More and more modern scientists are looking back to Pythagoras as to the one who first conceived the vision that they are laboring to realize. Whitehead, for example, has this to say:

> So today when Einstein, and his followers proclaim that physical facts, such as gravitation, are to be construed as exhibitions of local peculiarities of spatio-temporal properties, they are following the pure Pythagorean tradition. Truly, Pythagoras in founding European philosophy and European mathematics, endowed them with the luckiest of lucky guesses — or, was it a flash of divine genius, penetrating to the inmost nature of things. . . Finally, our last reflection must be, that we have in the end come back to a version of the doctrine of old Pythagoras, from whom mathematics and mathematical physics, took their rise. (10)

Ernst Cassirer also sees in Pythagoras the progenitor of modern science:

> In the times of Pythagoras and the first Pythagoreans Greek philosophy had discovered a new language, the language of numbers. This discovery marked the natal hour of our modern conception of science. . .
> The Pythagorean thinkers were the first to conceive number as an all-embracing, a really universal element. It use is no longer confined within the limits of a special field of investigation. It extends over the whole realm of being. When Pythagoras made his first great discovery, when he found the dependence of the pitch of sound on the length of the vibrating chords, it was not the fact itself but the interpretation of the fact which became decisive for the future orientation of philosophical and mathematical thought. Pythagoras could not think of this discovery as an isolated phenomenon. One of the most profound mysteries, the mystery of beauty, seemed to be disclosed here. To the Greek mind beauty always had an entirely objective meaning. Beauty is truth; it is a fundamental character of reality. If the beauty which we feel in the harmony of sounds is reducible to a simple numerical ratio it is number that reveals to us the fundamental structure of the cosmic order. "Number," says one of the Pythagorean texts, "is the guide and master of human thought. Without its power everything would remain obscure and confused." We would not live in a world of truth, but in a world of deception and illusion. In number, and in number alone, we find an <u>intelligible</u> universe. . .
> In this general methodological ideal we find no antagonism between classical and modern physics. Quantum mechanics is in a sense the true renaissance, the renovation and confirmation of the classical Pythagorean ideal. (11)

But Pythagoras is not the only one among the ancient Greeks to whom modern scientists and philosophers of sciences are looking back for inspiration. In the question of the mathematical interpretation of nature he is made to share his honors with Plato:

> An intense belief that a knowledge of mathematical relations would prove the key to unlock the mysteries of the relatedness within Nature was ever at the back of Plato's cosmological speculations. . .
> His own speculations as to the course of nature are all founded upon the conjectural application of some mathematical construction. . .

> Plato's mathematical speculations have been treated as sheer mysticism by scholars who follow the literary traditions of the Italian Renaissance. In truth, they are the products of genius brooding on the future of intellect exploring a world of mystery. (12)

The Platonic doctrine on the question of mathematical physics is considerably more difficult to define than the Pythagorean. For in the time that had elapsed between Pythagoras and Plato the development of the philosophical mind had gone a long way: it had gone far enough to reach a high degree of complexity, but not far enough to reduce this complexity to the clarity of an accurately defined and well articulated system. Historians have presented the position of Plato in a way which makes it appear extremely paradoxical. On the one hand, it is often identified with that of Pythagoras. It is in this way that it is presented by Emile Meyerson: "Pour Platon, le fin fond de la nature, ce que nous appelons actuellement, d'un terme kantien, la chose en soi, est mathématique et n'est que mathématique. Tout le réel se compose uniquement de figures de géométrie." (13) Since mathematics is in a sense the most perfect form of rationality for the human mind, it would seem to follow that for Plato nature was in itself something perfectly rational. And Meyerson seems to accept in substance this inescapable consequence, for he writes: "Platon. . . croyait fermement à l'explicabilite de l'univers. . . Pour lui, en effet, la régularité de la nature, sa légalité, n'était précisément qu'un corollaire de cette

rationalité. (14)

On the other hand, nature would seem to have been in a sense completely irrational for Plato, for he held that no true science (episteme) of it was possible. About the material universe man could have only opinion (doxa). (15) And it has been customary to draw a sharp contrast between the irrationality of the universe of Plato and the rationality of the universe of Aristotle, who made a science of nature possible by incarnating, so to speak, the Platonic ideas in the world of sense. The paradox could scarcely be more incisive; on the one hand the transparent intelligibility of mathematics, the most rational of all the sciences; on the other an unintelligibility so complete as to preclude the possibility of any true science.

We are evidently faced here with the traditional problem of the conflict between the rationality and the irrationality of the cosmos which had been so acute for the philosophers who had preceded Plato, especially Heraclitus and Parmenides. In a sense it is this conflict that is at the bottom of the problem we are undertaking to solve. But we feel that in so far as Plato himself is concerned the paradox has been rendered more acute than it actually is by the more or less arbitrary oversimplifications of historians.

In the first place, though it is true that Plato borrowed heavily from the Pythagoreans, his position cannot be identified with theirs. The impact upon the Platonic physics of other systems, especially that of Heraclitus, was too strong to allow such an identification. (16) For Plato the mathematical world was not realised as such in the world of sense; the ideal mathematical forms were not given in nature, but merely suggested by it, in so far as nature in some more or less obscure way participated in them. The world of mathematics was not simply immanent in the physical world, but to some extent transcendant from it. Yet it was not so far removed from it as the world of pure ideas. It occupied, in fact, a kind of intermediary position between the ideas and the world of changing things. That is why the mathematical forms were realised in nature more easily and more perfectly than the other ideas. But at the same time this realisation came from without.

The following passage of Aristotle brings out the difference between the position of Plato and that of the Pythagoreans:

> But he agreed with the Pythagoreans in saying that the One is substance and not a predicate of something else; and in saying that the Numbers are the causes of the reality of other things he agreed with them; but positing a dyad and constructing the infinite out of great and small, instead of treating the infinite as one, is peculiar to him; and so is his view that the Numbers exist apart from sensible things, while they say that the things themselves are Numbers, and do not place the objects of mathematics between Forms and sensible things. His divergence from the Pythagoreans in making the One and the Numbers separate from things, and his introduction of the Forms, were due to his inquiries in the region of definitions (for the earlier thinkers had no tincture of dialectic). . . .(17)

It is clear from this text that the reason why Plato separated the mathematical forms from the physical world was that the absolute, universal, and necessary definitions characteristic of mathematics could not be realised as such in the essentially mutable world of sense. Nevertheless, physical reality in some way participated in those mathematical forms, and it seems that for Plato our knowledge of nature could approximate to the true scientific knowledge that is characteristic of the intelligible world in so far as it could take on the form of precise measurement and mathematical formulation. In the Philebus (18) for example, he distinguishes between the arts "which have a greater participation in true scientific knowledge and those which have less." And to illustrate his point he says, "If we took away the numbering and measuring and weighing from all the arts, what would be left in each case would be called a poor thing..."

Ernst Cassirer has characterised the position of Plato in the following terms:

> It is rooted in Plato's interpretation of mathematics, which is for him the 'mediator' between the ideas and the things of sense. The transformation of empirical connections into ideal ones cannot take place without this middle term. The first and necessary step throughout is

to transform the sensuous indefinite, which as such cannot be grasped and enclosed in fixed limits, into something that is quantitatively definite, that can be mastered by measure and number. It is especially the later Platonic dialogues, as for example the Philebus, which most clearly developed this postulate. The chaos of sense perception must be confined in strict limits, by applying the pure concepts of quantity, before it can become an object of knowledge. We cannot rest with the indefinite 'more' or 'less', with the 'stronger' or 'weaker', which we think we discern in sensation, but we must strive throughout for exact measurement of being and process. In this measurement, being is grasped and explained (cf.Philebus, 16, 24f) Thus we stand before a new ideal of knowledge, one which Plato himself recognized as in immediate harmony with his teleological thought, and combining with it a unified view. Being is a cosmos, a purposively ordered whole, only in so far as its structure is characterized by strict mathematical laws. The mathematical order is at once the condition and the basis of the existence of reality; it is the numerical determinateness of the universe that secures its inner self preservation. (19)

Plato's doctrine here, as in so many questions, is far from being easily definable. But perhaps enough has been said to show that his position can be identified with that of Pythagoras only by considerable oversimplification. On the other hand, it is perhaps an even greater oversimplification to draw the contrast between him and Aristotle so incisively that the Peripatetic world appears as something completely rational and the Platonic world as something completely irrational. We shall point out later what a large part the paralogon played in the system of Aristotle. It was precisely because of the irrationality he saw in the cosmos that he conceived of mathematical physics as a scientia media, an intermediary science

in which it was necessary to reach out beyond the realm of physics to that of mathematics in order to rationalize nature. Paradoxical as it may appear, the Aristotelian cosmos is at once both less rational and more rational than the Platonic, and the solution of this antinomy lies in a distinction between two types of rationality. We consider this distinction to be of capital importance; it will, in fact, be one of the keys for the solution of our whole problem.

The first type of rationality is that proper to the physical world itself. It is a rationality that arises out of the existence of foci of intelligibility in the obscure mass of materiality, of rallying points of intellectual stability in the flux of contingency. Because the mind can discover and disengage these intelligible forms, in a confused way at least, a science of nature in the strict sense of the word, in the sense of episteme, is possible. It would seem that Plato never arrived at the realization of this possibility, and it remained for Aristotle to found the philosophy of nature. From this point of view, the Platonic cosmos was irrational; it was the Heraclitean cosmos of change and obscurity. Of it the mind could not have true episteme, but only doxa.

The second type of rationality is the mathematical rationality of which we have already spoken. From this point of view the Platonic world was extremely rational. For even though in the scheme of Plato nature was not composed intrinsically of mathematical forms, and the process of mathematization came in some way from without, nevertheless nature was profoundly mathematical in the sense of being highly amenable, perhaps indefinitely amenable, to this process of mathematization. Professor A. E. Taylor sums up Plato's doctrine on this point in the following terms:

> The identification of the forms (εἴδη) with numbers means that the "manifold" of nature is only accessible to scientific knowledge in so far as we can correlate its variety with definite numerical functions of "arguments". The "arguments" have then themselves to be correlated with numerical functions of "arguments" of higher degree. If this process could be carried through without remainder, the sensible world would be finally resolved into combinations of numbers, and so into the transparently intelligible. This would be the complete "rationalization" of nature. The process cannot in fact be completed, because nature is always a "becoming", always unfinished; in other words, because there is real contingency. But our business in science is always to carry the process one step further. We can never completely arithmetise nature, but it is our duty to continue steadily arithmetising her. "And still beyond the sea there is more sea"; but the mariner is never to arrest his vessel. The "surd" never quite "comes out", but we can carry the evaluation a "place" further, and we must. If we will not, we become "ageometretes". (20)

Plato seems to have considered this mathematization as the revelation of a <u>logos</u> that was proper to nature. That is why in his system mathematical rationality could supplant physical rationality, and his mathematical interpretation of nature become a philosophy of nature. From this

point of view, Aristotle's attribution of mathematicism to the Platonists would seem to apply to Plato himself: "Mathematics has been turned by our present day thinkers into the whole of philosophy".[21]

Aristotle's discovery of the physical rationality of nature did not make him lose sight of two important facts. The first fact was that this rationality is only partial, indeed extremely meager. He too recognized a <u>doxa</u> of nature along with the <u>episteme</u> he had discovered. As we have already suggested, and as we shall explain more fully later, it is only as long as the mind remains in generalities that it is able to lay hold of an objective <u>logos</u> of nature with certitude; and as it follows its natural development towards fuller concretion, this certitude very quickly fades into a dialectical knowledge that is similar to the Platonic <u>doxa</u>. The second fact was that Aristotle also recognized the part played by mathematical rationality in the study of nature. Indeed, one of the main objectives of this study is to show with what clarity and precision he recognized it. But we shall not take time out now in this brief historical sketch to set forth his position on this point. For besides the general fact that all that is to follow will be an explanation and development of it, we intend later in this chapter to give special attention to the question of the relevance of Peri potetician in the problem of mathematical physics. Let it suffice for the moment to have pointed out why the Aristotelian cosmos was at once both more rational and less rational than the Platonic. The universe of Plato seems to have been completely rational from the mathematical point of view, at least in the sense of being indefinitely amenable to mathematization. It was at the same time completely irrational from the purely physical point of view. The universe of Aristotle was at once partially rational and partially irrational from both points of view.

Another interesting paradox emerges from a comparison of the positions of Plato and Aristotle. In the doctrine of Plato the mathematical world is closer to the physical world and at the same time farther away from it than in the doctrine of Aristotle. It is closer to it for the reasons just indicated: for Plato the physical world is indefinitely amenable to mathematization, and this mathematization is a revelation of a logos that is proper to nature; for Aristotle only one aspect of nature is susceptible of the application of mathematics, and even with regard to this one aspect, the application always remains essentially extrinsic in the sense of providing only a substitute rationality.

The mathematical world is at the same time farther

away from the physical world in the position of Plato than in that of Aristotle. In separating the mathematical world from the physical world with which it was identified in the doctrine of the Pythagoreans, Plato gave to it an ontological existence that was independent of the material cosmos. Aristotle also separated the mathematical world from the physical world, but in doing so he gave it only a conceptual existence. For him mathematical forms are abstracted by the mind from the quantitative determinations of the material cosmos. As such they can exist only in the mind. In so far as ontological existence can be attributed to them at all, this existence must be found in the material cosmos. (22) But they can have this existence only at the expense of being robbed of the specific state of abstraction that is proper to them, and that is why, in themselves, they always remain essentially extrinsic to nature. Since, then, the mathematical forms of Aristotle have no ontological existence apart from sensible things and always have an essential physical reference they are closer to the physical world than those of Plato. But since the abstraction that is proper to them makes it impossible for their properties to be attributed to the things of nature, they are at the same time farther away from the physical world.

It is clear, then, why Aristotle was justified in claiming that the Platonists had turned mathematics into the whole of philosophy. For because of the closeness of the mathematical world to the physical world in the doctrine of Plato, his physics was a kind of mathematical physics. On the other hand, because of the ontological existence attributed to the mathematical world, his mathematics took on a metaphysical character, and to that extent his metaphysics was a kind of mathematical metaphysics. That is why so much of his speculation about reality, whether physical or metaphysical, is involved in mathematics. And that is why on the face of things his system might appear as the best philosophical explanation of the mathematical interpretation of nature. But we feel that a deeper analysis will reveal that this is not true. For his mathematical physics is far from being the mathematical physics of modern science. Strange as it may seem, the very proximity of his mathematical world to the physical world prevents his doctrine from being the true explanation of modern mathematical physics. On the other hand, the very fact that he invested the mathematical world with an ontological existence of its own drew mathematics out of its proper sphere and away from its proper function, and got it involved in intellectual situations alien to its true character and to the rôle it plays in modern science.

The following lines of Professor Strong are extremely

pertinent here:

> To substitute mathematical objects for the "fiction" of Forms makes ideal and mathematical number the same and destroys the distinction by which mathematical number is valid no matter what metaphysical theory of the universe is advanced; "for they state hypotheses peculiar to themselves and not to those of mathematics." (Aristotle: Met. λIII, 1086 a 9)
> The hypotheses in respect to the metaphysical status of number are peculiar to metaphysics and not to mathematics. To make ideal and mathematical number the same is a verbalism, a figurative way of speech disguising the fact that the ideal number is not the mathematician's science nor the use of mathematics in dealing with physical phenomena. Optics, music, and astronomy are open to mathematical treatment or involve a mathematical element. Their subject-matter is mathematically formulable, because objects can be designated by number and can present quantitative aspects. Further to posit mathematical objects and relations as having substantial existence not only does not advance mathematical science, but also results in a confusion of mathematical procedures and properties with the first principles of being. . .
> Plato, if we may judge from Aristotle's account proposes a scientific myth. Aristotle would object to identifying mathematics, the demonstrative science, with the conjectural theories of existential number; at least he objects to supposing that "ideal" mathematical number is, in fact, what mathematics is before going to the length of paying it metaphysical compliments.
> If we suppose that God is a geometer who geometrizes continually, we have carried mathematical certainty to the throne of metaphysical or theological certainty. It will thence be delivered back to us in the creation of things, by figure and number. It will enter into knowledge, since the soul itself will be a number. What actually returns in the philosopher's account is the discretion and classification of Intelligences, Ideas, the soul, and the existences which make up the world after the patterns, paradigms exemplars, divine or seminal numbers in the mind of God. The procedures without which there is no demonstrative science do not come back from this journey. Numbers and figures are valued in respect to their reality and this depends upon their status in respect to God and not to mathematical use. In the face of such a transformation, arithmetic and geometry are propaedeutic to theological arithmetic, ancillary sciences for a kind of superscience in which they become metaphores and analogues. (23)

As has already been noted, it is being frequently urged by contemporary philosophers of science that the doctrine of Plato and the Platonic tradition are the metaphysical forebear of modern mathematical physics. "In modern times," writes Cassirer, "mathematical physics first seeks to prove its claims by going back from the philosophy of Aristotle to that of Plato." (24) This claim might mean several things. In the first place, it might mean that historically it was the platonic tradition that actually gave birth to modern mathematical physics, that it provided the metaphysical basis and the intellectual impetus which brought about its origin and development. It is in this way that the claim is understood by many modern critics, and Professor Burtt, among others, has gone to some lengths in his _Metaphysical Foundations of Modern Physical Science_ to give it substance. (25) We do not think that the claim, understood in this sense, has as much importance as might first appear. For history is not logic; nor, generally speaking, is its development shaped by per-se determined causes. There is consequently no reason why a philosophical system which is wholly inadequate to explain the true meaning of mathematical physics might not have been the actual historical impetus which brought about the origin of modern physical science.

Yet it is interesting to note that an accurate and detailed study of this question recently undertaken by Professor Strong has made the claim that the Platonic tradition sired modern science appear extremely dubious. Strong undertook this study with the intention of consolidating the opinion of Burtt, but all the evidence that emerged from a close examination of the work of the scientists of the early-modern period forced him to arrive at the opposite conclusion. In his _Procedures and Metaphysics_ he writes:

> A Pythagorean-Platonic (or Neo-Platonic) conception of mathematics is regarded by some present-day critics as the realistic and rationalistic doctrine of a mathematical structure of nature. This may mean that we are today (in the light of contemporary Platonic scholarship) in a position to establish critically analogies between Plato's writings and prominent characteristics of modern science and philosophy. If, however, it is asserted that the early-modern mathematical investigators based their science upon metaphysical foundations, Platonic or otherwise, the weight of evidence gleaned from a survey of some of the Italian scientists is opposed to such an assertion. The historical problem should here be disentangled from modern critical exposition. By such exposition, it can be maintained that a Pythagorean-Platonic metaphysics is compatible with the mathematical treatment of nature. In the light of historical evidence, however, we may question whether the Platonism of the fifteenth and sixteenth centuries had at that time the role and significance which philosophers now critically assign to it in connection with modern science. The assertion that the Platonic metaphysics laid the foundations for the mathematical science of Galileo is at odds with the positive evidence already presented. Futhermore, it appears highly questionable when the tradition of Platonism is examined. The Neo-Platonic doctrines of Ficino, Giovanni

Pico, and Reuchlin, and of the mathematical writers — Zamberti, Domenico and Dee — express metamathematical doctrines carried over from Proclus and his predecessors with additional cabalistic embroideries. If this archaic tradition is characteristic, we are in a position to recall the objections and difficulties raised against Nicomachus, Theon, and Proclus. The main intention of this chapter is to expose the definitely archaic character of the Platonizing tradition of metamathematics preserved in several mathematical writers — archaic that is, in the sense of its ineptness and nonconnection with the scientific work of the period in which it is reinvoked...

The Neo-Pythagoreans and Neo-Platonists were impressed with the mathematical disciplines, particularly arithmetic. Mathematics is taken over and given a cosmological significance, but the doctrines presented, the metamathematics of Platonizing thinkers, are foreign to the method and use of mathematics. The role attributed to number satisfied the assertions of metaphysics, but these assertions could not be applied or substantiated by either the logic or the practice of the mathematician. The metamathematicians assume a being and function for mathematical objects superior to the subject-matter and procedures of the science proper and assume that this metaphysical status is more real and important. Mathematics and mathematical science could not and were not expected to substantiate the assertion that one could by mathematics mount to a knowledge of a superior realm of being; yet a propaedeutic value was supposed to lie in this initiative capacity of mathematical study. The converse of this assertion is equally unsubstantiated, namely, that he who knows the mysteries of ontological and cosmological number-forms is able to penetrate into the inner significance of natural things. This is not a hypothesis for mathematical procedure. The basic supposition is the notion that natural things are the created copies of a creating form, inferior effects in an individual of a superior, unitary cause. Thus, although the metamathematicians employed a number-symbolism, the symbolism stood for forms and efficacies not mathematically conceived. . .

It is a sobering reflection to consider how long the Pythagorean arithmology and its constitution in the Neo-Platonic system persisted in claims unsubstantiated in fact. Demands of logical and doctrinal consistency were satisfied so far as the purpose and end of the metaphysician were concerned. To suit a metaphysical purpose, mathematics was thrown into a status and assigned a role divorced from mathematical conception and meaningless for procedure. The metaphysical and of cosmological status and divine residence was assumed to be the goal for which mathematics was preparatory as an intellectual purification; and since the One is casual of the many and the archetypal number-form is the unity of the individual, created thing, the use of mathematics is supposed to depend upon the constitution of natural things by the metamathematical patterns. Modern mathematical-physical science established its method and achieved its results in spite of, rather than because of, this kind of metamathematical tradition. Had the early-modern mathematical investigators in general, rather than by exception, taken the philosophical tradition seriously, history might have seen more mixtures of metaphysics and science similar to Kepler's, without, perhaps, the saving conditions that brought Kepler's metaphysical predispositions to a scientific issue. (26)

But the modern critics' insistence upon the relevance of the doctrine of Plato for modern science might also be taken to mean that among all philosophical systems, or at least among those which have come down to us from antiquity, this doctrine provides the most adequate explanation of the true meaning of mathematical physics. Understood in this sense, the claim is of extreme importance. And it is the purpose of this study to dispute its validity. But in doing so we have no intention to minimise the genius of Plato or his contributions to the philosophy of science. In his doctrine the philosophical mind made a great advance towards providing the true explanation of the mathematical interpretation of nature. The concept of the world of mathematics as occupying a kind of intermediary position between the physical world and the world of pure ideas was a significant contribution. Even more significant was the corollary that naturally flowed from it:

the mathematization of the cosmos is in some sense imposed upon nature from without. Moreover, there are a number of striking analogies between prominent features of modern science and points of Platonic doctrine. The view now generally accepted by the best scientists and philosophers that experimental science can never give more than probable knowledge would seem to be a confirmation of the Platonic dogma. The increasingly evident fact that modern science is essentially constructed of idealizations, that is to say of ideal forms and limit cases which are not given in nature but merely suggested by it, that scientific laws are not discovered in the objective universe but imposed by the mind in its attempt to rationalize experience would seem to be reminiscent of the Platonic doctrine of the relation between ideas and physical reality. Out of this mathematization and rationalization of experience through the process of idealization has come the ever increasing use of hypothesis, which played such an essential role in the method of Plato.[27] And there would seem to be something kindred to Platonism in the a priori character of the modern scientific world which is made up so largely of constructs of the mind. All of these points are significant, but we do not feel that they suffice to constitute the doctrine of Plato as an adequate philosophy of science.

Continuing now our historical sketch, we find that in the Middle Ages the problem of the mathematical inter-

pretation of nature received comparatively little attention, though, as we shall see, its true nature was far from being ignored by the Thomistic school. Grosseteste at Oxford seems to have had considerable interest in the possibilities of mathematical physics. We are told that he tried to reduce all the sciences of nature to the one universal science of optics, that he considered mathematical principles as the key to all knowledge of the physical universe, and consequently tried to explain natural phenomena in terms of geometrical lines, figures and angles. This same interest is found in Roger Bacon, who in this, as in so many ways, anticipated the so-called modern mind. Bacon held that the book of nature is written in the language of geometry, and that mathematics is "the alphabet of all philosophy." How accurately he had conceived the mathematico-observational method of modern physics may be gathered from the following lines:

> It is true that mathematics possesses useful experience with regard to its own problems of figure and number, which apply to all the sciences and experience itself, for no science can be known without mathematics. But if we wish to have complete and thoroughly verified knowledge, we must proceed by the methods of experimental science. [28]

With the dawn of the early modern period a new, spontaneous enthusiasm for mathematics began to make itself manifest. And this gravitation of the mind towards mathematical science soon became all of a piece with the general pattern of Renaissance philosophy, which was so profoundly humanistic. For, as we shall explain later on, mathematics is the most "human" of all the sciences, in the sense that it has the greatest connaturality with the human intellect. It is also the science in which the mind can in some way imitate the a priori and creative character of divine knowledge, and as a consequence it offers to the mind a great measure of autonomy. That is why it was almost inevitable that there should be a natural gravitation towards mathematics in the period of humanism in which the intellect of man tended to become the measure of all things and to that extent necessarily divine, and in which there was such a universal vindication of the complete autonomy of the mind. "Through Copernicus', Kepler's and Galileo's great discoveries," writes Dilthey, "and through the accompanying theory of constructing nature by means of mathematical elements given a priori was thus founded the sovereign consciousness of the autonomy of the human intellect and of its power over nature; a doctrine which became the prevailing conviction of the most advanced minds." (29)

This gravitation towards mathematics is already found in the doctrine of Cardinal Nicholas of Cusa, in whom were burgeoning practically all the trends which were subsequently to give direction to the development of the modern mind. He held that "knowledge is always measurement", (30) that "number is the first model of things in the mind of the Creator", (31) and that "there is nothing certain in our

knowledge except mathematics." (32) From these principles he derived the idea of a universal mathematical structure and determination of reality, or a reality whose spiritual core and origin is revealed in its being the subject of universal laws, laws of number and magnitude." (33)

In the early modern period the one who grasped most clearly the significance of mathematics for the study of nature was undoubtedly Leonardo da Vinci. For Leonardo science was genuine only in the measure in which it was mathematical. "No human investigation can call itself true science unless it proceeds through mathematical demonstrations." "There is no certainty in sciences where one of the mathematical sciences cannot be applied, or which are not in relations with these mathematics." (34) "Oh, students, study mathematics, and do not build without a foundation." This enthusiasm for mathematics did not, however, lead him to believe that nature itself is mathematical; he attributed to the mathematical world only conceptual existence: e tuta mentale. And he was insistent upon combining observation with mathematical speculation. "Those sciences are vain and full of errors which are not born from experiment, the mother of all certainty, and which do not end with one clear experiment. (35) That all this was not pure theory in the mind of Leonardo is well known. His important contributions to the development of mechanics, hydraulics, and optics were an impressive confirmation of his belief in the fruitfulness of the mathematico-observational method.

This method was taken up by Kepler and applied with great success to problems of astronomy. "Astronomy is subordinate to the genus of Mathematical discipline and uses Geometry and Arithmetic as two wings: through them, it considers quantities and figures of mundane bodies and movements, and enumerates times, and in this way prepares its own demonstrations: and it brings all speculations into use or practice." (36) We have already remarked that there is no conclusive evidence to show that Platonic philosophy provided a foundation for the scientific work of any of the early-modern scientists. It might seem, however, that a case could be built up for Kepler. For his writings are saturated with a deep conviction that the cosmos is made up of hidden mathematical harmonies, a conviction that seems impregnated with the quasi mystical attitude of the Pythagoreans and Neo-Platonists, which attached a recondite religious significance to the mathematical character of reality. "Geometry," he writes, "was the form of creation and entered into man with the image of God" (37) There can be no doubt that a great deal of philosophical reflection distinctively Neo-Platonic in tone accompanied the scientific work of Kepler, but it remains extremely questionable to what extent, if any, the former provided a foundation

for the latter, or exercised any true causal influence upon it. (38)

In the work of Galileo the mathematico-observational method became a well-defined scientific procedure. In his famous experiment of rolling a ball down an inclined plane at the tower of Pisa and of describing the phenomenon in terms of a mathematical equation, modern scientific method was clearly crystallized. And he pointed out the fundamental principle of this method when he wrote: "To be placed on the title-page of my collected works: Here it will be perceived from innumerable examples what is the use of mathematics for judgments in the natural sciences and how impossible it is to philosophize correctly without the guidance of Geometry, as the wise maxim of Plato has it." (39) "Philosophy is written in that great book which ever lies before our eyes -- I mean the universe -- but we cannot understand it if we do not first learn the language and grasp the symbols, in which it is written. This book is written in the mathematical language, and the symbols are triangles, circles, and other geometrical figures, without whose help it is impossible to comprehend a single word of it; without which one wanders in vain through a dark labyrinth." (40)

All scientific method involves selection, and it was inevitable that the growing consciousness of the fruitfulness of mathematics in the explanation of natural phenomena should result in an increasing concentration of attention upon the quantitative aspects of nature. But scientific methods all too easily tend to become tyrannical, and what begins as a mere selection for the purpose of explaining phenomena often issues into an explaining away of the elements left out of the selection. Galileo was probably the first in modern times to call into question the existence of the non-quantitative aspects of reality. Kepler seems to have supposed that the non-mathematical properties of nature were in some way less real, but he did not deny their objective existence. This denial is found explicitly in Galileo, for whom the qualitative properties of nature had existence as such only in the faculties of man.

> I feel myself impelled by necessity, as soon as I conceive a piece of matter or corporal substance, of conceiving that in its own nature it is bounded and figured by such and such a figure, that in relation to others it is large or small, that it is in this or that place, in this or that time, that it is in motion or remains at rest, that it touches or does not touch another body, that it is single, few or many; in short by no imagination can a body be separated from such conditions. But that it must be white or red, bitter or sweet, sounding or mute, of a pleasant or unpleasant odour, I do not perceive my mind forced to acknowledge it accompanied by such conditions; so if the sense were not the escorts perhaps the reason or the imagination by itself would never have arrived at them. Hence I think that those tastes, odours, colours, etc. on the side of the object in which they seem to exist, are nothing else but mere names, but hold their residence solely in the sensitive body; so that if the animal were removed, every such quality would be abolished and annihilated. (41)

This quantification of nature found its full realization in the philosophy of René Descartes.

It has been customary to consider Descartes as <u>the</u> philosopher of modern mathematical physics. Meyerson writes: "C'est Descartes, incontestablement, qui a été le véritable législateur de la science moderne." (42) This opinion is shared by Maritain:

> ...il (Descartes) a eu la claire vue intellectuelle du constitutif propre et des droits de la science physico-mathématique du monde, avec toutes ses exigences et, si je puis dire, sa férocité de discipline originale, d'habitus irréductible. Il mérite vraiment, à ce point de vue, d'être regardé comme le fondateur de la science moderne, non qu'il l'ait créée de toutes pièces, mais parce que c'est lui qui l'a tirée à la lumière du plein jour et établie à son compte dans la république de la pensée. (43)

We believe that this passage is filled with errors and ambiguities. It will eventually become clear, we hope, that Descartes' intellectual view of the "constitutif propre" of mathematical physics was extremely confused and profoundly erroneous. As a consequence he could have no just notion of its rights and exigencies. As a matter of fact, the extent to which he exaggerated them was nothing less than monstruous. Since mathematical physics is, as we shall see, an intermediary science, and since it is, in fact, not a science in the strict and formal sense of the word, but dialectics, nothing could be more false than to apply to it the terms "discipline originale" and "habitus irréductible". Much could be said, moreover, in criticism of the expression "république de la pensée" for taken as it stands it could easily lead to a false notion of the independence of the sciences, but this is not the place to develop such a criticism.

We do not believe that Descartes deserves to be called the founder of modern science. Nevertheless, his doctrine had an extremely important historical influence upon the development of mathematical physics and for that reason it merits considerable attention.

For Descartes the mathematization of nature was not a mere scientific method; it was a world vision. The story of how that vision came to him on that winter's night at Newburg on the Danube is one of the best known events in the history of philosophy. It had been preceded by another great discovery which was to play an all important part in the fruitful development of mathematical physics — the discovery of Analytical Geometry. Having succeeded in reducing geometry to arithmetic and algebra, in spite of the fact that the aristotelians had always insisted on their formal distinction, the next step was to reduce physics completely to mathe-

matics. It was a tremendous step, but Descartes did not hesitate to take it. In actual fact he went much farther than this and reduced the whole of philosophy to mathematics in the sense that his universal method was the geometrical method of beginning with a clear and distinct intuition and proceeding by means of deduction. All this lay behind the "Cogito." That is why his whole philosophy may be considered a kind of mathematicism. But we are not interested in this aspect of Cartesianism here.

The vision of which we have spoken is summed up in the epitaph written by his closest friend, Chanut: "In his winter furlough comparing the mysteries of nature with the laws of mathematics he dared hope that the secrets of both could be unlocked with the same key." And he has himself described this vision for us in the following terms:

> As I considered the matter carefully it gradually came to light that all those matters only are referred to mathematics in which order and measurement are investigated, and that it makes no difference whether it be in numbers, figures, stars, sounds, or any other object that the question of measurement arises. I saw consequently that there must be some general science to explain that element as a whole which gives rise to problems about order and measurement, restricted as these are to no special subject matter. This, I perceived, was called universal mathematics.
> Such a science should contain the primary rudiments of human reason, and its province ought to extend to

the eliciting of true results in every subject. To speak freely, I am convinced that it is a more powerful instrument of knowledge than any other that has been bequeathed to us by human agency, as being the source of all others. (44)

Having once laid down this principle, Descartes did not hesitate to follow its consequences to the very end. "My whole physics," he wrote to his friend Mersenne, "is nothing but geometry." (45) "I accept no principles in physics which are not at the same time accepted in mathematics." And he goes on to explain;

Nam plane profiteor, me nullam aliam rerum corporearum materiam agnoscere, quam illam omnimode divisibilem, figurabilem et mobilem quam Geometrae quantitatem vocant et pro objecto suarum demonstrationum assumunt; ac nihil plane in ipsa considerare, praeter istas divisiones, figuras et motus; nihilque de ipsis ut verum admittere, quod non ex communibus illis notionibus de quarum veritate non possumus dubitare, tam evidenter, deducatur, ut pro mathematica demonstratione sit habendum. "Et quia sic omnia Naturae phaenomena possunt explicari, ut in sequentibus apparebit, nulla alia Physicae principia puto esse admittenda, nec alia etiam optanda." (46)

The immediate and necessary consequence of the transformation of physics into mathematics was the identification of the nature of bodies with extension, of matter with quantity. What is matter, asks Descartes in the <u>Principia</u>. And his answer is that "Its nature consists neither in hardness, nor in weight, nor in heat, nor in any other qualities, but only in extension in length, breadth, and depth, which the geometricians call quantity." "Those

who distinguish between material substance and extension or quantity, either have no real idea corresponding to the name of substance, or else have a confused idea of material substance." (47)

Motion had traditionally been the main stumbling block for those who had tried to mathematicize nature. Aristotle's criticism of the Pythagoreans and Platonists had been that mathematization means the exclusion of movement, and he who is ignorant of movement cannot understand nature. And Saint Thomas had said: "Ex mathematicis non potest aliquid efficaciter de motu concludi." (48) This problem proved no obstacle to Descartes. He was convinced that even movement could be mathematicized, not in the sense in which it would be mathematicized later by the calculus of Newton and Leibniz, but in a sense far more radical. Descartes thought that motion was in its very essence mathematical, that in the last analysis it could be reduced to the displacement of a point on a plane. And this seemed so evident to him, and the nature of motion seemed so immediately clear that he scorned the definition of Aristotle whose profundity appeared to him to be nothing but the obscuration of something essentially simple and transparent.

Some modern philosophers find in this difference in the concept of motion the best expression of the difference between the ancient and the modern mind. Thus, M. Brunschvicg believes that in the modern concept of motion "une forme de l'intelligence apparait, qui remplace une autre forme de l'intelligence, avec qui elle est sans aucun rapport." (50) Whatever may be thought of this view, it is certain that in this difference between the obscurity of the Aristotelian definition of motion and the clarity of Cartesian motion we have a striking symbol of the vast change wrought by Descartes in the history of philosophy. Reality, which for the Greeks and the medievalists had always been something profoundly complex, suddenly became transparently clear. This is a very significant point.

But in a particular way, we find in this question of motion the sharpest contrast between Aristotelian and Cartesian physics. In fact, a more incisive antinomy could hardly be imagined. For Aristotle movement was a becoming; for Descartes it was a state; for Aristotle it was a process; for Descartes it was a relation. For Aristotle it was self-evident that because of the principle of inertia the cessation of a body in motion demanded a cause. We shall return to this antinomy in the course of our analysis.

With these two clear intuitions of matter and motion as points of departure, Descartes set out to deduce the whole cosmos even to its smallest detail. He felt confident that with matter and motion alone he could construct the world. In commenting upon this attempt of Descartes, Duhem writes:

> Ainsi, dans tout l'univers, est répandue une matière unique, homogène, incompressible et indilatable dont nous ne connaissons rien sinon qu'elle est étendue; cette matière est divisible en parties de diverses figures, et ces parties peuvent se mouvoir les unes par rapport aux autres; telles sont les seules propriétés véritables de ce qui forme les corps; à ces propriétés doivent se ramener toutes les apparentes qualités qui affectent nos sens. L'objet de la Physique cartésienne est d'expliquer comment se fait cette réduction.
> Qu'est-ce que la gravité? L'effet produit sur les corps par des tourbillons de matière subtile. Qu'est-ce qu'un corps chaud? Un corps 'composé de petites parties qui se remuent séparément l'une de l'autre d'un mouvement très prompt et tres violent.' Qu'est-ce que la lumière? Une pression exercée sur l'éther par le mouvement des corps en flammes et transmise instantanément aux plus grandes distances. Toutes les qualités des corps, sans aucune omission, se trouvent expliquées par une théorie où l'on ne considère que l'entendu géométrique, les figures qu'on y peut tracer et les divers mouvements dont ces figures sont susceptibles. 'L'univers est une machine en laquelle il n'y a rien du tout à considérer que les figures et les mouvements de ses parties.' Ainsi la science entière de la nature matérielle est réduite à une sorte d'arithmétique universelle d'où la catégorie de la qualité est radicalement bannie." (51)

When he had finished his task, Descartes stopped to contemplate it with pride and satisfaction, and he declared that nothing was lacking, that his work was perfect.

One of the last paragraphs in the Principia has as its title: "That there is no phenomenon that is not included in what has been explained in this treatise." (52) It was no slight claim on the part of Descartes to pretend to have a direct intuition of the inner essence of physical reality and to be able to embrace all its phenomena in a type of knowledge that was clear and exhaustive.

The proclamation of Descartes as the founder or legislator of modern mathematical physics is susceptible of a variety of interpretations. It may, in the first place, be taken to mean that his philosophical system affords the truest explanation of the meaning of physico-mathematical knowledge. We believe that any claim of this kind is far from being justified, but it would be premature to embark upon a discussion of this point here. It may also be taken to mean that he formulated with accuracy and clarity the method that has been responsible for the development of modern physics. We do not think that even this claim is admissable. Cartesian physics as a system was extremely short-lived. This in itself is not necessarily a condemnation of cartesian method, for it is possible for a thinker to work out a true scientific method, and yet in spite of it be led into numerous errors in the order of application, and this faulty application may be due to circumstances beyond con-

trol. But in the case of Descartes the errors were for the most part because of his method rather than in spite of it. His physics is a tissue of arbitrary assumptions precisely because he refused to recognize the inductive character of physical science. Modern science is constituted essentially of both a priori and a posteriori elements, and Descartes was as blind to the latter as Francis Bacon was to the former.

Nevertheless there is something to be said for Descartes. His discovery of analytical geometry provided an extremely useful instrument for the mathematization of nature, even though he failed to recognize the true nature of his own creation. But more than that, his ambition of a completely mathematicized physics bequeathed to physicists a dialectical goal towards which they would never cease to strive: to bring all the phenomena of nature under the control of number. That is why it may be said that in the philosophy of Descartes the mathematical interpretation of nature seemed to have received its official charter. From then on there was never any question of the road that physics would follow in its development.

Added to the general inspiration given to mathematical physics by cartesian philosophy, was the tremendous impetus coming from the new discoveries in mathematics:

No picture however generalized of the achievements of scientific thought in this century can omit the advance in mathematics. Here as elsewhere the genius of the epoch made itself evident. Three great Frenchmen, Descartes, Desargues, Pascal, initiated the modern period in geometry. Another Frenchman, Fermat, laid the foundations of modern analysis, and all but perfected the methods of the differential calculus. Newton and Leibnix, between them, actually did create the differential calculus as a practical method of mathematical reasoning. When the century ended, mathematics as an instrument for application to physical problems was well established in something of its modern proficiency. (53)

As a result of the philosophical influence that stemmed from Descartes and of the discovery of more powerful mathematical instruments, the role of mathematics in physics continued to grow with ever increasing fruitfulness. There were a few reactionary attempts made, particularly in Germany by Goethe, Schelling and Hegel, but they had no lasting success, and left behind them no positive trace in science.

In the physics of Newton the mathematical interpretation of nature seemed to have reached its crowning achievement. "The outstanding fact that colors every other belief in this age of the Newtonian world," writes Randall, "was the success of the mathematical interpretation of nature." (54) The part that mathematics played in the work of Newton himself is aptly expressed by the title he chose for his classical work, The Mathematical Principles of Natural Philosophy, and by the brief interpretation he gave of its significance in

the preface:

> We offer this work as mathematical principles of philosophy . . . By the propositions mathematically demonstrated in the first book, we then derive from the celestial phenomena the forces of gravity with which bodies tend to the sun and the several planets. Then, from these forces, by other propositions which are also mathematical, we deduce the motions of the planets, the comets, the moon, and the sea...(55)

Although throughout his work Newton acted as though in nature there were a possibility of infinite determination, it may be doubted perhaps just what significance he attached to this methodological principle. "To Newton, at any rate," says J.W.N. Sullivan, "the attempt to describe nature mathematically was an adventure that might or might not be successful." (56) And Dingle writes:

> In the matter of fitting observations into a mathematical framework, Newton was both more and less thoroughgoing than Galileo. He himself enlarged the framework considerably, so that while to Galileo mathematics was mainly geometry, to Newton geometry occupied only a subordinate place. Thus he was able to conduct a mathematical treatment of the phenomena of colour which Galileo had relegated to the rank of a subjective quality. On the other hand, he did not regard the whole of external Nature as necessarily mathematical in character, although he hoped it might prove to be so.(57)

It would be too long and tedious to trace the subsequent development of mathematical physics in full detail. Much could evidently be said about Leibniz whose doctrine, in so far as it related to the physical universe, was, in the last analysis, a kind of mathematicism. Much could be said in particular about Kant, whose Transcendental Aesthetics deals with the question of pure mathematics, and whose Transcendental Analytic is an explanation of the mathematical science of nature. One of the greatest contemporary philosophers of physical science, Sir Arthur Eddington, has this to say about the doctrine of Kant:

> If it were necessary to choose a leader from among the older philosophers, there can be no doubt that our choice would be Kant. We do not accept the Kantian label; but, as a matter of acknowledgment, it is right to say that Kant anticipated to a remarkable extent the ideas to which we are now being impelled by the modern developments of physics. (58)

We shall not stop to evaluate this statement now, nor to discuss in detail the relation of mathematical physics to the philosophy of Kant. This we hope to do in Chapter XII. By that time we shall be in a position to see how many large concessions must be made to Kantianism if we are to understand the true nature of physico-mathematical knowledge. For the present let it suffice to point out that Kant considered Newtonian physics as the only genuine type of science, and that there is a sense in which it is true to say that he made it the foundation of his whole elaborate philosophical system. From the following lines it is evident that for him the physical world can be known scientifically only through mathematics:

> Les suppositions de la géométrie ne sont pas des déterminations d'une simple création de notre fantaisie poétique, ne peuvent ainsi être rapportées avec certitude

> à des objets réels, mais elles sont nécessairement valables pour l'espace, et par suite pour tout ce qui peut se rencontrer dans l'espace, parce que l'espace n'est pas autre chose que la forme de tous les phénomènes extérieurs sous laquelle des objets des sens peuvent nous être donnés. La sensibilité sur la forme de laquelle se fonde la géométrie, est ce dont dépend la possibilité des phénomènes extérieurs; ceux-ci ne peuvent donc jamais renfermer autre chose que ce que la géométrie leur prescrit." (59)

For Kant space and time which are the a priori forms that determine all our scientific knowledge of the material world are reducible to the abstract concepts of continuous and discrete quantity. In his First Metaphysical Principles of the Science of Nature he writes: "In every particular theory of nature the only thing that is scientific in the strict sense of the word is the quantity of mathematics it contains." (60)

The progress of physics in recent years, particularly since the advent of the theory of relativity, the quantum theory and wave-mechanics, has resulted in a mathematization of nature never dreamed of by even the most enthusiastic of the classical physicists. (61) In one sense at least, the mathematical element seems to be supplanting more and more the purely physical. An obvious example of this is the way in which the problem of gravitation, which in classical physics was a question of dynamics involving the notion of force, has in Einsteinian physics been reduced to a problem of pure geometry. Moreover, in the comparison with classical physics, the conceptual mathematical implements now being used are of a much more abstract nature, and are taken from what is sometimes known as "pure mathematics". Sir James Jeans sees in this application of "pure mathematics" to the physical universe a new epistemological phenomenon which constitutes a major difference between contemporary and classical mathematical physics. (62)

On the other hand, paradoxical as it may seem, Relativity and Quantum physics are at the same time less mathematical and more physical than classical physics. Cartesian and Newtonian physics were in many ways extremely simplicist. They attempted to impose upon the physical universe absolute quantitative determinations such as they may be conceived of by a mathematician who does not have to worry about concrete physical processes of observation and concrete physical procedures of measurement. Einstein brought to light the vast difference between a pure mathematician and a mathematical physicist by showing how much is involved in the concrete procedures of observation and measurement. As a result, science has been brought closer to the objective physical universe. Moreover, contemporary physics has become less mathematical and more physical in the sense that it has come to realize more clearly that nature overflows any geometrical frame that we may attempt to impose

upon it, that there is a greater irrational element in nature than was suspected before. However, underneath this revolutionary character of contemporary physics there is, of course, a fundamental continuity with the past, as we shall try to make clear later on. (63)

One of the characteristic features of recent physics which is of particular interest to us is its self-consciousness. Classical physics was self-conscious but it was, so to speak, the naive self-consciousness of adolescence. In recent years physical science has begun to achieve the self-consciousness of maturity, which consists chiefly in a detached self-criticism. All of the greatest contemporary mathematical physicists, those who have contributed most to the advancement of science, such as Einstein, Planck, De Broglie, Weyl, Dirac, Heisenberg, Schrodinger, Eddington and Jeans, have felt the need of doing some serious reflective thinking about the nature of their science. This thinking is of unequal philosophical value, to be sure, but out of it has come a wealth of helpful insights into the nature of physical science. At this point we can do no more than select from these contributions a few typical observations on the general nature of mathematical physics. These will be sufficient to situate our problem accurately in its contemporary context, and that is all that interests us for the moment.

But before indicating the characteristic positions taken by some of the more recent mathematical physicists as to the general nature of their science, perhaps it would be worth while to consider here a highly significant passage of one of the most outstanding of nineteenth century biologists, Claude Bernard. Bernard was one of those who made the greatest contributions to the growth of the critical view of science, and his observations on the general character of natural science are of the greatest value:

> The absolute principle of the experimental sciences is a necessary and conscious determinism in the conditions of the phenomena. It is of such a sort that a natural phenomenon, whatever it is, being given, the experimenter can never admit that there is a variation in the expression of this phenomenon, unless at the same time there be the intervention of new conditions in its manifestation; moreover, he has an a priori certitude that these variations are determined by rigorous and mathematical connections. Experience simply shows us the form of the phenomena; but the connection of the phenomenon to a determined cause is necessary and independent of experience, and it is necessarily mathematically absolute. We thus see that the principle of the criterion of the experimental sciences is in reality identical with that of the mathematical sciences, since in each of them this principle is expressed by a necessary and absolute relation of things. However, in the experimental sciences these connections are surrounded by numerous, complex, and infinitely varied phenomena, which hide the connections from our view. By the aid of experience we analyse, we dissociate the phenomena, in order to reduce them to relations and conditions that are more simple. We wish in this way to seize the form of scientific truth, that is to say, to find the law which should give us the key to all the variations of the phenomena. This experimental analysis is the only means that we have for searching out the truths in the experimental sciences; and the absolute determinism of the phenomena, of

> which we have an a priori consciousness, is the sole criterion or the sole principle which directs and supports us. In spite of our efforts, we are still very far from this absolute truth; and it is probable, especially in the biological sciences that we shall never see it in its nudity. (64)

When scientists speak of the general question of determinism in nature, it is sometimes difficult to know whether they are talking of determinism as a methodological principle or as a physical principle. In fact the two are often enough confused in the mind of the scientists themselves. Determinism is, of course, legitimate and necessary as a methodological principle. Without it there could be no science. But it is evident from the passage just quoted that for Bernard determinism is not merely a method existing in the mind of the scientist and in the process through which he studies nature, but a reality existing in nature itself. In the physical universe is objectively realized the infinite rigor of the mathematical world. This view of Bernard seems to have been the generally accepted opinion of the classical physicists, though among them there was this difference that while for some the infinite determination of nature could be arrived at by science, at least theoretically, for others it was an objective limit towards which science must ever move. The ever increasing success of the application of mathematics to nature tends almost inevitably to lead scientists to some position of this kind, for as Professor Bridgman has pointed out:

> ...it is a result of every day experience that as we refine the accuracy of our physical measurements the quantitative statements of geometry are verified within an ever decreasing margin of error. From this arises that view of the nature of mathematics which apparently is more commonly held; namely that if we could eliminate the imperfections of our measurements, the relations of mathematics would be exactly verified. Abstract mathematical principles are supposed to be active in nature, controlling natural phenomena, as Pythagoras long ago tried to express with his harmony of the spheres and the mystic relation of numbers. (65)

And although Heisenberg's principle of uncertainty, which expresses the high degree of indeterminism recently discovered by scientists on the level of microscopic phenomena, has thrown wide open the whole problem of the determination of nature, there are still many scientists who hold that this indeterminism is purely subjective and that it gives no reason for doubting the objective existence of a mathematical determination in the universe.

In the annals of modern science there is no greater name than that of Albert Einstein, and consequently his opinion on the nature of mathematical physics is of the utmost interest. Of the many important statements he has made on the subject the following is perhaps the most significant for us and the most relevant to our present purpose.

> On the contrary, the scientists of those times were for the most part convinced that the basic concepts and laws of physics were not in a logical sense free inventions of the human mind, but rather that they were derivable by abstraction, i.e. by a logical process, from experiments.

> It was the general theory of Relativity which showed in a convincing manner the incorrectness of this view. For this theory revealed that it was possible for us, using basic principles very far removed from those of Newton, to do justice to the entire range of the data of experience in a manner even more complete and satisfactory than was possible with Newton's principles. But quite apart from the question of comparative merits, the fictitious character of the principles is made quite obvious by the fact that it is possible to exhibit two essentially different bases, each of which in its consequences leads to a large measure of agreement with experience. This indicates that any attempt logically to derive the basic concepts and laws of mechanics from the ultimate data of experience is doomed to failure. If then, it is the case that the axiomatic basis of theoretical physics cannot be an inference from experience, but must be free invention, have we any right to hope that we shall find the correct way? Still more -- does this correct approach exist at all, save in our imagination? Have we any right to hope that experience will guide us aright, when there are theories (like classical mechanics) which agree with experience to a very great extent, even without comprehending the subjects in its depths? To this I answer with complete assurance, that in my opinion there is <u>the</u> correct path, and, moreover, that it is in our power to find it. Our experience up to date justifies us in feeling sure that in Nature is actualized the idea of mathematical simplicity. It is my conviction that pure mathematical construction enables us to discover the concepts and laws connecting them which give us the key to the understanding of the phenomena of Nature. Experience can of course guide us in our choice of serviceable mathematical concepts; it cannot possibly be the source from which they are derived; experience of course remains the sole criterion of the serviceability of a mathematical construction for physics but the truly creative principle resides in mathematics. I a certain sense, therefore, I hold it to be true that pure thought is competent to comprehend the real, as the ancients dreamed.(66)

This passage is so lucid and precise that it scarcely needs a commentary. The important point to be drawn from it is that although the mathematical concepts and principles used in physics are not derived directly from nature, but come from the productive activity of the mind, nevertheless there exists in the common a basic mathematical structure and through the progress of science the mathematical construction of the mind can ultimately be brought into exact conformity with it.

Allusion has already been made to the views of Sir James Jeans on the significance of the application of mathematics to nature. For Jeans recent developments in physics have produced a new and highly significant epistemological phenomenon: the successful application of "pure mathematics" to the physical universe. In classical physics the use of mathematics had been large and fruitful, but the mathematics used was something that had been previously drawn from nature; it was not "pure mathematics" deriving solely from the creative activity of the intellect. "By 'pure mathematics' is meant those departments of mathematics which are creations of pure thought, or reason operating solely within her own sphere, as contrasted with 'applied mathematics' which reasons about the external world, after first taking some supposed property of the external as its raw material." (67) It is this "pure mathematics" which is now used in Relativity and Quantum physics. And the great mystery is that nature seems to conform to these free creations of pure thought:

We could not of course draw any conclusion from this if the concepts of pure mathematics which we find to be inherent in the structure of the universe were merely part of, or had been introduced through, the concepts of applied mathematics which we used to discover the workings of the universe. It would prove nothing if nature had merely been found to act in accordance with the concepts of applied mathematics; these concepts were specially and deliberately designed by man to fit the workings of nature. Thus it may still be objected that even our pure mathematics does not in actual fact represent a creation of our own minds so much as an effort, based on forgotten or subconscious memories, to understand the workings of nature. If so, it is not surprising that nature should be found to work according to the laws of pure mathematics. It cannot of course be denied that some of the concepts with which the pure mathematician works are taken direct from his experience of nature. An obvious instance is the concept of quantity, but this is so fundamental that it is hard to imagine any scheme of nature from which it was entirely excluded. Other concepts borrow at least something from experience; for instance multidimensional geometry, which clearly originated out of the experience of the three dimensions of space. If, however, the more intricate concepts of pure mathematics have been transplanted from the workings of nature, they must have been buried very deep indeed in our sub-conscious minds. This very controversial possibility is one which cannot be entirely dismissed, but it is exceedingly hard to believe that such intricate concepts as a finite curved space and an expanding space can have entered into pure mathematics through any worth of unconscious or subconscious experience of the workings of the actual universe. In any event, it can hardly be disputed that nature and our conscious mathematical minds work according to the same laws. She does not model her behaviour, so to speak, on that forced on us by our whims and passions, or on that of our muscles and joints, but on that of our thinking minds. This remains true whether our minds impress their laws on nature, or she impresses her laws on us, and provides a sufficient justification for thinking of the universe as being of mathematical design. Lapsing back again into the crudely anthropomorphic language we have already used, we may say that we have already considered with disfavour the possibility of the universe having been planned by a biologist or an engineer; from the intrinsic evidence of his creation, the Great

> Architect of the Universe now begins to appear as a pure mathematician. (68)

It is to be noted that for Jeans the mathematical interpretation of nature gives exhaustive knowledge of it, for he says: "The final truth about a phenomenon resides in the mathematical description of it; so long as there is no imperfection in this, our knowledge of the phenomenon is complete." (69)

If we were to stop at this point and look back over the historical sketch we have been giving, we would find this one central thought running through the various opinions discussed: the fundamental reason why mathematics can be applied to nature is that nature is ultimately mathematical, that in the physical universe there is realized a basic mathematical structure; mathematical physics simply means that in the last analysis mathematics and physics are in some sense identified. Most of the authors we have mentioned would suscribe to the opinion of Juvet: "Sans préciser davantage notre pensée, nous dirons que le monde physique n'est qu'un reflet ou une section du monde mathématique." (70)

But at the present time a large number of authors are advancing an opinion which on the surface at least seems to be directly opposed to the position just stated. For many

modern philosophers of science, mathematics is nothing but formal logic, and the part that it plays in physics has no other significance than the part that logic plays in all the sciences. Vassily Pavlov has summed up this position in the following terms:

> It were well, then, to introduce briefly the claim that mathematics at bottom is only logic. To many this claim has been demonstrated for all time in the work of Frege, Peano, Bertrand Russell, A.N. Whitehead, and others, who developed the subject of "symbolic" or "mathematical" logic. Mathematics and formal logic have been declared to be identical. Both have been pictured as vast systems of so-called "tautologies", substitutions, identities, possessing novelty only in a psychological sense. The entire system of mathematics (or logic) is said to be contained in its postulate sets, which are nothing but the "rules of the game", a game conventional to the core, possibly derived from reality but vastly indifferent to it. In short, there has occured an apotheosis of the rules, the rules without the game.
>
> Many of us are very uncomfortable over the sharp separation which has occurred between the rules of the game and the game itself. Every application of mathematics-logic to nature, then, seems to us a promise of a happy reunion. We return to nature only that which belonged to it in the first place. The mystery, if any, lies in the original separation, rather than in the application. (71)

Taken as it is presented here, this opinion means that mathematics is used in physics merely as an instrument that remains extrinsic to the essence of the science in which it is employed, just as logic is a mere instrument that remains essentially extrinsic to the inner constitution of the sciences which employ it. But it must be noted that not all the authors who teach that mathematics is only a tool in physics necessarily hold that it is a purely extrinsic instrument. For, as we shall explain presently, it is possible to hold that the mathematics employed in physics constitutes an essential part of the object of physical science and still consider it as purely instrumental in the sense that the whole purpose of physical science is to know the physical universe and not the mathematical world, and consequently the whole raison d'être of the use of mathematics is to enable the mind to come into closer contact with the objective cosmos. Perhaps it is in this light that we must interpret the opinion of Dirac:

> From the mathematical side the approach to the new theories presents no difficulties, as the mathematics required (at any rate that which is required for the development of physics up to the present) is not essentially different from what has been current for a considerable time. Mathematics is the tool specially suited for dealing with abstract concepts of any kind and there is no limit to its power in this field. For this reason a book on the new physics, if not purely descriptive of experimental work, must be essentially mathematical. All the same the mathematics is only a tool and one should learn to hold the physical ideas in one's mind without reference to the mathematical form. (72)

It seems quite probable that it is also in this light that the position of Sir Arthur Eddington must be understood. Contrary to the opinion of Jeans, he holds that the physical universe is not mathematical, and that if mathematics enters into physical science it is only because the mind has introduced it from without. Nor can the role of mathematics be

reduced to a question of mere symbolism. Mathematics is able to get a grip on the cosmos because physical reality can by processes of measurement be transformed into series of measure-numbers, and the relation between these measure-numbers can be built up into a mathematical system principally through the instrumentality of the theory of Groups. In The Philosophy of Physical Science he has this to say:

> Theoretical physics to-day is highly mathematical. Where does the mathematics come from? I cannot accept Jean's view that mathematical conceptions appear in physics because it deals with a universe created by a Pure Mathematician; my opinion of pure mathematicians, though respectful, is not so exalted as that. An unbiased consideration of human experience as a whole does not suggest that either the experience itself or the truth revealed in it is of such a nature as to resolve itself spontaneously into mathematical conceptions. The mathematics is not there till we put it there. The question to be discussed in this chapter is, At what point does the mathematician contrive to get a grip on material which intrinsically does not of itself render a subject mathematical. If in a public lecture I use the common abbreviation No. for a number, nobody protests; but if I abbreviate it as N, it will be reported that "at this point the lecturer deviated into higher mathematics". Disregarding such prejudices, we must recognize that the allocation of symbols A, B, C,... to various entities or qualities is merely an abbreviated nomenclature which involves no mathematical conceptions. (73)

And he goes on to explain how the Theory of Groups is employed in transforming physical science into a mathematical system. (74)

There is still another opinion which in the mind of many of the authors who advance it may not represent anything substantially different from the position of those who hold that mathematics is nothing more than a logical tool, but which if taken literally amounts to something quite different. It is the view that the role played by mathematics in physics, is that of a universal and extremely convenient language. In so far as it is used in physics, mathematics is just a code, a kind of symbolic language, a sort of esperanto of science. (75) "Mathematics," says Herzfelt, "is only a tool, a short-hand way of expression, but cannot add anything to the physical concept, although it might occasionally suggest a physical law because its mathematical expression might be particularly simple."

For some who hold this opinion, the role of mathematics in physics is reduced to that of a stenographic method; and just as short-hand is a mere substitute for long-hand, and everything it expresses can be expressed with equal fulness and accuracy, though not with equal convenience, by the ordinary mode of writing, so everything contained in a world geometry could, strictly speaking, be expressed in purely "physical language." For others the symbolism of number has advantages over the symbolism of ordinary language which reach far beyond mere convenience, and which are the source of the fruitfulness of the application of mathematics to physics. For the symbolism of ordinary language can represent reality only in a dispersed and isolated way, whereas the symbolism

of number is essentially a relational symbolism and that is why it is able to represent the structure of the universe and thus open up its secrets. Perhaps the clearest expression of this opinion is found in Ernest Cassirer:

> The symbols of language themselves have no definite systematic order. Every single linguistic term has a special "area of meaning". It is, as Gardiner says, "a beam of light, illumining first this portion and then that portion of the field within which the thing, or rather the complex concatenation of things signified by a sentence lies." But all these different beams of light do not have a common focus. They are dispersed and isolated. In the "synthesis of the manifold" every new word makes a new start.
> This state of affairs is completely changed as soon as we enter into the realm of number. We cannot speak of single or isolated numbers. The essence of number is always relative, not absolute. A single number is only a single place in a general systematic order. It has no being of its own, no self-contained reality. Its meaning is defined by the position it occupies in the whole numerical system... We conceive it as a new and powerful symbolism which, for all scientific purposes, is infinitely superior to the symbolism of speech. For what we find here are no longer detached words but terms that proceed according to one and the same fundamental plan and that, therefore, show us a clear and definite structural law. (76)

This view, which at first glance, at least, seems to reduce the role of mathematics in physics to a question of language, differs from the opinion of those who identify mathematics with formal logic to the extent that language differs from logic, though perhaps the distance between logic and mathematical language would not be so great as that between logic and ordinary language, it might be argued that in the measure in which mathematics would be considered a universal language it would be lifted out of the materiality of individuality and brought closer to the universal laws of thought. At first sight, this position would seem to be at the other extreme from the opinion which sees the mathematical world realized in the physical world, but perhaps if we looked deeper we might find ourselves in the presence of a case where extremes meet, for, if mathematical language is but a substitute for "physical language" might not the reason be that the mathematical world and the physical world are really one?

5. Relevance of Thomism

In undertaking to establish the significance of Thomism (77) for the problem of mathematical physics we are not insensible to the fact that such an undertaking calls for an apologia. For historians almost without exception have represented the rise and development of modern physics as something completely antithetical to the whole structure of peripatetic philosophy. Speaking of Galileo Bertrand Russell says: "His few facts sufficed to destroy the whole vast system of supposed knowledge handed down from Aristotle, as even the palest morning sun suffices to extinguish the stars." (78) And Professor Burtt writes:

> But now, of course, the question which Copernicus has thus easily answered carries with it a tremendous metaphysical assumption. Nor were people slow to see it and bring it to the forefront of discussion. Is it legitimate to take any other point of reference in astronomy than the earth? Mathematicians who were themselves subject to all the influences working in Copernicus' mind, would, so he hoped, be apt to say yes. But of course the whole Aristotelian and empirical philosophy of the age rose up and said no. For the question went pretty deep, it meant not only, is the astronomical realm fundamentally geometrical, which almost any one would grant, but is the universe as a whole, including our earth, fundamentally mathematical in its structure? Just because this shift of the point of reference gives a simpler geometrical expression for facts, is it legitimate to make it? To admit this point is to overthrow the whole Aristotelian physics and cosmology (79)

We are dealing here not merely with those who hold it as an indisputable methodological principle that enlightenment first dawned upon the world at the time of the Renaissance. Such as these we could afford to ignore. But there are many others who while they have a sincere admiration for all that Greek and medieval culture has to offer us in the way of art, of metaphysics, and of morals, nevertheless believe that if there is one field in which both Aristotle and the Medievalists are completely barren, it is the field of science. Most of these might be willing enough to concede to Professor Whitehead that scholastic logic and theology prepared the soil in which modern science took its roots,[60] but this could scarcely serve as a sufficient basis to constitute Thomism as a significant philosophy of science.

Among contemporary philosophers of science few have won for themselves wider recognition and a greater name than Emile Meyerson, particularly in questions of the relation between modern science and its historical background. Yet if there is one theme which runs through all of Meyerson's voluminous works it is that Peripateticism has absolutely nothing to offer to science. In *Identité et Réalité* he writes: "Le retour au péripatéticisme, préconisé avec tant de force et de savoir par Duhem nous paraît impossible.

Il ne nous semble pas, en effet, que la pure doctrine d'Aristote ait été une doctrine véritablement scientifique."[61] And again in *Du Cheminement de la Pensée*, he says: "La science péripatétique, assurément, a péri et, quoi qu'en pensent certains partisans extrêmes du retour au moyen âge, péri totalement et irrémédiablement. Il est aussi impossible de la maintenir en face du triomphe de la physique moderne qu'il l'est de la concilier, fût-ce même partiellement avec celle-ci."[62]

In recent years, a few historians have, indeed, come to recognize the eminence of the scientific spirit and method of Aristotle, and the worthwhile significance of the accomplishments which were the fruit of that spirit and method; but the tributes of these few are entirely restricted to the field of biological science. That these tributes are merited is evident to anyone who has ever taken the pains to read the physical treatises of Aristotle, but they leave unsolved the question in which we are directly interested. In fact some have seen in the intense devotion of the Stagirite to research in the field of biology an argument against the contention we have set out to substantiate. Dopp, for example, writes:

> Il est arrivé qu'Aristote s'est senti peu de gout pour les mathématiques, ne s'est point consacré à ces sciences qui les utilisaient, mais s'est donné surtout à ses recherches

> d'histoire naturelle et de biologie, lesquelles consistaient essentiellement en descriptions ou en analyses de qualités ou d'activités plus ou moins discontinues, donc qualitatives. . .
> Cette doctrine avait en somme pour portée de libérer le physicien à l'égard de la pensée mathématique. Elle pèsera sur toute la tradition philosophique du Moyen Age et, par certaines de ses conséquences, sur la philosophie moderne jusqu'à nos jours. (83)

The view is now being advanced by more than one philosopher of science that there is a direct connection between Aristotle's predominant interest in biological sciences and the type of logic he evolved, and that Aristotelian logic is not only of little use for the development of mathematical physics, but in some sense an obstacle to it. For biology is essentially qualitative and classificatory, that is to say, it attempts to classify living beings in a schema of genera and species that is based upon qualitative characteristics. And that explains, we are told, why Aristotelian logic is essentially classificatory, and not relational like modern mathematical logic. Professor Whitehead has laid considerable emphasis on this point:

> In a sense, Plato and Pythagoras stand nearer to modern physical science than does Aristotle. The two former were mathematicians, whereas Aristotle was the son of a doctor, though of course he was not thereby ignorant of mathematics. The practical counsel to be derived from Pythagoras is to measure, and thus to express quality in terms of numerically determined quantity. But the biological sciences then and till our own time, have been overwhelmingly classificatory. Accordingly, Aristotle by his Logic throws the emphasis on classification. The popularity of Aristotelian Logic retarded the advance of physical science throughout the Middle Ages. If only the schoolmen had measured instead of classifying, how much they might have learnt. (84)

Professor Etienne Gilson, who is considered by many to be one of the most eminent modern champions of Thomism, has gone far beyond either Dopp or Whitehead by claiming that peripateticism has been utterly sterile in the realm of physics because Aristotle attempted to biologize the whole of physical reality, that he actually made physical bodies into so many animals. In his essay, "Concerning Christian Philosophy" (85) we find the following devastating criticism:

> ...We are bound to condemn the scientific sterility of the Middle Ages for those very reasons which to-day make us condemn the philosophic sterility of "scientism". Aristotle also had exaggerated the scope of one science and the value of its method, to the detriment of the others; and in a sense he was less excusable than Descartes, for in this he came into open contradiction with the requirements of his own method, whereas Descartes was only carrying his through. And yet, philosophically, Aristotle's was the less dangerous error, for it was an error of fact, and left the question of principle untouched; to biologize the inorganic as he and the mediaeval philosophers did, was to condemn oneself to ignorance about those sciences of the inorganic world whose present popularity comes chiefly from the inexhaustible fertility which they display in things practical; but to mathematize knowledge entirely, and on principle, was to set strange limits to physics and chemistry, and to make impossible biology, metaphysics, and consequently moral theory. . . Aristotle's error lay in not being true to his principle of a science of the real for every order of the real, and the error of mediaeval philosophy lay in following him in this. Committing the opposite mistake to that of Descartes, Aristotle set up the biological method as a physical method. It is generally admitted that the only positive kinds of knowledge

in which Aristotelianism achieved any progress are those which treat of the morphology and the functions of living beings. The fact is that Aristotle was before everything a naturalist just as Descartes was before everything a mathematician; so much so indeed that instead of reducing the organic to the inorganic like Descartes, Aristotle claimed to include the inorganic in the organic. Struck by the dominance of form in the living being, he made it not only a principle of the explanation of the phenomena of life, but even extended it from living beings to mobile beings in general. Hence the famous theory of substantial forms, the elimination of which was to be the first care of Descartes. For a scholastic philosopher, as a matter of fact, physical bodies are endowed with forms from which they derive their movement and their properties; and just as the soul is a certain species of form — that of a living being — so is form a certain genus of soul — the genus which includes both the forms of inorganic beings and the forms or souls of organized beings.
This explains the relative sterility of the scholastic philosophy in the order of physics and even chemistry, as well as the inadequacy of Cartesianism in the order of the natural sciences. If there is in the living being anything other than pure mechanism, Descartes is foredoomed to miss it; but if there is not in physical reality that which defines the living being as such, then the scholastic philosophy will not only fail to find it there, but will never discover even what is there. Nevertheless it wasted its time in looking for what was not there; and as it was convinced that all the operations of inorganic bodies are explained by forms, it strove with all its might against those who claimed to see there something else, and clung to that impossible position until, in losing it, it lost itself. Three centuries spent in classing what must be measured, as to-day some persist in measuring what must be classed, produced only a kind of pseudo-physics, as dangerous to the future of science as to that of the philosophy which imagined itself bound to it; scholasticism was unable to extract from its own principles the physics which could and should have flowed from it. . . Formae naturales sunt actuosae et quasi vivae, said the Scholastics: between the Cartesian artificialism which makes animals into so many machines, and the Aristotelian vitalism which makes physical bodies into so many animals, there must be room for a mechanism in physics and a vitalism in biology. (86)

To this criticism Gilson appends the following inte-

resting footnote:

> It is clear that Aristotle's error, less serious than that of Descartes from the point of view of philosophy, was more serious from the point of view of science. To extend, like Descartes, a more general science to the less general sciences, leaves it possible to reach in these last what they have in common with the first; hence a mechanization, always possible though always partial, of biology; but to turn the method of a more particular science back upon a more general science amounts to leaving the more general without an object. Now, in missing the real objects of physics and chemistry, Aristotle missed at the same time all that bio-chemistry teaches us concerning biological facts — which, although it is neither the whole nor the most important part, is possibly the part which is most useful. And this, as well as being a serious gap in his theory, is the thing that human utilitarianism will never forgive him. (87)

It is to be noted that these lines are written by an historian who does not cite so much as one text to substantiate his criticism. Moreover, the only thing that presents the semblance of a reason for the assertions made is that Aristotle extended his doctrine of substantial form to inorganic as well as organic bodies, "and just as the soul is a certain species of form — that of a living being — so is form a certain genus of soul — the genus which includes both the forms of inorganic beings and the forms or souls of organized beings." The sophistry of this argument is so obvious that it does not have to be pointed out.

Gilson holds that Peripatetic sterility in the realm of physics derives from the fact that Aristotle failed to recognize or at least to follow the principles that were inherent in his doctrine, but he admits that these principles could provide a fruitful philosophy of science. This, however, has been denied by M. Augustin Mansion, who in a long article entitled "La Physique Aristotélicienne et la Philosophie," (88) has tried to show not only why nothing of any consequence for mathematical physics is found in the doctrine of Aristotle, but even why it was theoretically impossible for it to be found therein. According to Mansion, mathematical physics could find no proper place in the doctrine of Aristotle because by an unfortunate and highly arbitrary division of the sciences he created an abyss between physics and mathematics by placing them in formally different degrees of abstraction. Having once made this fatal blunder, he could not but be embarrassed by the actual existence of certain physical sciences already to some extent mathematicized, such as astronomy, optics, etc., and recognizing the utter impossibility of finding a special place for them in the scheme he had conceived a priori, he was forced to class them among the mathematical sciences, while at the same time attempting to save the situation in so a fashion by pointing out that they were "more physical" than pure mathematics. In this way he removed these sciences from the realm of physics proper. This, added to the fact that Aristotle had a personal aversion for mathematical speculation, explains why peripateticism is completely barren from the

point of view of mathematical physics.

"Voilà donc écartées de l'oeuvre d'Aristote, avant tout physicien et naturaliste, — quand il n'est pas logicien et métaphysicien, — les sciences mathématiques proprement dites. Mais il est allé plus loin, et, cette fois, il a, de façon expresse, fait appel à ses principes, pour alléger son programme de certaines sciences auxquelles on ne peut guère dénier le caractère de sciences physiques. Ce sont celles précisément qui, de son temps, se trouvaient être les plus avancées et qui avaient pris déjà la forme qui leur fait reconnaître la qualité de sciences au sens moderne du mot: astronomie, optique, harmonique ou acoustique, mécanique. La supériorité caractéristique de ces disciplines, comparées à d'autres encore moins développées, provenait du fait que le côté quantitatif des phénomènes envisagés était non seulement reconnu et décrit en termes généraux, mais était étudié en détail, par l'application poussée aussi loin que possible. Dès lors, il fallait une compétence suffisante en mathématiques pour aborder ces branches du savoir, qui par le fait même étaient devenues l'apanage des mathématiciens. Aussi Aristote les classe-t-il sans hésitations parmi les μαθήματα — les sciences mathématiques, — tout en leur attribuant un caractère "plus physique" qu'aux mathématiques pures (Physic, B.2, 194 a 7 - 12)
On touche du doigt ici les conséquences de la doctrine des deux premiers degrés d'abstraction, en même temps que de l'éloignement qu'éprouvait Aristote pour la spéculation mathématique. Les sciences ou branches de la physique déjà mathématisées auraient dû constituer pour lui le type le plus achevé des sciences physiques particulières, à condition, bien entendu, d'assigner à chacune d'elles l'étude complète des phénomènes d'un domaine bien délimité, celui de l'astronomie ou de la mécanique par exemple...
On voit donc comment, en écartant de la physique, pour les assigner au domaine mathématique, les sciences mentionnées à l'instant, Aristote a manqué l'occasion de traiter à fond sur des cas concrets parfaitement adaptés, le problème de la différence entre une étude philosophique et une étude purement scientifique de telle ou telle portion du monde matériel. Ses vues sur le degré d'abstraction de l'objet mathématique en sont responsables pour une part; mais, d'un autre côté, une fois admises, elles eussent aussi bien postulé une astronomie ou une mécanique complète, à la fois mathématique et physique, en effet, de l'aveu même du Stagirite, les entités mathématiques sont ἐξ ἀφαιρέσεως ; ce sont des abstraits ou des extraits d'un ensemble plus complexe, qui constitue précisément l'objet physique. Donc elles en font partie et pour étudier ce dernier objet de

façon intégrale, le physicien lui-même n'en peut négliger l'aspect quantitatif jusque dans ses dernières déterminations.
Nous savons donc pourquoi, — touchant la question de fait, — nous ne trouvons pas et nous ne pouvons pas trouver, dans l'oeuvre d'Aristote, des exposés ou des traités ressortissant au domaine physique et répondant à des sciences particulières assez avancées pour avoir revêtu une forme mathématique quelque peu développée. (89)

Some authors have sought for a source of this barrenness in the Aristotelian doctrine on sensible knowledge which establishes an absolute identity between the sensible and the physical, thus precluding the possibility of a physical science that would be based not on the sensible qualities of nature, but upon its quantitative relations. Speaking of the physico-mathematical sciences in relation to the system of Aristotle, Salman writes:

> Elles ne dérivent pas en effet normalement de la théorie des degrés d'abstraction, mais sont des données de fait, assez gênantes d'ailleurs, que le théoricien intègre comme il le peut dans une synthèse qui ne les prévoyait pas.
> Pour les auteurs scolastiques il n'y avait donc qu'une physique unique, homogène et uniforme, qui expliquait tout, depuis le Premier Moteur jusqu'à la salure des mers, et le régime des vents. Et ses conceptions épistémologiques étaient fondées sur une doctrine délibérée de la connaissance sensible, qui identifiait résolument le physique et le sensible. (90)

Salman makes much of this scholastic identification between the physical and the sensible. He finds in it a reason to reject not only that part of Scholastic natural doctrine which corresponds to modern physics, but even the whole philosophy of nature:

> Les scolastiques croyaient déboucher de plain-pied dans le réel, en percevoir d'emblée et par les sens l'organisation intime. Gratifiés d'une donnée immédiate et parfaitement simple, ils pouvaient édifier une scientia naturalis unique et homogène qui épuisait la connaissance de l'univers sensible. Les modernes sont moins bien partagés. Ils savent qu'il leur faut traverser la zone du sensible, qui est physiquement impure avant de retrouver un monde matériel vraiment objectif; ce n'est qu'ensuite, lorsqu'une pénible reconstruction leur aura rendu des données authentiquement physiques, qu'ils pourront songer à en faire la philosophie. La "Philosophie de la Nature" si éventuellement elle se reconstitue, sera l'analogue de la philosophia naturalis médiévale; tandis que la science physique moderne, malgré ses ressemblances superficielles avec l'ancienne, est d'un type épistémologique radicalement nouveau, dont il serait naïf de chercher la formule chez les auteurs du moyen âge.
> On peut mesurer du même coup la portée véritable de la physique scolastique, et ses possibilités d'adaptation. Il est manifestement futile, en effet, de multiplier les "objets formels", dont les nuances plus subtiles devraient remplacer les vues insuffisamment différenciées des anciens. Car, pour user de ce langage scolastique, c'est l'"objet matériel" lui-même qui se dérobe. Ces qualités sensibles, sur lesquelles repose toute la construction médiévale, n'ont point la portée ontologique qu'on leur accordait. Elles n'existent pas dans les corps de la nature, mais seulement dans la perception de qui les connaît. La physique ancienne n'est donc pas seulement erronée dans telle ou telle de ses conclusions, elle est atteinte, dès son point de départ, d'un subjectivisme radical dont se ressent profondément le système dans son ensemble. Plusieurs de ses thèses essentielles conservent sans doute une valeur permanente, et seront peut-être sauvées. Mais elles ne pourront revivre qu'après de nouvelles démonstrations fondées sur de nouvelles données, exprimées surtout dans un langage et avec une technique conceptuelle inspirés du réel physique et non par la vaine imagerie du sensible. Le seul parti raisonnable dès lors est de renoncer définitivement aux rapprochements superficiels et de reprendre l'élaboration d'une philosophie naturelle sur les bases toutes nouvelles que nous imposent une connaissance plus nuancée du monde physique et de son difficile accès. (90 a)

Other arguments of this kind could be easily adduced. One of the most telling consists in this that for Aristotle

Physics is the study of mobile being (*ens mobile*), and everything it considers must be studied in the light of mobility; yet the Aristotelians have always taught that mathematics necessarily excludes motion. As we have already pointed out, Aristotle himself used this argument against the mathematization of nature taught by the Pythagoreans and the Platonists and St. Thomas stated explicitly: "ex mathematicis non potest aliquid efficaciter de motu concludi." It would seem impossible, then, for a science to exist which would be at once physical and mathematical.

Montaigne once said of Aristotle that he had an "oar in every water and meddled with all things." However, the arguments we have just considered seem cogent enough to force the conclusion upon us that there was one expanse of water in which the Aristotelian oar never dipped: that of mathematical physics.

These are serious charges. They question the competence of Thomism in the whole realm of thought where philosophy comes to grips with science and with the multitudinous epistemological problems which have arisen out of its modern development. They go far deeper than even those who proffer them may suspect. In a sense they touch Thomism at its heart. For if there is one thing upon which Thomism prides itself, it

is its preeminence in that part of philosophy that is truly wisdom. Now it pertains to wisdom not only to have a critical knowledge of its own nature, but also to have that same critical knowledge of all the other sciences and of all their manifold interrelations. If Thomism cannot find within itself the principles which will be able to open up the inner meaning of mathematical physics and to situate it accurately in the whole epistemological scheme, it must renounce its claim to the possession of integral wisdom.

We do not propose to answer here all the charges indicated above. The whole study we are undertaking will be an answer to them. Yet it seems necessary at this point to purify the atmosphere of irrelevant considerations so that the real issue will be thrown into sharper focus.

In the first place, it must be pointed out that in seeking to establish the significance of Thomism as a philosophy of science we hold no brief for the decadent Scholasticism which first felt the impact of the rise of modern science and which has persisted in so many ways down to our own day. It is a sign of a singular lack of discernment on the part of historians to confuse true Thomism with this grotesque caricature. Galileo, who has traditionally been held up as the direct antithesis of all that peripateticism stands for, realized the necessity of distinguishing between them. In his <u>Lettere Intorno Alle Macchie Solari</u> he says: "Nec sum ignarus, quam haec opinio sit inimica philosophiae Aristotelicae; sectae magis quam principi est diversa. Da mihi redivivum Aristotelem."[91]

This does not mean that the advancement of physical science has not resulted in the liquidation of a good many of the theories proposed by Aristotle in his treatises which deal with nature in its concretion. But only those who are utterly ignorant of the meaning of experimental science can find in this a reason to condemn him. In dealing with nature in its concretion error is normal. As we pointed out in considering the philosophy of Descartes, it is important, when one wishes to evaluate the work of a thinker of the past, to distinguish between the errors for which his system and method are intrinsically responsible, and those over which he had no control. The historians who are so eloquent in ridiculing the physics of Aristotle fail to realize that the only goal that experimental science can attain is, in the last analysis, to "save the phenomena", and that the physics of Aristotle saved the phenomena that were known in his time just as accurately and as perfectly as the theory of Relativity saves the phenomena that are known today. And we may well wonder how much of Einstein's work will be still standing after as many

thousands of years have passed over it as have elapsed since the time of Aristotle.

We think that the following passage of Charles Singer is extremely discerning:

> Against Aristotle it has been urged that he obstructed the progress of astronomy by not identifying terrestrial and celestial mechanics, and by laying down the principle that celestial motions were regulated by peculiar laws. He placed the heavens beyond the possibility of experimental research, and at the same time impeded the progress of mechanics by his assumption of a distinction between "natural" and "unnatural" motion. On the other hand, we should remember that Aristotle gave an interest to the study of Nature by his provision of a positive and tangible scheme. It seems unfair to bring his own greatness as a charge against him. All our conceptions of the material world — "scientific theories" as we call them — are but temporary devices to be abandoned when occasion demands. That the scheme propounded by Aristotle lasted more than two thousand years is evidence of its symmetry and beauty and of the greatness of the mind that wrought it. That it received no effective criticism is no fault of Aristotle's, but is evidence of what dwarfs the men who followed him were by comparison with him. (92)

It is significant that the first one to call into question Aristotle's theory of the heavens seems to have been Thomas Aquinas, who considered Aristotle's doctrine as a mere opinion.[93]

It is clear, then, that in attempting to establish the relevances of Thomism for mathematical physics, we are not seeking to revive outmoded physical theories. Nor are we presuming to maintain that Aristotle or any of the Medievalists were great mathematical physicists The point is that

Aristotle was something greater than a mathematical physicist; he was a great philosopher. Unquestionably, a full and exact knowledge of mathematical physics is indispensable for any philosopher who attempts to come to grips with the highly specific and concrete epistemological problems that arise out of the advanced development of physical science. But this knowledge is not necessary in order to discover the key which will open up a clear and precise view of the true nature of mathematical physics and its relations to all the other sciences. We believe that Aristotle discovered that key. We believe that that key is necessary today if we are to find our way out of the epistemological maze into which the progress of science has led us.

It may readily be admitted that from a purely material point of view Aristotle has very little to say about mathematical physics. The few passages in which he touches upon the subject are almost swallowed up in the great bulk of his writings. But that point of view is entirely irrelevant. Moreover, there are other reasons to explain this phenomenon other than the purely extrinsic reasons which delight so many historians. It has often been maintained that Aristotle knew very little mathematics and that he had a

particular aversion for mathematical speculation. Gilson, for example, tells us that if Aristotle did not get very far with scientific enquiry in terms of quantity and measurement, "it may be simply because of his ignorance of mathematics, of which he seems to have known only simple proportion. It is possible that this fact had a considerable influence on the general trend of his labours." (94) This is also the opinion of Mansion, as we have seen. Gilson gives us neither reasons nor references to support his assertion. And all that Mansion has to offer is an allusion to a text in the twelfth book of the *Metaphysics* where Aristotle, speaking of the movements of the heavenly bodies, writes:

> That the movements are more numerous than the bodies that are moved is evident to those who have given even moderate attention to the matter; for each of the planets has more than one movement. But as to the actual number of these movements, we now — to give some notion of the subject — quote what some of the mathematicians say, that our thought may have some definite number to grasp; but, for the rest, we must partly investigate for ourselves, partly learn from other investigators, and if those who study this subject form an opinion contrary to what we have now stated, we must esteem both parties indeed, but follow the more accurate. (95)

Of this text Mansion says: "témoin la confession à peine voilée qu'il en a fait au XIIe livre de la Métaphysique à propos des astronomes, traités comme des spécialistes, devant la compétence duquel il s'incline sans vouloir discuter ni leur titre ni leurs hypothèses." (96) Even a casual reading of the text of Aristotle reveals the utter gratuity of Mansion's inference. No one who is at all acquainted with the writings of Aristotle is unaware of the fact that it is customary for him to introduce a question by considering what authorities in the field have had to say about it, and that he always has respect for the opinions of these authorities unless his own reasoning has produced evidence to induce him to differ from them. In this case, it is evident from the text and context in question that he is interested merely in arriving at some probable opinion about the number of the movements of the heavenly bodies so that the mind will be able to fix itself upon a definite number. (97) And since the opinions of Eudoxus and Callipus seem probable to him he accepts them.

As a matter of fact, scholars are now coming to recognize that Aristotle's knowledge of mathematics was far advanced for his day. "It was knowledge, rather than ignorance of the mathematics of his time," writes F.S.C. Northrop, "which supported Aristotle in the formulation of his logic." (98)

Aristotle's polemic against the mathematicism of the platonists was not a polemic against the existence of mathematical science, as some seem to think, but against the ontological existence of mathematical entities. By dissipating

the confusion of mathematics with both physics and metaphysics that was characteristic of the doctrine of the platonists, Aristotle established its true epistemological status. He thus freed it of all the associations which tended to draw it away from its proper function, and made of it a more apt instrument for the use of scientists. Professor Strong has brought out this point with remarkable clarity, and we cannot refrain from quoting the following passage in spite of its length:

> Critics can criticize Aristotle for his refusal to accept the doctrine of Form as metaphysical number, but certainly not upon the ground that he failed to consider the meaning of mathematics. Rather, one may say, it was because Aristotle refused to confuse mathematical science with metaphysical principles, and because he insisted upon the operational character and physical reference of mathematics that he refused to identify mathematical number with ideal number existing in a separate realm of reality. This means that Aristotle did not advocate the formulation of a metaphysics in mathematical terms and relations and saw such a metaphysics as a confusion of the notion of mathematics with ontological realities. Hence Aristotle held no doctrine of the universe framed in mathematical universals of relation, for he regarded the ratios and proportions of mathematics as constituting no class of existences-in-themselves. They are relational only of entities of a mathematical character in arithmetic, geometry, or some more physical science such as mechanics.
> The Physica, De Caelo, and Problemata reveal passages in which he used mathematics in connection with physical problems. This is of course not equivalent to saying that the basic principles of Aristotle's physical science were mathematical. Aristotle recognized mathematics as a self-contained science and as an instrument in the physical sciences. So far as he mainly directed his own treatment of nature to the problem of growth where mathematical formulation was not relevant, so far we may say that his in-

terest and approach were directed to other than the quantitative aspects and concepts of nature.
It is characteristic of Aristotle's approach to his predecessors that he regards them as men striving for the theoretical view. His analyses of his predecessors are thus a source of knowledge with respect to their "metaphysics." His own inquiry ends in a position opposed to the views of Democritus and Plato. The opposition, in accordance with the view presented in the foregoing analysis, is not to mathematics or to the use of mathematics in natural science, but to the role which number and mathematical objects are supposed to have as ontological existences. To insist upon the distinction between the mathematician's subject-matter and the substantial and ideal number attributed to Plato, does not involve a rejection of mathematics proper. It does involve a rejection of theories about the "real" existence of number-forms. Those who assume that a mathematical metaphysics is fundamentally important in a regulative and interpretative role to the development of mechanics and mathematical physics charge Aristotle, upon the basis of his different conclusion in metaphysics, with having obstructed the progress that would supposedly have followed from his acceptance of the Platonic theories of existential number. So far as Plato and the Academy were actually engaged in mathematical work, the argument appears to carry weight. Nevertheless, the assumption that metaphysics is important in respect to subject-matter and procedure must first be established before Aristotle can be held responsible for obstructing the development of mathematical science. (99)

It is clear, then, that there must be other reasons besides a lack of knowledge of mathematics to explain why Aristotle, having once discovered the true principles of mathematical physics, did not devote himself to their development. In the first place, in order for any substantial progress to be made in the application of mathematics to nature two kinds of instruments are essential; conceptual mathematical instruments, and physical instruments of exact experiment and measurement. Without these only extremely meager progress can be made, and Aristotle lacked both. It was only after the Renaissance that the necessary physical instruments were invented, and the conceptual instruments which were to prove so fruitful, such as analytical geometry and the calculus, were discovered. The development of mathematical physics depends completely upon these instruments, and, as Meyerson has pointed out, "si les mathématiques accomplissaient à l'heure actuelle un progrès comparable, ne fût-ce que dans une certain mesure, à celui qui a été effectué par la création du calcul infinitésimal, la physique à son tour ferait, presque immédiatement, un bond en avant immense." (100)

Another possible explanation of why Aristotle failed to give more attention to the exploitation of the fruitful principles he had discovered may be that he was far from realizing the vast extent of the applicability of his own principles. But before considering this possibility it is necessary to examine the major texts in which these principles are laid down.

There are two capital texts in which Aristotle deals explicitly with the nature of mathematical physics. These will constitute the seed out of which our whole study will grow:

The first of these two texts is found in the Posterior Analytics. This whole work is devoted to a discussion of the principles that are common to all the sciences. In chapter thirteen of the first book Aristotle explains how knowledge of the fact (scientia quia) differs from knowledge of the reasoned fact (scientia propter quid). After showing how they differ within the same science, he goes on to explain how they differ when they are found in different sciences; and in making this explanation he brings in the question of the subalternation of the sciences which we consider the key to the whole problem of mathematical physics.

> But there is another way too in which the fact and the reasoned fact differ, and that is when they are investigated respectively by different sciences. This occurs in the case of problems related to one another as subalternate and superior, as when optical problems are subalternated to geometry, mechanical problems to stereometry, harmonic problems to arithmetics, the data of observation to astronomy. Some of these sciences are almost synonymous, e.g. mathematical and nautical astronomy, mathematical and acoustical harmonics. Here it is the business of the empirical observers to know the fact, of the mathematicians to know the reason for the fact. For the latter are in possession of the demonstrations giving the causes, and are often ignorant of the simple fact: just as those who know universals are often ignorant of some of its particular instances through lack of observation. Such are all the sciences which, though differing by their essence, use forms. For the mathematical sciences have to do with forms; they are not concerned with a subject, since, even though geometrical properties are predicable of a subject, it is not as predicable of a subject that they consider them. As optics is related to geometry, so another science is related to optics, namely the theory of the rainbow. Here it pertains to the physician to know the fact, but

> to the optician to know the reason for the fact, either qua optician or qua mathematician. Many sciences, though not subalternated, are mutually related in a similar way, e.g. medicine and geometry: it is the business of the student of medicine to know that circular wounds heal more slowly, but it pertains to the geometer to know the reason why. (101)

The second important text is found in chapter two of the second book of the Physics. Since some historians have failed to see why this passage should be in this particular place and have preferred to seek for some extrinsic reason to explain its presence here, it is worthwhile to point out its connection with the context. After having discussed in book one the problem of the principles of nature, Aristotle takes up in book two the principles of the science of nature. The general principles common to all science had already been considered in the Posteriora Analytica. But each science has its own proper method, and consequently it was necessary for Aristotle after having determined upon the principles of nature to discuss the method to be used in the investigation of nature. It was necessary to consider the causes according to which demonstration may be had in natural science. Now it happens that the natural scientist in seeking for the cause of natural phenomena often turns to mathematics for light. Aristotle had to explain the significance of this recourse to mathematics. In other words, after having discussed in the Posterior Analytics the general principles governing the subaltermation of one science to another, he now applies these principles to the subalternation of physics to mathematics.

> Having determined the different ways in which the term "nature" is used, we must now consider how the mathematician differs from the physicist. For physical bodies contain surfaces and volumes, lines and points, and these are the object of the mathematician. Moreover, astronomy is either different from physics or a part of it. For it seems strange that it should pertain to the physicist to know the nature of the sun or the moon, but not to know any of their accidents, especially since writers on physics obviously do discuss their shape also and whether the earth and the world are spherical or not. Now the mathematician, though he treats of these things, nevertheless does not treat of them as the limits of a physical body; nor does he consider the accidents precisely as accidents of such bodies. That is why he abstracts them; for in thought they are abstractable from motion, and it makes no difference, nor is any falsity involved if he so abstracts them. The holders of the theory of Forms are unaware of this. For they abstract physical things, even though these are less abstractable than mathematical things. This becomes plain if one tries to state in each of the two cases the definitions of the things and of their attributes. 'Odd' and 'even', 'straight' and 'curved', and likewise 'number', 'line', and 'figure', do not involve motion; not so 'flesh' and 'bone' and 'man' - - these are defined like 'snub nose', not like 'curved'. Similar evidence is supplied by the sciences which are more physical than mathematical, such as optics, harmonics, and astronomy. These are in a way the converse of geometry, for while geometry investigates physical lines but not *qua* physical, optics investigates mathematical lines, but *qua* physical, not *qua* mathematical. (102)

The central idea that emerges from these two texts is that mathematical physics is a hybrid science in which physics is subalternated to mathematics. It is, to use the technical Thomistic expression, a *scientia media*, an intermediary science between physics and mathematics; it involves a kind of noetic hylemorphism in which the material element is drawn from

physics and the formal element from mathematics. The purpose of this study is to analyze the unique type of knowledge that is born of this union. As we have already indicated, it is not our intention to attempt to come to grips with all the complicated epistemological problems which have evolved out of the development of mathematical physics. Rather we have in mind to take this one idea of a *scientia media* and explore all of its implications. But we hope to draw out these implications far enough to make it clear that in this one idea is found the central key which will open up the meaning of all the other problems encountered in physics.

Before undertaking the detailed analysis of these texts several general considerations are in order. In the first place, for the purpose of indicating the direction that this analysis will follow, it is helpful to try to orientate the position of Aristotle in relation to the other positions outlined earlier in this chapter. As we have already suggested, most of these opinions can be reduced to two categories: the role of mathematics in physics is either considered to be that of a pure instrument (whether logical or merely linguistic,) that is employed by the scientist in order to work more effectively upon his sole direct object which is nature; or it is considered [illegible] to be that of the direct object of the science itself in the sense that the mathematical world is identified with or realized in the physical world. Now the position of Aristotle is located squarely between these two extreme positions.

In the first place, the role of mathematics in physics is essentially instrumental in the sense that the whole raison d'etre of its introduction into physics is to enable the mind to get to know the physical universe better. The goal at which the whole of mathematical physics aims is not to know the mathematical world (for that is already known) but the physical world. Mathematics is employed as a means to that end.

On the other hand, mathematics is much more than a mere tool in physics, that is to say, it does not remain extrinsic to the science; on the contrary it enters intrinsically into its very constitution. And it enters into it intrinsically not merely in the sense of providing the principles from which physics may draw conclusions concerning its own proper object which in itself remains untouched by mathematics, but in the sense of entering into the very object of the science. For, as we shall see in chapter three, the type of subalternation found in mathematical physics is not merely subalternation according to principles, such as is found in the dependence of theology upon the science of the blessed, but subalternation according to

the object. This means that the formal object of mathematical physics is constituted by a combination of both a mathematical and a physical element.

But the nature of this combination must be rightly understood. It does not mean that mathematical physics studies as such the quantitative determinations found in nature from the point of view that is proper to them. Such a study is possible, but it will be either pure physics (if the quantitative determinations are considered in relation to mobility) or metaphysics (if the nature of quantity and its properties are considered). Mathematical physics studies the quantitative determinations found in nature, not just in the light of their ontological status, but in the light of the status that is proper to mathematical abstraction. For example, when the physicist says that light is propagated in a straight line, the line he is talking about is neither a mere physical, sensible line, such as is found in nature, nor is it merely a mathematical line; it is a combination of the two: the sensible line is considered in the light of a mathematical line.

In this way mathematics enters into the very essence of the object of physics, but it does so in such a fashion that the mathematical world is not identified with the physical world. It retains the extrinsic character that is proper to it.

And this is extremely important. For only by remaining extrinsic can it fulfill its essentially functional and instrumental role, by retaining all the pliancy and inexhaustible virtuosity that is proper to mathematical abstraction.

This brings us to a delicate point that must be touched upon before proceeding further in our analysis. It would seem that for Aristotle and the medieval Thomists the combination between the mathematical and the physical element in the object of mathematical physics was in a sense more intimate than it is possible to admit today. Because of a lack of refinement in their means of observation, they seem to have held that there are quantitative determinations in nature which come sufficiently close to the absolute state of perfection that they enjoy in the mathematical world to allow for a true scientific (103) handling of them in terms of mathematics. The heavenly bodies, for example, were for them perfect spheres, and consequently there was sufficient conformity between them and mathematical spheres to allow the mathematical properties of sphericity to be applied to them directly and adequately. This does not mean, of course, that mathematical entities were realized as such in the physical universe, for that would involve a confusion of mathematics and physics, and Aristotle and St. Thomas go to great lengths in inveighing against those who proposed such a confusion. (104) But it does mean that some physical entities possessed a determination which was in close enough conformity with the perfect determination of mathematical entities for mathematics to give an adequate explanation of them. That is why Aristotle and St. Thomas could look upon the combination of mathematics and physics as giving rise to a science in the strict sense of the term.

It would seem that this particular aspect of their doctrine is open to modification. Because of our more highly refined instruments of research, we are not longer inclined to believe that such a conformity exists between physical and mathematical entities. As a consequence, the mathematical interpretation of nature is never more than an extrinsic approach to nature. And that is why from this point of view mathematical physics cannot be considered a science in the strict Aristotelian sense of the term, but a species of dialectics.

There is another closely related point that must be underscored here in order to establish accurately the connection between Thomistic doctrine and modern mathematical physics. When Aristotle and the medieval Thomists speak of mathematics they understand it in the sense in which it was generally understood until recent years — that is to say, as a science which deals with quantitative relations that are capable of realization in the sensible world though not in the state of abstraction that is proper to them — "oportet salvari principia mathematica in omnibus naturalibus, ut dicitur III Caeli et Mundi." As is well known, modern mathematics is no longer restricted to these limits. It now embraces a great range of conceptual constructions which reach far beyond these quantitative relations. Now it is bootless to dispute about names, but it is extremely important to keep in mind what they are meant to signify. And in so far as our problem is concerned, it is necessary to recognize the fact that from the point of view of Thomistic terminology, the part of modern mathematics which does not deal with quantitative relations abstracted from the sensible world is not mathematics, but a tissue of dialectical constructions. Now these dialectical constructions have been employed with great success in the recent developments of physics. The obvious example which immediately suggests itself is the use of non-Euclidian geometry in the theory of relativity. Does this mean that the Thomistic doctrine of scientia media has no relevance for recent mathematical physics? We do not believe that such a conclusion is legitimate. For the application of the dialectical constructions of modern mathematics to nature follows the same general pattern as the application of mathematics in the restricted sense in which it was understood by Aristotle and the Medievalists, and is go-

verned by many of the same general principles. Nevertheless it is necessary to keep in mind that in so far as these conceptual constructions are employed, mathematical physics is dialectical in a sense never envisaged by Aristotle and St. Thomas, that is to say, although their notion of dialectics is applicable, they never envisaged this application.

In connection with this question of the meaning of the term "mathematical" it will be helpful to determine here what breadth of meaning the phrase "mathematical physics" will have throughout this study. This is a double problem, involving the range of applicability of both the term "mathematical" and the term "physics." In so far as the first aspect of the question is concerned, it is to be noted that some authors restrict the phrase "mathematical physics" to those parts of physics which have attained the highest degree of mathematization. Professor Lenzen, for example, divides physics into experimental physics, theoretical physics, ideal theoretical physics, and mathematical physics.(105) The Thomistic acceptance of the phrase is much broader. It includes any part of physics in which a mathematical element is introduced to determine the object in such a way that new significant truths result which would not arise without this determination.

The second question which must be determined is the meaning of the term "physics". A reading of the texts of Aristotle cited above raises a problem about the range of applicability which the principles laid down in them had for Aristotle and the medieval Thomists. The examples given in these passages are restricted to a very few especially priviledged cases in which the presupposition of all mathematization, namely, order and regularity, is found in a particularly high degree - whether it be the geometrical order that is found in astronomy, for example, or the arithmetical order that is found in music. It would seem that the examples given are more than examples, that they are an exhaustive indication of the fields in which physics had to some extent been subalternated to mathematics. Did Aristotle or the medieval Thomists look beyond these fields? Did they conceive the possibility of a universal interpretation of nature in terms of mathematics? It seems quite possible that they did not. It is probable that the honor of this discovery must be accorded to the scientists and philosophers of the Renaissance. But this admission in no way compromises the objective applicability of these principles, nor their real fecundity.

Mathematics is almost synonymous with determination, and as a consequence nature is refractory to mathematization to the extent in which it participates in some form of indetermination. That is why it is necessary to understand the ways in which nature is subject to indetermination if we are to see the extent to which mathematics may be applied to nature. Now there are two types of indetermination: passive indetermination which is an imperfection arising out of the potentiality of matter, and active indetermination which is a perfection deriving from the actuality of form. Passive indetermination is found in all beings which have any share in potentiality; active indetermination is found in its fullness only in the liberty proper to spiritual beings, but it is also found anticipated to a greater or lesser degree in the spontaneity of all living things.

Now in Aristotle's and the medieval Thomists' concept of the cosmos, the heavenly bodies occupied a very priviledged position. Though mobile, they were incorruptible, and they consequently occupied a position between the metaphysical realm of immobile beings and the terrestrial world of corruptible beings.(1[illegible]) Though inanimate they were in a sense more perfect than the living beings of the earth, even than man, in that they were subject to no intrinsic corruption, but only to the extrinsic mobility of local motion. (107) They were thus free of both the passive indetermination that is proper to corruptible things, and the active indetermination that is found in living beings. That is why for the ancients they constituted the part of nature that was most highly amenable to mathematization. It would be difficult to say just what possibilities of mathematization Aristotle and St. Thomas saw in the terrestrial world of corruptible things in which both passive and active indetermination play such a large part. But at least this much can be said: they would readily grant the possibility of a mathematical interpretation of the corruptible world to the extent in which definite regularity and order could be discovered in its phenomena (108).

But whatever Aristotle or Saint Thomas may have thought about the extent to which nature may be mathematized, there is no doubt that their principles are applicable to the whole range of mathematization which modern physics has achieved. And that is all that is of any real importance. This universal applicability of Thomistic

principles is so true that in this study we shall, when speaking of mathematical physics, take the term "physics" in its primitive Aristotelian meaning in which it is coterminous with the whole of nature. In this sense it includes not only chemistry but even biology and psychology. As we shall see, according to Thomistic principles of the unity and distinction of the sciences, all of the sciences which deal with nature, whether it be inanimate, animate, or even psychic nature, constitute one indivisible science. In recent years there has been an attempt made by many Thomists to depart from this doctrine, but we shall point out in Chapter Two the error involved in this attempt. That mathematics has been successfully and fruitfully applied to all of these different fields of study is well known. And all of these applications (and whatever new applications the future may discover) constitute the scientia media of which Aristotle and Saint Thomas speak. (109)

Not all fields in the study of nature are equally amenable to mathematization. This is evident <u>a posteriori</u> from the history of science. It is even more evident <u>a priori</u>. For the objective basis of mathematization is, as we shall see, the homogeneous exteriority found in nature. In the measure, then, in which the object of a certain branch of natural doctrine has to do with homogeneous exteriority and in the measure in which it excludes heterogeneous interiority, to that extent mathematization is possible. The field in which this condition is found in its highest degree is, of course physics, in the modern sense of the term. And that explains not only why mathematization is possible to such a large extent in physics, but also why it is necessary. For, to the extent in which heterogeneous interiority is excluded, physical rationality loses ground. That is why, if scientific investigation in the realm of physics is to advance at all, it must proceed in the light of mathematical rationality.

For experimental scientists, physics realizes the ideal type of science. And it is perfectly legitimate and natural for them to make every effort to bring the other branches of natural doctrine into as close conformity with physics as possible. As we shall see later, homogeneity is from one point of view more knowable than heterogeneity, and as Aristotle and St. Thomas point out, it is natural for the intellect to reduce the less knowable to the more knowable. But there is no doubt that

(110)

this conformity will never be complete. Mathematics is not competent to treat adequately of all natural being. For the subject of mathematics is quantity, which is the order of the parts of the substance in which it inheres. But the parts in question are always material parts, and hence must not be confused with the form of the substance. This confusion would lead to a denial of what is best in natural things.

In other words, in the measure in which beings are ontologically more perfect, they lend themselves less to mathematical interpretation. For a being is perfect in proportion to the extent that its form emerges above the potentiality of matter, that is to say, triumphs over the potentiality of matter. Now, in the structure of material being, while quantity follows upon matter, quality follows upon form. That is why as we ascend the scale of material being qualitative determinations assume an ever increasing importance. This is particularly true of living beings. For the formal principle of life is form, and if a thing is living it is because its form has emerged to a sufficient extent above the potentiality of matter. That is why qualities and classification play such an important role in biology. Moreover, in living beings we find not only the passive indetermination common to all material things, but also the active indetermination of their vital spontaneity. This double indetermination will always provide great resistance to mathematization.

All this amounts to saying that as we ascend the scale of being heterogeneous interiority constantly increases. Within the cosmos it finds its fullest realization in man, the most perfect cosmic being. And we are referring here not merely to the psychic side of man, but also to the somatic part of his make-up. Of all the bodies in the universe, the body of man has the greatest heterogeneous interiority; it is the farthest removed from the Cartesian body, which is the ideal of an autonomous and self-sufficient physics. It is this heterogeneity of living beings that makes it possible for us to have a valid science of biology without mathematization—a science of classification.

It is interesting to note here in passing that whereas for physical science (in the modern sense of the term) heterogeneity is an irrational element, for philosophy it is homogeneity that is in some sense irrational. Here we are touching upon an important point to which we shall

return in chapter nine: the difference in the measurement that is proper to each science. For every science, even metaphysics, is in a way based upon measurement, but in each science there is a vast difference in the measure which provides the norm in relation to which everything that falls within its object is determined.

The important point to be borne in mind for the present is that in spite of the great heterogeneity found in nature, all natural things are spatio-temporal beings and consequently subject to a common measure. In discussing the problem of Indeterminism, Professor DeKoninck has emphasized this point:

> Qu'on ne crois pas échapper à cette conséquence en disant que l'animal et la plante sont hétérogènes et rebelles à une mesure homogène. Ne peut-on mesurer leur durée par une même horloge? Cependant, puisque l'existence est proportionelle à l'essence - - quantum unicuique inest de forma, tantum inest ei de virtute essendi — la durée des êtres cosmiques est aussi de plus en plus simple, de moins en moins temporelle; il existe ainsi toute une hiérarchie de durées cosmiques. Mais cette hétérogénéité ontologique n'empêche pas le temps physique, que l'on définit par la description de son procédé de mesure, d'enlacer tous les êtres spatio-temporels par ce qu'ils ont d'homogène entre eux au point de vue durée. Cette commune mesure est fondée sur le genre commun de corporéité dans lequel conviennent tous les êtres naturels. Le temps physique n'atteint que leur bas-fond, et encore n'y touche-t-il que du dehors. L'homogénéité est fondement de toute mesure quantitative; ce genre physique

commun explique suffisamment l'unité spécifique du temps expérimental et pourquoi l'hétérogénéité des durées échappe aux prises d'une métrique calquée sur l'extériorité homogène. La science expérimentale débouche là où tous les êtres se touchent et se confondent; l'échelle graduée sur la balance n'indique aucune différence entre 150 livres d'homme et 150 livres de briques. Si maintenant le temps physique touchait les êtres dans leur fond ontologique et spécifique, si ce temps épuisait le réel, ne fût-ce qu'au point de vue de la durée, les différents degrés d'êtres ne seraient que des épiphénomènes de complexité matérielle croissante. Même si les choses sont plus que du dehors, cela n'empêche pas que la mesure de leur extériorité homogène soit commune et vraie. Ces deux perspectives ne sont point contraires, elles se complètent l'une l'autre. Sans connaître la complexité expérimentale d'une chose on ne peut saisir la richesse de son unité ontologique. (111)

The same author has elsewhere summed up the question at issue:

La biologie expérimentale est une science exacte. Les sciences expérimentales peuvent être appelées exactes dans la mesure où elles nous permettent de faire des prédictions. C'est en ce sens que la physique peut être dite la plus exacte des sciences expérimentales. En astronomie on peut prédire des éclipses qui n'auront lieu que dans plusieurs siècles, à une fraction de seconde près. La science expérimentale est essentiellement métrique. Elle ne peut définir les propriétés que par la description de leur procédé de mesure. Aucune loi expérimentale - - relation algébrique entre des nombres-mesures - - n'est absolument rigoureuse. Cependant, dans l'ensemble, les lois strictement physiques sont plus rigoureuses que les lois biologiques. Nulle raison de s'en étonner. Nous venons de dire qu'il y a dans les êtres vivants une spontanéité toujours croissante qui dans

l'homme aboutit à une veritable liberté. Il est absolument impossible à un physicien de prédire d'avance quel mouvement de bras je ferai dans les cinq minutes à venir, si j'y prête attention. Il peut mesurer le mouvement que je fais quand je le fais. Mais de cette mesure il ne peut pas déduire le mouvement suivant. Chaque mouvement que j'effectue librement est quelque chose d'absolument nouveau dans le monde. Dès lors on peut dire que plus un être vivant est parfait, plus il échappe à la rigueur métrique. Plus il est concentré au-dessus de l'espace-temps, plus il échappe aux prises de la science expérimentale. Ainsi, de toutes les sciences expérimentales, la psychologie expérimentale est la plus imparfaite, la plus inadéquate, bien qu'elle étudie la plus haute forme d'organization naturelle.
En philosophie, c'est le contraire qui est vrai. Plus nous nous éloignons de l'homme pour descendre l'échelle des vivants, plus leur vie devient obscure. Ainsi, la vie des plantes est plus obscure pour nous que la vie animale. Nous reviendrons là-dessus. Il suffit de remarquer pour le moment qu'il existera une certaine complémentarité compensatrice entre ces deux ordres de connaissance si profondément distincts. Et par cette complémentarité compensatrice, je n'entends pas qu'à un certain point ces deux ordres de connaissance se fusionnent l'un dans l'autre. Non, ils ne sont jamais plus éloignés l'un de l'autre qu'au point où ils se touchent; comme des points sur une droite non euclidienne qui sont infiniment proches, mais aussi infiniment éloignés." (112)

In chemistry we already find an element which is refractory to complete mathematization. For the part that qualitative diversity plays in chemistry is essential. (113) and even though history has made short shrift of Comte's rejection of the possibility of the mathematization of chemistry, as it has of many another Comtian theory, it is safe to conclude that in this science there will always remain a margin impenetrable to complete mathematization.[114]

In biology this margin will always be immeasurably larger than in chemistry, for the reasons indicated above. Nevertheless, the attempts already made towards mathematization in this field have been surprisingly fruitful, and there is no way of laying down any well defined limits beyond which this mathematization may not go. As Whyte has pointed out, "if the laws of life were independent of the physical laws, life could neither exist within the physical universe nor discover its laws."[115] And just as it is the duty of every scientist to proceed in practice as though there were no limit to the determination coming from per se causality, that is to say, as though there were no chance in nature, so it is the duty of the biologist to act as though there were no limit to mathematization in biology, even though he may realize that the immanence that is characteristic of life will always remain superior to pure corporeality, and thus to some extent escape measurability.

It does not fall within the scope of this study

to discuss in detail the various ways in which mathematics have been applied to biology.[116] But the work already carried on in biomathematics by such men as D'Arcy Thompson, W.R. Thompson, Janisch, A.J. Lotka, Vito Volterra, and R.A. Fisher, for example, has been sufficient to demonstrate how promising this line of research in biology is. To cite only a few typical examples, mathematics have been applied successfully and fruitfully to problems of organic structure,[117] laws of growth, laws of reproduction, etc.

Of particular interest are the attempts being made to relate biological phenomena with the discoveries of modern physics. In this connection the experiments carried on by Timofeef-Ressovsky, Zimmer and Delbruck on the relation between genes and molecules, and those carried on by Stanley on the relation between virus individuals and molecules seem especially suggestive. Moreover, recent experimentation on the biological effects of radiation seem to indicate some promise of the general usefulness of an atomic-physical and quantum-physical interpretation of fundamental life processes. And it is interesting to note that Bohr has lent the great weight of his name to the belief that the new physics will ultimately have profound repercussions upon biological science. There can be no doubt that by abandoning the mechanism of the nineteenth century in favor of the analysis of phenomena in terms of constituent functional relationships, physics has immeasurably increased its significance for biology, and opened up in the latter science great possibilities of mathematization.

As we have already suggested, experimental psychology is of all the fields of natural doctrine the least congenial to mathematical interpretation. Yet even here the application of mathematics has been large and fruitful.[118] The use of mathematical formulations in the intelligence tests of Binet and his followers is well known.[119]
The Weber-Fechner law for the intensity of sensation, the logarithmic laws governing rote memory and forgetting, the Spearman factorial analyses of mental abilities are only a few of the results of the application of mathematics to experimental psychology. And what we have said of biology applies here as well: there is no way of laying down definite limits beyond which this mathematization may not go.

4. Some Implications of the Problem

In the beginning of this essay we alluded to

the importance of the philosophical study of the nature of mathematical physics. Perhaps it would be well, before bringing this chapter to a close, to try to round out our introductory considerations by indicating briefly some of the major issues involved in the study we are undertaking.

In the first place, this study is of vital importance for physical science itself. There was a time when philosophy was hermetically sealed off from science. Even when scientists did not feel it necessary to be inimical to philosophy, they thought that they could remain completely aloof from it. That time has passed.
"It is a well-founded historical generalization," says Whitehead in a somewhat different context, "that the last thing to be discovered in any science is what the science is really about." Men go on groping for centuries, guided merely by a dim instinct and a puzzled curiosity, till at last 'some great truth is loosened.'"[120]
Great truths have been loosened in modern physics and they have made us realize that in order to carry on the progress of science it is necessary to find out what science is really about. We have already pointed out

how all of the greatest contemporary physicists have been forced by the very needs of their science to invade the realm of philosophy. This is a highly significant phenomenon. It means that science is beginning to recognize a need for wisdom. In this connection Heisenberg writes:

> Many of the abstractions that are characteristic of modern theoretical physics are to be found discussed in the philosophy of past centuries. At that time these abstractions could be disregarded as mere mental exercises by those scientists whose only concern was with reality, but today we are compelled by the refinements of experimental art to consider them seriously.(121)

Of the many great physicists who have felt the need of turning to philosophy, no one has contributed more to scientific epistemology than Sir Arthur Eddington. In his <u>Philosophy of Physical Science</u> Eddington discusses the significance of the need that science has of philosophy:

> It is however, important to recognize that about twenty five years ago the invasion of philosophy by physics assumed a different character. Up till then traffic with philosophy had been a luxury for those scientists whose dispositions happened to turn that way. I can find no indication that the scientific researches of Pearson and Poincare were in any way inspired or guided by their particular philosophical outlook. They had no opportunity to put their philosophy into practice. Conversely, their philosophical conclusions were the outcome of general scientific training, and were not to any extent dependent on familiarity with recondite investigations and theories. To advance science and to philosophise on science were essentially distinct activities. In the new movement scientific epistemology is much more intimately associated with science. For developing the modern theories of matter and radiation a definite epistemoglogical outlook has become a necessity; and it is the direct source of the most far-reaching scientific advances.
>
> We have discovered that <u>it is actually an aid in the search for knowledge to understand the nature of the knowledge which we seek.</u> Theoretical physicists, through the inescapable demands of their own subject, have been forced to become epistemologists, just as pure mathematicians have been forced to become logicians. The invasion of the epistemological branch of philosophy by physics is exactly parallel to the invasion of the logical branch of philosophy by mathematics. Pure mathematicians, having learnt by experience that the obvious is difficult to prove — and not always true — found it necessary to delve into the foundations of their own processes of reasoning; in so doing they developed a powerful technique which has been welcomed for the advancement of logic generally. A similar pressure of necessity has caused physicists to enter into epistemology, rather against their will. Most of us, as plain men of science, begin with an aversion to the philosophic type of inquiry into the nature of things. Whether we are persuaded that the nature of physical objects is obvious to common sense, or whether we are persuaded that it is inscrutable beyond human understanding, we are inclined to dismiss the inquiry as unpractical and futile. But modern physics has not been able to maintain this aloofness. There can be little doubt that its advances, though applying primarily to the restricted field of scientific epistemology, have a wider bearing, and offer an effective contribution to the philosophical outlook as a whole.

> Formally we may still recognize a distinction between science, as treating the content of knowledge, and scientific epistemology, as treating the nature of knowledge of the physical universe. But it is no longer a practical partition; and to conform to the present situation scientific epistemology should be included in science. We do not dispute that it must also be included in philosophy. It is a field in which philosophy and physics overlap. (122)

Scientists are becoming increasingly conscious of the fact that what they get to know of reality is inextricably bound up with the way they get to know it, and that as a consequence they cannot be sure of what they know except by studying the way in which they get to know it. To use the happy expression of Leon Brunschvieg, they are no longer satisfied with giving an artificial communique of their victories over reality, as was their wont in the past; they are finding it necessary to give an account of their battles.

But philosophy has as much to draw from scientific epistemology as physics has — and more. For the philosopher few undertakings are more rewarding than the study of the mystery of knowledge. And of all the different types of knowledge none presents greater epistemological complexity than mathematical physics.

In physico-mathematical knowledge there are implications that are deep and far-reaching. A false view of its nature leads inevitably to a false view of the nature of human knowledge in general or to a false view of the nature of reality, or to both. It would be interesting to point out the connection between modern physical science and the many modern theories of knowledge, but that would take us too far afield. We have already alluded in a general way to this connection in Cartesianism and Kantianism, and this must suffice for the moment.

Because the true nature of physico-mathematical knowledge has been generally misunderstood, it has been almost universally substituted since the time of the Renaissance for the philosophy of nature. And the results have been disastrous for both philosophy and physics. Out of this substitution has arisen the great historical misunderstanding of the relation between Aristotelian and modern physics.
Looking back at the physics of Aristotle through the eyes of modern mathematical physics, and not taking the trouble to find out what Aristotle was actually talking about, scientists and philosophers of science have become a prey

to the fallacy of ignoratio elenchi. They have not suspected that when Aristotle was talking about motion his approach to the question was something entirely different from that of Descartes. If this study should accomplish no other purpose than to help to clear up this unfortunate misunderstanding, our efforts will be more than justified.

But even when mathematical physics has not been substituted for the philosophy of nature, the failure to grasp its true epistemological character has led to abortive and extremely unhappy attempts to integrate it directly with philosophy. These attempts have been numerous both inside and outside Scholastic circles. Before the true relation between philosophy and science can be worked out, an immense epistemological task of purification and clarification of notions must be undertaken. It is hoped that this study will contribute something to the furtherance of this task.

As we have said, the consequences of a false view of the nature of mathematical physics are far-reaching. It would be easy to show for example how it leads (and de facto has led) to a deterministic view of

the whole of nature. In this connection Boutroux writes:

> Telle est la racine du déterminisme moderne. Nous croyons que tout est déterminé nécessairement, parce que nous croyons que tout, en réalité, est mathématique. Cette croyance est le ressort, manifeste ou inaperçu, de l'investigation scientifique. (123)

But the implications are even deeper that this. In the course of history the human mind has often been turned on the dialela of materialism and idealism. It is significant that a false notion of the nature of mathematical physics leads to both of these diametrically opposed extremes.

The reason for this derives from the peculiar character of mathematical science. As we shall see there is something necessarily material about mathematics in the sense that it deals with quantity, which, while it abstracts from sensible matter does not abstract from intelligible matter, and even intelligible matter implies homogeneity. In so far as mathematics has reference to reality, that reality can be nothing but material. Hence any possible real mathematical order is necessarily material. That is why universal mathematicism can lead and has led to materialism. On the other hand, mathematics is the most abstract of all the sciences, in a sense even more abstract than metaphysics. For mathematical entities are considered by the mathematician in their very state of abstraction, and as a consequence they are indifferent to reality. Moreover, these mathematical entities in their abstract state are prior to the sensible reality to which we apply them. That is why universal mathematicism can lead and has lead to idealism.

During the years when mechanism held complete sway over mathematical physics the tendency of mathematicism was towards materialism. In recent years, however, since the breakdown of classical physics, the tendency has largely been towards idealism. Professor Joad has described the dialectic by which mathematicism leads to idealism:

> But if the entities of which the universe is on a naively realistic view supposed to consist: substance and space-time, turn out to be mathematical, that is completely resolvable into mathematical formulae, and if to be mathematical is to be mental, more will be implied by the various statements asserting the mathematical nature of things than that the universe is describable in terms of mathematics; it will be implied that the universe somehow is mathematics. And, since mathematics is thought, to be mathematical will also be to be mathematical thought. (124)

Of all the modern mathematical physicists who have been drawn towards idealism, Sir James Jeans is perhaps the most outstanding example:

> The terrestrial pure mathematician does not concern himself with material substance, but with pure thought. His creations are not only created by thought but consist of thought, just as the creations of the engineer consist of engines. And the concepts which now prove to be fundamental to our understanding of nature ... seem to my mind to be structures of pure thought, incapable of realisation in any sense which would properly be described as material ... The universe cannot admit of material representation, and the reason, I think is that it has become a mere mental concept.(125)

And elsewhere he writes:

> Broadly speaking, the two conjectures are those of the idealist and realist — or, if we prefer, the mentalist and materialist— view of nature. So far the pendulum shows no signs of swinging back, and the law and order which we find in the universe are most easily described— and also, I think, most easily explained — in the language of idealism. Thus, subject to the reservations already mentioned, we may say that our present-day science is favourable to idealism. In brief, idealism has always maintained that, as the beginning of the road by which we explore nature is mental, the chances are that the end also will be mental. To this present-day science adds that, at the farthest point she has so far reached, much, and possibly all, that was not mental has disappeared and nothing new has come in that is not mental. Yet who shall say what we may

find awaiting us round the next corner? (126)

We must try to see whether it is necessary to choose between materialism and idealism.

CHAPTER TWO

THE SPECIFICATION OF THE SCIENCES

1. The Problem

The expressions "mathematical physics" and "physico-mathematical science" immediately suggest an epistemological dualism which implies both a distinction and a union. And the crux of our whole problem lies in analyzing accurately the nature of that distinction and that union. In the present chapter we shall endeavour to lay bare the basic principles which determine the distinction between mathematics and physics; in chapter three we shall consider the principles which govern the union of the two. And the principles laid down in these two chapters will serve as the foundation upon which the entire superstructure of the chapters which are to follow will be built; they will guide and shape the whole subsequent analysis.

Our first concern, then, is to see how physics and mathematics are distinguished from each other. The mere recognition of the dualism implied in the expression "mathematical physics" does not of itself predetermine the solution of our problem. For a dualism may be only nominal; it may be only the superficial expression of a basic identity. As a matter of fact, the dictionary of modern science is filled with expressions which suggest epistemological dualism: bio-chemistry, astro-physics, etc. And the very creation of these apparently hybrid sciences seems to have come from a recognition of a basic identity between the branches of knowledge joined together. As science progresses, this basic identity seems to be growing increasingly evident. Barrier after barrier between the sciences is being broken down; there is steady progress towards epistemological homogeneity. And on the face of things this seems to hold for mathematical physics as well as for the other hybrid sciences. Recent developments seem to be wearing pretty thin the traditional distinction between physics and mathematics. The most abstract conceptions of pure mathematics are being "incarnated" in the physical universe; the most concrete elements of the physical universe are finding a mathematical explanation. And perhaps few would hesitate to deny that there is a greater dichotomy between mathematics and physics than between biology and chemistry.

Our problem, then, is to try to discover how deep this dichotomy is between physics and mathematics. It is a problem which has innumerable ramifications, and which cannot be dealt with adequately in isolation from its epistemological context. In order to get at the nature of the distinction between physics and mathematics we must see how they fit into the whole epistemological scheme of things. In other words, we are faced with the question of a classification of the sciences . And we must explore this general question at least to the extent in which it is necessary to throw light upon the specific problem we have in hand.

It has often been remarked that the human mind has an instinctive tendency towards monism. It is an extremely significant tendency, and one which reveals the inner nature of the intellect. The history of philosophy has been a constant manifestation of this tendency under a great variety of forms. There have for example been countless attempts at some kind of ontological monism. But this is not the aspect of the tendency in which we are interested here; we are concerned with what might be called epistemological monism: the attempt to reduce all human knowledge to one homogeneous type; the failure to recognize the radical heterogeneity of the ways in which the human mind enters into contact with reality. It would hardly be an exaggeration to say that one of the greatest intellectual evils of modern times has been this persistent attempt to homogenize knowledge. It is an evil which has had far reaching consequences, notably in the field of education. But these consequences are not particularly relevant here.

In this connection, positivism and scientism readily come to mind. But even philosophical circles which have rejected positivism and scientism (including the majority of modern Scholastic circles) have been affected by this evil in a number of ways. Typical examples are: the identification of speculative and practical knowledge; the identification of metaphysics and the philosophy of nature; the identification of dialectical knowledge true scientific knowledge, and the identification of mathematical and physical knowledge. This last example is obviously the one which affects us most directly. But all the others have definite repercussions upon our problem as we shall eventually see. It is worth while

pointing out here that the unification of knowledge has historically been associated with mathematicism. And the reason is that in no science can this tendency be carried so far as in mathematics.

Now it is extremely significant to note that homogeneity is at once the source of unity and the source of multiplicity — infinite multiplicity. That is why the melting down of human knowledge to one standard type has almost inevitably resulted in the breaking up of the sciences into almost innumerable branches. One has only to study the classification of the sciences attempted by Bacon, Comte, Spencer, Bain, Karl Pearson and Huxley, to mention only a few, in order to see how highly arbitrary the distinctions between the sciences must necessarily be if all knowledge is of one homogeneous type. And because these distinctions are arbitrary, the advancement of science has made short shrift of many of them. That is why some have come to the conclusion that all distinctions between sciences are purely capricious. And in this connection the following lines of Max Planck are significant: (1)

> Looked at correctly, science is a self-contained unity; it is divided into various branches, but this division has no natural foundation and is

due simply to the limitations of the human mind which compel us to adopt a division of labour. Actually there is a continuous chain from physics and chemistry to biology and anthropology and thence to the social and intellectual sciences; a chain which cannot be broken at any point save capriciously. (2)

In the sixteenth century two contemporary philosophers wrote on the question we are discussing. The one represented the birth of a new philosophical movement; the other represented the end of an old philosophical tradition that was passing away. The first was Rene Descartes, and the second John of Saint Thomas. Descartes was the principal source of what Maritain has justly called "the radical levelling of the things of the spirit" that is so characteristic of modern times. (3) In his famous page in the Regulae on the unity of knowledge, modern epistemological monism received its first explicit formulation. And the source of this formulation was the mathematization of nature, about which we spoke in Chapter One. (4) Around the time that Descartes wrote this page in Regulae, John of St. Thomas wrote an article on the unity and distinction of the sciences at the end of his Ars Logica. (5) - an article which summed up and synthesized with admirable clarity and precision all of the fundamental Thomistic principles governing the classification of the

sciences. Though it must be admitted that in his philosophical writings he neglected the order of concretion, and that he seemed completely unaware of the great scientific discoveries that were going on around him, no one ever achieved a better exposition of the fundamental notions of science and the principles which determine the unity and distinction of the sciences. It is principally to him that we shall look for a guide in our discussion of the present question. At the same time it must be noted that he merely synthesized principles already found in Aristotle and St. Thomas; he in no way changed or added to these principles, as some have maintained.

But before embarking upon this discussion it is important to point out that there are two fundamentally distinct aspects to the question of epistemological pluralism. For the problem may be considered either from the point of view of the plurality of formally distinct objects that the mind lays hold of in reality, or from the point of view of the plurality of the means of knowing employed by the mind, namely the intelligible species. In other words there are two distinct problems of the one and the many. (6) Because we are engaged here with human knowledge, both aspects enter into our problem. But it is important to keep in mind that a plurality on the part of the objects does not necessarily imply a plurality on the part of the means of knowing. In fact, in proportion as an intelligence is more perfect, the plurality of its means of knowing decreases while the distinctness with which it knows objective reality increases. The divine intelligence sees the whole of reality exhaustively in its ultimate distinction in the one intelligible species which is His essence. At the other extreme of the scale of intelligences, the human mind needs as many intelligible species as there are natures to be known. If the human intellect were in a state of perfection, the problem of the distinction of the sciences would be easily solved: there would be as many species of science as there are species of things. Saint Thomas explains that in the infused knowledge of Christ there were as many species of science as there were species of things known by Him. (7) But because of the imperfection of the human intellect, it is necessary for it to know a plurality of objects which in themselves are specifically distinct in the light of a common scientific species. This commonness, however, is something quite different from the commonness of the intelligible species possessed by the higher intelligences which enables them to grasp reality in its distinction.

It is a commonness of potentiality which hides rather than reveals the distinction of reality.

In connection with the question of epistemological monism mentioned above it seems necessary to point out here that if the monistic tendency consists merely in an <u>attempt</u> to reduce the plurality of the means of knowing, as is done in the method of limits, it is a legitimate and laudable thing. It is reprehensible, however, when it consists in a reduction on the part of the objects.

These remarks should suffice to show that the question of the distinction and specification of the sciences is an extremely complicated thing, which depends essentially upon the nature of the intellect in question. For God, for example, there is no speculative science distinct from His one science which is wisdom, since He necessarily must see all reality in terms of Himself, the First Cause. This does not mean, of course, that He fails to grasp the <u>ratio mobilitatis</u>, for example, which, as we shall see presently, is the formal <u>ratio</u> of all natural things, but He sees it <u>sub ratione Deitatis</u>.

For all created intelligences there is a distinction of speculative sciences even though all of them must remain essentially subordinated to wisdom. And the nature of this distinction depends upon the nature of the intelligence in question. That is why there is a plurality of sciences peculiar to the human intellect which, unlike the angelic intellect whose knowledge is prior to things in so far as it is derived from the species divinae rerum factivae, (8) is dependent upon things for its knowledge. This dependence, plus the fact that its object is necessarily material things, make human knowledge essentially abstractive. And that is why the plurality of the human speculative sciences is determined by abstraction. No other principle of division is possible.

But before we come to the question of how the speculative sciences are distinguished by the different degrees of abstraction, it is necessary to go back further in our analysis of the heterogeneity of knowledge. For reasons which will become apparent later, particularly in Chapter IV, we must begin with the primordial distinction between speculative and practical knowledge.

2. Speculative and Practical Knowledge

The implications of this distinction are manifold,

and it would take us too far afield to consider even the more important ones. We shall content ourselves with a summary consideration of those implications which have a particular relevance for the understanding of mathematical physics. (9)

Briefly, then, speculative and practical knowledge differ by their end. (10) The end of speculative knowledge is truth; the end of practical knowledge is an operation, that is, a work to be done or made. (11) When we say that the end of practical knowledge is an operation, or a work to be done or made, we mean an operation or a work that is outside the intellect. (12) For as Saint Thomas points out, an operation may be either exterior or interior to the intellect. In the latter case the operation is a mere contemplation of truth, and in this speculative knowledge consists. Moreover, within the intellect there may be a kind of opus consisting in an ordering and a construction. In this case we have an art, but only a speculative art, and not a practical art, for the opus remains interior to the mind. Both logic and mathematics are arts of this kind. This distinction between speculative and practical art is of some importance, since both of them have a vital part to play in the construction of mathematical physics.

The object of all practical knowledge, then, is something outside the limits of the intellect.(13) It is, in fact, primarily and essentially the object of an appetite, for the intellect can have practical knowledge only because it submits itself in some way to an appetite (even though practical knowledge in itself does not consist in a mere extrinsic submission). Hence it follows that practical knowledge has as its object the good as good (bonum ut bonum), and not the good as true (bonum ut verum) which is the object of speculative knowledge. That is why in order to have true practical knowledge it is not sufficient that the object be in itself an operabile, i.e. something that in itself is "makeable"; it is necessary that this object be considered precisely in ordine ad operationem, or per modum operandi.(14) Now whereas the object of speculative knowledge is something within the intellect, and that of practical knowledge something outside the intellect, if we consider the principles of these two types of knowledge, the situation is exactly the reverse (at least in so far as human knowledge is concerned). The principles of speculative knowledge are in things, and

the movement is from things to the mind; the principles of practical knowledge are in the mind and the direction is from mind to things. That is why St. Thomas writes: "Practicus intellectus est de his quorum principia sunt in nobis, non quomodocumque, sed in quantum sunt per nos operabilia."(15)

Consequently, the mind is the measure of the things of which it has practical knowledge, whereas it is measured by the things of which it has speculative knowledge, as St. Thomas explains in the following passage:

> Res aliter comparatur ad intellectum practicum, aliter ad speculativum. Intellectus enim practicus causat res, unde est mensuratio rerum quae per ipsum fiunt; sed intellectus speculativus, quia accipit a rebus, est quodammodo motus ab ipsis rebus; et ita res mensurant ipsum. Ex quo patet quod res naturales, ex quibus intellectus noster scientiam accipit, mensurant intellectum nostrum, ut dicitur X Metaphys. (com.9): sed sunt mensuratae ab intellectu divino; in quo sunt omnia creata, sicut omnia artificiata in intellectu artificis. Sic ergo intellectus divinus est mensurans non mensuratus; res autem naturales, mensurans et mensurata; sed intellectus noster est mensuratus, non mensurans quidem res naturales sed artificiales tantum. (16)

Now there is an analytical connection and a direct proportion between the operabilitas (the "makeableness") of a thing and its degree of immateriality. Here it must be noted immediately that we are taking the term "immateriality" in its broadest significance, in the sense in which it is opposed to any kind of potentiality, and hence to any form of contingency. The first condition required for a thing to be the object of practical knowledge is that its essence be not identified with its existence. For the practical knowledge is knowledge of things to be brought into existence. That is why God is the only being who cannot be the object of practical knowledge (except in the sense that He is attainable by intelligent creatures through practical knowledge).[17] As John of St. Thomas points out,[18] the speculative abstracts in some way from the existential (ab exercitio existendi), whereas the practical considers its object in its existential state (ut stat sub exercitio existendi). Yet it would be highly ambiguous to say, as some authors have done, that speculative knowledge has to do with the essential order, and practical knowledge with the existential order. For there is an operabilitas in the essential order as well as in the existential order. All beings which have potency in their essence, i.e. matter in the strict sense of the terms have an intrinsic ontological plasticity, a "formability" which pure forms do not have. In all material creatures,

"formability" touches the very substance. In their very essence is found the reason for their intrinsic physical contingency.

Viewing the hierarchy of being dialectically, we may say that in the measure in which we get farther and farther from pure immateriality in which the essence is identified with existence, in the measure in which we get deeper and deeper into materiality, the closer we approach to pure operabilitas and hence the greater becomes the scope of practical knowledge. We are getting deeper and deeper into contingency and hence farther and farther away from the necessary, which is the object of speculative knowledge. In this dialectical process we start with the Being of which only speculative knowledge is possible, and we tend towards a limit which would be an object that would be purely practical. This object does not exist, nor can it exist, but there is something like it in moral knowledge. Saint Thomas points out that the study of morals is not for the contemplation of truth.[19]

It should be pointed out, perhaps, that we are considering this descending scale from the point of view of natures, for if other points of view were introduced, such as the large place that fortune plays in human life, and the immense amount of contingency involved in the supernatural order, what we have just said might be open to modification. Perhaps some might be tempted to take exception to the last paragraph on the score that the ultimate elements might very well prove to be few in number and highly determined in their constitution. But even if this should prove to be true what we have said would still hold. For elements are by their very nature for the whole, and from this point of view they would possess indefinite malleability and "formability" and serviceability because of the fact that everything in material creation would be made out of them.

Now all this has an extremely important bearing upon the nature of physics. For the object of physics is down very far in the scale we have been considering. This is particularly true of that part of physics which is far advanced towards concretion. And the farther physics advances the deeper it gets into materiality. That is why the things with which physics deals are principally operabilia, more operabilia than speculabilia. And as physics progresses, the things with which it deals become less and less amenable to speculative knowledge and more

and more amenable to practical knowledge.

Moreover, in order to possess fully the speculative knowledge of which these things are capable, it is necessary to have practical knowledge of them. For even though speculative knowledge always remains something distinct from practical knowledge, in order to have perfect speculative knowledge of things that are in their very nature operabilia, it is necessary to have practical knowledge of them. And the more things are operabilia in their very nature, the greater becomes the necessity of having practical knowledge of them in order to possess with any kind of adequacy the speculative knowledge that it is possible to have of them.

Now the difficulty is that this practical knowledge is not open to us. For we cannot make natures. We can only imitate them by making artificial things.(20) Natures are, in fact, essentially "rationes artis divinae," as Saint Thomas points out in the second book of the Physics.(21) In other words, art is essentially an extrinsic principle, and it is only in divine art that this extrinsic principle can be the cause of the intrinsic principle. The reason is that whereas all created art presupposes a subject, divine art does not, and as a consequence it can reach the very first principle of the things it makes.

But even though man cannot have a practical knowledge of natures which alone would make it possible for him to have perfect speculative knowledge of them, he can have practical knowledge in relation to natures, and by means of it acquire a more perfect speculative knowledge of them. As a matter of fact, in order for man to have a profound speculative knowledge of natural things in their concretion it is necessary for him to have recourse to an immense amount of practical knowledge. He must operate upon nature with instruments devised by himself. And the deeper he plunges into concretion the more highly complex and subtle must these instruments and operations become. In this way practical knowledge becomes more and more an implement of speculative knowledge. Man must construct before he can contemplate, and it is precisely because of the weakness of his speculative knowledge that he must have recourse to practical knowledge.

Not only must physical construction enter into physics in an increasingly large measure as it advances, but mental construction as well. In theory-building, which still falls within the genus of art, though it be a

speculative art, the scientist makes, as it were, an <u>ersatz logos</u> which can never do more than connote objective nature. Moreover, in order to rationalize nature the physicist is forced to borrow heavily from mathematics which is also a speculative art.

Thus in a number of ways construction enters into the object of physics — enters into it so profoundly that it becomes impossible to distinguish between what is derived from nature and what comes from art. All this is necessary but it constitutes a danger. For it is all to easy for man to come to look upon nature as a mere malleable matter to be worked upon and used. Moreover, the knowledge we acquire by having recourse to this construction makes possible such extensive mastery over nature that the practical power that is derived from this knowledge all too easily becomes confused with the purely speculative knowledge of nature which is the basis of the practical knowledge. In other words there is the danger of confusing the speculative knowledge we have of natural things with the knowledge of what we can do with them, or at least of subordinating the speculative knowledge of nature to the practical knowledge we are able to have in relation to it, in somewhat the same way as is found in the case of the artist who is concerned with the nature of the material he uses only to the extent to which that is necessary for the achievement of his work of art. Then the practical knowledge is no longer the instrument of the speculative knowledge, but just the contrary. And even when the confusion between speculative and practical knowledge, or the perversion of the right order that should exist between them does not occur, there is at least the danger that the abundant use that we can make of nature might lead us to cease to wonder at nature, and without this wonderment, as Aristotle has pointed out, speculative knowledge cannot thrive.

That the tendencies we have just mentioned have been prevalent in modern times is all too evident. Already in Descartes we find the following:

> Mais sitôt que j'ai eu acquis quelques notions générales touchant la physique, et que, commençant à les éprouver en diverses difficultés particulières, j'ai remarqué jusques où elles peuvent conduire et combien elles diffèrent des principes dont on s'est servi jusqu'à présent, j'ai cru que je ne pouvais les tenir cachées sans pécher grandement contre la loi qui nous oblige à procurer autant qu'il est en nous le bien général de tous les hommes: car elles m'ont fait voir qu'il est possible de parvenir à des connaissances qui soient fort utiles à la vie;

> et dans les écoles, on en peut trouver une pratique, par laquelle, connaissant la force et les actions du feu, de l'eau, de l'air, des astres, des cieux et de tous les autres corps qui nous environnent, aussi distinctement que nous connaissons les divers métiers de nos artisans, nous les pourrions employer en même façon à tous les usages auxquels ils sont propres, et ainsi nous rendre comme maîtres et possesseurs de la nature.(22)

These tendencies have continued to grow since the time of Descartes, and today it is not rare to find even in the writings of those who have otherwise made valuable contributions to the philosophy of science passages in which the important distinction between speculative and practical knowledge seems to have faded to a large extent. The following lines of F.C.S. Schiller are fairly typical:

> The mental attitude which entertains hypotheses.. feels free... to rearrange the world at least in thought, to <u>play</u> with it, and with itself. For hypothesis is a sort of game with reality, akin to fancy and make-believe, fiction and poetry ... It is by this hypothesis - building habit that science touches poetry on the one side, and action on the other; for it is akin to both. The play of fancy and the constructive use of the imagination reveal the creativeness of human intelligence; by their use the scientist becomes a "maker" like the poet... Yet on the other side, this hypothetical attitude mediates between thought and action, and helps to break down the superficial distinction between the theoretic and the practical. It drives the scientist out of the

purely receptive attitude, and makes him a doer. For to entertain a hypothesis is to hold a mental content hypothetically, and this is to hold it experimentally, which, again is to operate on it and to manipulate it. (23)

From many points of view it is in Marxism that the tendencies of which we have been speaking have found their fullest expression. Marx' eleventh thesis on Feuerbach states that "the philosophers have only interpreted the world differently; the point is to change it." At the heart of Marxism is a revolt against the humble state of being measured by things that is characteristic of speculative knowledge and a desire to become their measure through practical knowledge. There is a seeking to transform nature completely, to reconstruct it to one's own image and likeness, to subject it entirely to the exigencies of one's life of praxis. In his Introduction to Dialectical Materialism, Edward Conze, a faithful disciple of Marx, has this to say:

> Dialectical materialism is surrounded by the glamour of being something specially strange, mysterious and startling. To the extent to which this new method of thinking becomes better known, the charm of the unknown will vanish. It will be seen that it is not a nice piece of decoration, but a very prosaic and practical tool. It has more the functions

> of an axe than of a Chinese vase...
> Not the mere understanding, but an increased control of the world, is the ultimate purpose of scientific method. (24)

But all this is an anticipation of what is to come in subsequent chapters. Consequently, we must leave this point, and having seen the nature of the distinction between speculative and practical knowledge, we must pass on now to a consideration of the hierarchy of speculative science. This will bring us directly to the central point around which the whole of the present discussion is revolving; the nature of the distinction between physics and mathematics.

3. The Hierarchy of Speculative Science.

Science, writes Professor Urban, "is the most ambiguous concept in the modern world." (25) In order to avoid confusion it seems necessary to point out immediately that at the beginning of this discussion and until further notice we shall take the term "science" in its strict Aristotelian sense. Both Aristotle and Saint Thomas sometimes use the expression "scientific knowledge" in a fairly loose fashion. (26) Thus, in the Posterior Analytics "quaelibet certitudinalis cognitio" is called scientific knowledge. In the Summa[27] St. Thomas sometimes uses the word "scire" in terms of knowledge of particular contingent facts. But outside of a few exceptions of this kind, "science" in the peripatetic tradition has consistently meant a knowledge that is universal and necessary, a knowledge that has been arrived at by demonstration, and a knowledge that has been fixed and determined in an intellectual habitus.[28]

Now, in coming to grips with the problem of the distinction and classification of the sciences, it is extremely important to discover the true criteria by which one type of scientific knowledge is distinguished from another. One cannot select these criteria in an arbitrary fashion, for, as we have already pointed out, this inevitably leads to epistemological confusion. What, then, will reveal to us the true criteria of an objective and necessary classification. Obviously, the nature of knowledge itself.

Knowledge is essentially objective, for, in Thomistic terminology, to know is to be the thing known in its very "otherness." But human knowledge, because of its limitations, is never completely objective under every aspect. Potentiality always involves some kind of

subjectivity, and the intrinsic potentiality of man's nature necessarily limits the objectivity of his knowledge. Quidquid recipitur ad modum recipientis recipitur; hence if the knowing faculty is very imperfect, the objectivity of its knowledge, however true it may be, must necessarily be very imperfect. It would seem to follow from this that the segmentation of scientific knowledge into specifically distinct types must be based on something which is fundamentally objective, but which has, at the same time, a subjective determination.

As we have already remarked, if human knowledge were in a state of perfection the problem of the distinction of the sciences would be simple, since there would be as many species of science as there are species of things. But because man is incapable of grasping things perfectly, it is necessary for him to know a plurality of objects which in themselves are specifically distinct in the light of a common scientific species. Now in order to grasp clearly the nature of this common scientific species we must introduce here the distinction between "thing" and "object". By "thing" we understand what is commonly known as the material object of knowledge, i.e. that which is known, the res in se, considered purely in its entitative status. By "object" we understand what is commonly known as the formal object of knowledge, i.e. the particular determination or formality by which the cognitive power lays hold of the "thing". For a thing can become the object of knowledge only in so far as it is orientated to a cognitive power in a certain determined way. Thus, an eye can perceive a wall only because the wall is orientated to the eye by means of its color. From what has already been said about the nature of human knowledge it must be evident that the specification of scientific knowledge must come from reality,[29] not however in so far as reality is a "thing", but in so far it is constituted as a scientific object.[30]

Consequently, whenever St. Thomas uses such expressions as "scientiae secantur quemadmodum et res,"[31] he understands "res" in the sense of formal object; for in the text just cited he immediately adds: "nam omnes habitus distinguuntur per obiecta, ex quibus speciem habent."

In relation to the formal object, Cajetan introduces a further distinction which will be extremely useful for us, not only for our present purpose, but also for the final explicit formulation of the nature of physico-

mathematical knowledge which we shall attempt in Chapter XIII.[32] He points out that there are two kinds of formal object; one is the formality existing in the thing itself which directly terminates the act of cognition, and by means of which the "thing" is made apprehensible by the cognitive power; the other is a formality which actualizes the first formality. The concrete example usually given to illustrate this distinction is borrowed from the realm of sense cognition: in visual cognition there are two formalities: the color existing in the wall, and the light which plays upon the wall and actualizes its color. By transposing this example to the realm of intellectual cognition we discover that the second formality is a kind of objective spiritual light[33] which manifests and actualizes a determined formality existing in the thing, which in turn renders the thing intelligible by constituting it as an object. The first of these two formalities is known in Thomistic terminology as the "objectum formale quod" or the "ratio formalis quae", or the "ratio formalis objecti ut res." The second is known as the "objectum formale quo," or the "ratio formalis sub qua," or the "ratio formalis objecti ut objectum." This distinction may appear extremely subtle, but Cajetan rightly insists upon its necessity:

> Necessitas autem, qualitas et distinctio harum rationum sumenda est ex distinctione duorum generum, in quibus oportet locare obiectum scientiae. Oportet enim quod formaliter sit talis res, taliter scibilis. Et ideo oportet quod habeat et rationem formalem constituentem formaliter ipsam in tali esse reali, et rationem formalem constituentem formaliter ipsam in tali esse scibili. (34)

Now, from what has been said thus far it should be evident that the point of departure of the whole question of the specific distinction of the sciences must be an attempt to discover in the entire realm covered by scientific knowledge specifically distinct "rationes formales sub quibus." For, as we have just seen, it is the "ratio formalis sub qua" that actualizes the ratio formalis quae . In other words, what we are trying to decide is whether or not there are specifically distinct ways in which reality is scientifically knowable, and it is precisely ratio formalis sub qua which constitutes reality as scientifically knowable, i.e. in esse scibili . But where shall we turn to discover the specifically distinct rationes formales sub quibus by which one science will be distinguished from another. Once again our answer will be found in the nature of knowledge in general, and the nature of scientific knowledge in particular. (35)

The root of all knowledge is immateriality. (36) This immateriality is required first of all on the part of the knower which, in order to be open to other forms besides its own, must enjoy a certain independence of the restrictions of matter which is essentially a subject and hence entirely closed in upon itself. It is also required on the part of the thing known, for a thing can be known only in the measure in which it is constituted as an object, that is to say in the measure in which it is lifted out of the state of being a pure subject. When sufficient immateriality is not possessed by the object in the state in which it is found in nature, the knower must operate upon it and lift it to the state of immateriality required.

Because of this dependence of knowledge upon immateriality, if in the realm of speculative knowledge different levels of immateriality are discernible, there will be a stratification of the sciences corresponding to these different levels. Moreover, necessity pertains to the essence of science, for no truly scientific knowledge is possible of things in their contingency. (37) Consequently there will be as many different sciences as there are different types of necessity; that is to say, the sciences will be distinguished by the specifically different levels according to which the scientific object can be lifted out of the flux of contingency. Hence St. Thomas concludes: "Et ideo secundum ordinem remotionis et a materia et a motu scientiae speculativae distinguuntur."(38) But the sciences will not be specified by the degree of immateriality and necessity of the object considered in its entitative state in such a way that the species of science will correspond to the degrees of being. If this were the case, the specification would be coming from the material object, which as we have seen, is impossible. It is the degree of immateriality and necessity arising out of the way in which the object is known by the intellect that is the principle of specification.

Now the means by which the intellect lifts its object out of the opacity of matter and the flux of change is called abstraction. Hence it will be the specifically different degrees of abstraction that will give us the rationes formales sub quibus we are looking for, and these in turn will actualize in the object different rationes formales quae. But before pursuing the discussion of the diverse degrees of abstraction, it is necessary to point out that we are concerned here not with total but with formal abstraction. This distinction is of capital im-

portance for the philosophy of science, and no one has probed its profound implications with greater acuteness than Cajetan.(39) Since all positive abstraction involves some kind of separation, the basis of this dual abstraction is a dual composition: the composition of matter and form, and the composition of a universal whole and its subjective parts. Abstraction is called formal when it consists in disengaging a form from the matter in which it is concretized; it is called total when it consists in laying hold of a universal whole apart from the subjective parts in which it is distributed. When a mathematician abstracts a certain quantitative concept, such as the notion of line, from the sensible matter in which it is concretized in the real world, he is practising formal abstraction. For "line" stands in relation to "sensible" as form to matter. When, however, one abstracts the concept of animal from its subjective parts, man and brute, to consider it apart, he is using total abstraction.

In order to avoid confusion it is necessary to point out that when we say that formal abstraction consists in abstracting a formal element from its material concretion it is never a question of abstracting the substantial form from the matter to which it is united, for as St. Thomas points out,[40] the interdependence existing between a substantial form and its corresponding matter is such that one cannot be understood without the other. Thus, the student of nature never abstracts the substantial form from its matter; he merely prescinds from the contingent materiality proper to individuals. This point is of extreme importance for a proper appreciation of the nature of physics, and it is usually misunderstood by scholastic writers. Similarly, the mathematician does not abstract the substantial form, but the accidental form of quantity. The metaphysician lays hold of substantial form only in so far as it is a co-principle of material being.

There is a world of difference between the two intellectual processes involved in formal and total abstraction. In the first case the separation results in a double concept each of which is complete by itself. The notion of line does not involve the notion of sensible matter, nor does the notion of sensible matter necessarily involve the notion of line. Hence each can be perfectly conceived in separation from the other. But in total abstraction only one complete concept results: the idea of animal is conceivable without the notion of either man or brute; but neither man nor brute is intelligible without

the notion of animal. Because formal abstraction reveals a form that is purified of the potentiality of its material concretion, it gives rise to greater objective intelligibility. In fact, this greater intelligibility is the very reason for the separability of the form. The notion of line, for example, is much more intelligible in its state of abstraction from sensible matter, than in its state of concretion. And let it be noted in passing that here we are touching upon the pivotal point of the whole problem of mathematical physics. In fact, we have given here the fundamental reason why physics in its development must necessarily become mathematical physics. But lest any confusion arise, it is necessary to emphasize the fact that we have been speaking here of greater objective intelligibility. Because of the imperfect condition of our intellect, greater objective intelligibility (<u>intelligibilitas natura</u>, or, <u>intelligibilitas in se</u>) does not necessarily mean greater intelligibility for us. In fact, there is ordinarily an inverse proportion between the two, as we shall have occasion to point out in chapter IV. We say "ordinarily," because mathematical science presents a unique case which we shall study in chapter VI. And this unique case will have an extremely important role to play in the solution of our problem.

From the point of view of actuality, the movement of total abstraction is the reverse of that of formal abstraction. For, in ascending from brute and man to animal, and from there to higher genera in the Porphyrian tree, we are moving from what is more determined and more actual, and hence more intelligible objectively, to what is more confused, more potential, and hence less intelligible objectively. For the mind can abstract a universal whole from the subjective parts of which it is predicable only by retreating from the actuality and determination of these subjective parts into a state of greater potentiality. But it happens that in moving from what is less intelligible to what is more intelligible objectively we arrive at what is more intelligible for our imperfect intellects. The only kind of mind that can be realized in a being composed of matter and form is one which must acquire its knowledge through experience, and which must, therefore, begin with pure noetic potentiality, a tabula rasa, and move on gradually to greater and greater noetic actuality. That is why things are more intelligible for us in the potential and confused state of their

universality, than in the state of concretion. It is much easier for us to understand what a living being is than to understand what a cow is. We shall discuss this important point in considerable detail in Chapter IV, but it was necessary to bring it out here because, as we shall see in a few moments, a number of modern Thomists, while admitting in the abstract the distinctions we have laid down, have allowed themselves to arrive at erroneous conclusions about the nature of science because of a confusion between the two kinds of intelligibility we have just mentioned.

It should be clear from what has been said why the degrees of abstraction which specify the sciences are necessarily degrees of formal abstraction. Total abstraction is, in fact, common to all the sciences, since scientific knowledge deals necessarily with universals. But before leaving the question of abstraction in general to discuss the degrees of formal abstraction, there is another distinction to be made which will be of considerable usefulness for us as our analysis proceeds. We have in mind the distinction between positive and negative abstraction. In discussing total and formal abstraction we have been dealing with positive abstraction. Negative abstraction is something quite different, and since its use in experimental science is extensive, it will be necessary to describe its nature briefly.

There are two distinct types of negative abstraction. The first type is called negative because it does not achieve a noetical separation in the strict sense of the word. Just as a sense can pick out a certain quality existing in an object, e.g. the color, and leave aside all the other qualities coexisting with it, so the mind when confronted by a plurality of formalities can concentrate its attention on one of them to the neglect of all the others with which it is connected. In thus concentrating its attention on one formality, the mind does not lift this formality out of its context, set it forth by itself, and consider it formally as separated. Hence the term at which it arrives remains tied to the context from which it has been abstracted. That is why this type of abstraction does not achieve even one complete and independent concept, and in this it differs from both formal and total abstraction, as is evident from what was said above. Negative abstraction is like total abstraction in that it arrives at a common notion, but it differs from it in that this common notion is not considered in relation to its inferiors. (41) It is

like formal abstraction in that it lays hold of a certain formality, but it differs from it in that the seperation is only negative, and consequently it does not consider the formality formally as separated.

The second type of negative abstraction is that used in logic. (42) It gives rise to an object which, though related to something in nature, does not have being in nature, but only in the mind. Positive abstraction always gives rise to an object that has being in reality, even though, as in the case of mathematical abstraction, the mind separates it from something to which it must be united if it is to have its being in reality. It is of great importance to distinguish carefully between mathematical abstraction and this second type of negative abstraction. In mathematical abstraction the mind does not supply the object but merely the conditions of the object, whereas in negative abstraction the mind supplies the very object. This type of negative abstraction plays an important role in experimental science because of its dialectical character and because of the extensive use of mental constructs. The first type mentioned above is employed in the formation of the universals that are characteristic of experimental science, for since the universals are merely hypothetical they cannot be the result of positive abstraction.

We are now in a position to pursue our discussion of the degrees of formal abstraction. They are brought out with admirable clarity by Saint Thomas in his <u>Commentary on the De Trinitate</u> of Boethius,(45) and we scarcely need to do more than paraphrase his treatment of them. There are three kinds of matter, and consequently three distinct levels in the process by which the mind lifts its scientific object out of the potentiality in which it is concretized. First there is individual matter, i.e. the matter which sets individual things off from each other with all the particular individualizing characteristics proper to each. As long as these individualizing characteristics are retained no science is possible, for: <u>omne individuum ineffabile.</u> The reason is that a thing is intelligible only to the extent to which it is in act. Matter is being in potency and everything that is dependent upon it essentially and inseparable from it is not intelligible in act. Hence it is from the knower that intelligibility in act must come. Consequently the first step in the process of scientific abstraction is to slough off these particular characteristics and by so doing arrive at a specific intelligible essence. This first step is called physical abstraction, and it is used by all the

disciplines which study nature. The second kind of matter is known as common sensible matter. By sensible matter is meant matter that is apprehensible by the senses, and hence something that involves material qualities. This common sensible matter remains untouched by the first degree of abstraction, for the biologist, for example, studies flesh and blood, even though he does not study this particular flesh and blood, the flesh and blood of Socrates, for example.(44) The second step in scientific abstraction consists in disengaging an intelligible form from this sensible matter. This is known as mathematical abstraction, for it is the abstraction employed by the mathematical sciences. In spite of its high degree of abstraction, mathematics does not succeed in freeing itself of all matter, for whatever is quantitative is necessarily material. But the matter which it retains though apprehensible by the intellect is no longer apprehensible by the senses, since all sensible qualities have been refined away. Hence it is known as intelligible matter. The last step in our intellectual purification succeeds in freeing the scientific object of this last vestige of matter and in setting it forth in its pure intelligibility. This is known as metaphysical abstraction.

There is another and more profound way of presenting these three degrees of abstraction. Some scientific objects depend upon sensible matter both for their being and for their "being known", that is to say, both for their objective existence outside the mind and for their subjective existence in the mind. As a consequence, they can neither exist nor be conceived or defined except in terms of sensible matter. These constitute the objects of the disciplines which study nature. All of the natural sciences study the material cosmos in its state of concretion in sensible matter. And they study it precisely from the point of view of sensible matter; that is to say, all the definitions of natural science are in terms of sensible matter. One may be tempted to contest this statement, since sensible matter means, as we have said, sensible qualities invested in matter, and physics seems to prescind from all qualitative determinations and to study the universe only in terms of the category of quantity. The answer to this objection is, of course, that modern physics is mathematical physics, and consequently not a pure natural science. Other scientific objects depend upon sensible matter for their being, but not for their "being known." That is to say, in order for them to exist outside the mind

in the world of reality they must be invested in sensible matter. But they are conceived and defined independently of it. The notions of line, triangle, number three, etc. contain no sensible matter, nor are they ever defined in terms of it; yet if they are to exist at all in the objective world, they must be concretized in it. These form the objects of the mathematical sciences. Still other scientific objects depend upon sensible matter neither for their being, nor for their "being known". Not only are they conceived and defined independently of all matter, but they can exist in objective reality independently of all matter, either because they necessarily do not exist in matter, as for example God and the Angels, or because they do not necessarily exist in matter, as the concepts of substance, quality act and potency, etc. Here we have the objects of metaphysical science. (45)

St. Thomas points out that this threefold division is exhaustive. For the only other possible case that might be imagined would be that of objects that would be independent of sensible matter in their objective existence, but dependent upon it for their subjective existence in the mind. Though completely immaterial in their being, they would have to be materialized in order to be conceived and defined by the intellect. The inadmissibility of such a case is evident, since it implies that the intellect is essentially material and supposes the primacy of matter. Moreover such a process of materialization would be just the opposite of abstraction.

It is necessary to point out here in passing something that will be of considerable significance for us later. Even a casual consideration of the three degrees of abstraction brings to light the fact that there is something peculiar about the type of abstraction used in the mathematical sciences. In it alone do we find a deep dichotomy between the way things exist in reality and the way they are conceived by the mind. In both physical and metaphysical abstraction there is a correspondence between the way things exist objectively and the way they are conceived and defined by the mind. This correspondence is lacking in the second degree of abstraction. In order for mathematical objects to exist at all outside the intellect they must be immersed in sensible matter, whereas inside the intellect they are conceived and defined in complete independence of it. Hence in this case abstraction involves a separation that is not found in the other degrees. Later on in our analysis this dichotomy will throw a great deal of light upon the nature of mathe-

matical physics.

This threefold level of formal abstraction provides us with the specifically different rationes formales sub quibus that we set out to find. We have three different grades of immateriality, three different ways of abstracting and defining the scientific object. In metaphysics everything is defined without relation to matter of any kind. In Mathematics everything is defined in terms of intelligible matter alone. In the study of nature everything is defined in terms of sensible matter. Now these three rationes formales sub quibus in turn actualize and light up three specifically distinct rationes formales quae: being in metaphysics; quantity in mathematics; mobility in the study of nature. The first of these three objects is not of any special interest for our problem. We shall remit the question of the second object to Chapter VI where we shall discuss in some detail the nature of mathematical science. Since we are particularly concerned with physics, the scientific object which has the greatest interest for us is the one that is born of the first degree of abstraction. Thomists have traditionally insisted that the proper object of the study of nature is ens mobile: mobile being. (46) For those who approach the question for the first time it is not immediately evident perhaps why this should be so. There are a number of other ways of expressing the object studied by natural science which would seem to suggest themselves more spontaneously than "mobile being;" such as: "natural body", "natural substance", "sensible being", "physical body", "natural being", etc. In fact some modern Thomists have seen fit to substitute "sensible being" for the traditional "mobile being". (47) This question has been studied with great profoundity and acuteness by Cajetan (48) and John of St. Thomas, (49) and though it would be too long and tedious to summarize all of their arguments, there are a few points which must be insisted upon with special emphasis. The reason why mobilitas is taken as the formal object of the study of nature is that better than any other point of view that might be selected, it opens up the inner essence of natural things. In other words, it is only in terms of mobility that nature can be truly explained. The history of philosophy gives us an extrinsic reason for this: from Heraclitus down to Bergson and Whitehead, the problem of mobility has been the crucial point in the study of nature. In his Commentary on the Physics, St. Thomas suggests an intrinsic reason:

> De his vero quae dependent a materia non solum secundum esse sed etiam secundum rationem est Naturalis, quae Physica dicitur. Et quia omne quod habet materiam mobile est, consequens est quod ens mobile sit subiectum naturalis philosophiae. Naturalis enim philosophia de naturalibus est; naturalia autem sunt quorum principipium est natura; natura autem est principium motus et quietis in eo in quo est; de his igitur quae habent in se principium motus, est scientia naturalis. (50)

The expression "sensible being" which some modern Thomists have attempted to substitute does not bring out the true objective formality in terms of which nature must be studied. For, things in nature are not sensible for the separated substances, but only for us. Hence "sensible" does not directly explain what things are in themselves, but only how they are known by us. Of course, every mobile being is at the same time a sensible being, for there is an analytical connection between motion and sensible matter in that both of them involve material potency. But sensibility does not explain the objective nature of things; it merely explains how we know them. Mobility, on the other hand, is something objective. Even the separated substances know natural things as mobile beings, not, indeed, as we do, merely in terms of the general formality of mobility, but in terms of the specific type of mobility proper to each ontological species.

And just as no other word may be substituted for "mobile", so no other expression can adequately take the place of "being": not "substance", for that would exclude the consideration of accidents; not "body", for as St. Thomas points out, (51) it pertains to the science of physics to prove that all mobile beings are bodies, and no science proves its own subject. John of St. Thomas clearly indicates the positive reason why the expression must be "mobile being:" Motion is not defined in relation to substance or body, but in relation to being, for it is: "actus entis in potentia in quantum huiusmodi":

> Fundamentum huius conclusionis sumitur ex his, quae paulo ante sunt insinuata, quia videlicet propria et adaequata passio, quam physicus demonstrat de suo subiecto, est motus. Motus autem non definitur explicando ordinem ad corpus vel substantiam, sed ad ens mobile; definitur enim, quod est "actus entis in potentia", ut patet in hoc tertio libro. Ergo formalis ratio subiecti physici non explicat rationem corporis. Nam quod formaliter est subiectum scientiae, explicatur etiam in formali definitione propriae passionis tamquam id, ad quod passio dicit habitudinem. Ergo cum non explicetur in definitione motus ratio corporis, sed ratio entis in potentia, non pertinet ad formalem rationem subiecti corpus, licet in re verum sit, quod non sit mobile motu physico nisi id, quod est corpus. Sed tamen sub formalitate, qua pertinet ad Physicam, ita se habet, quodsi per impossibile daretur aliquod mobile, quod non esset corpus, adhuc tractaretur a Physica, sicut si per impossible daretur aliquod coloratum extensum, quod non esset corpus, adhuc videretur ab oculo. (52)

It is extremely important to insist upon the unity and indivisibility of the object of the study of nature. The composition found in the expression "ens mobile" is only verbal. It does not imply a composition of two objective formalities, the formality of being and the formality of mobility. Mobile being does not mean "being" with the addition of a specific difference: "mobile". If this were true, philosophy of nature would be a part of metaphysics or at least a science subalternated to it. Both Cajetan (53) and John of St. Thomas (54) lay considerable stress upon this point, and we shall see its importance in a few moments.

The assigning of "mobile being" as the object of the science of nature gives rise to a difficulty, the solution of which will enable us to penetrate more deeply into the nature of physical science. We said above that science is possible only in so far as its object is lifted above the flux of change, for science is about necessary and not contingent things. The etymological root of the word *episteme* means firmness and stability. Consequently a science of mobile being would seem to be a contradiction in terms.

... de permutante, idest de eo quod movetur... nihil verum dicitur inquantum mutatur. Quod enim mutatur de albedine in nigredinem, non est album nec nigrum inquantum mutatur. Et ideo si natura rerum sensibilium semper permutatur, et omnino, idest quantum ad omnia, ita quod nihil in ea sit fixum, non est aliquid determinata verum dicere de ipsa.(55)

In raising this question we are touching upon one of the most persistent antinomies in the whole history of philosophy. Ever since the time of the ancient Greeks philosophers have sought to reconcile the fluidity of nature, clearly revealed by the senses, with the necessity of science. In the doctrines of Heraclitus, Parmenides, Plato, and their followers philosophy and nature were in some measure in constant conflict. Sometimes it was philosophy that suffered from the conflict, and at other times it was nature. It took the genius of Aristotle to reconcile the two and to give birth to a philosophy of nature.(56) It is true that natural things are in a constant state of flux, of generation and corruption. Yet in the midst of this fluidity of phenomena there is in nature a permanent, general structure which the mind can lay hold of through the process of abstraction described above.

Contingentia dupliciter cognosci possunt. Uno

modo secundum rationes universales; alio modo secundum quod in particulari. Universales quidem igitur rationes contingentium immutabiles sunt, et secundum hoc de his demonstrationes dantur et ad scientias demonstrativas pertinet eorum cognitio. Non enim scientia naturalis solum est de rebus necessariis et incorruptibilibus, sed etiam de rebus corrputibilibus et contingentibus. Unde patet quod contingentia sic considerata ad eandem partem animae intellectivae pertinent ad quam et necessaria, quam Philosophus vocat hic scientificum.(57)

It is not necessary to transcend nature in order to find immutable types. Basic regulations in the stream of phenomena reveal the fact that there are immutable types immanent in nature itself. It is only in their individual composite existence, not in their universal essences that the things of nature are fluid. As Aristotle points out in the eighth book of the Metaphysics, it is only an individual house that is brought into existence, not the nature of house in general. In like manner when an individual man dies, the definition of man does not perish. "Etsi enim ista sensibilia corruptibilia sint in particulari, in universali tamen quamdam sempiternitatem habent."(58) It is in this way that definitions of natural things are possible, and wherever definitions are possible, science is possible. These definitions give the universal essences that are concretized in nature, shorn of their individual matter existence in reality are mobile, and which in their existence in the mind are from one point of view mobile and from another immobile: mobile in the sense that they are conceived of as mobile; immobile in the sense that they are conceived in an immobile way by virtue of a retreat into universality. Mathematical science deals with objects which have mobility in their objective existence, but absolute immobility in intellect. Metaphysical science considers objects which are absolutely immobile in both their objective and subjective existence.

In order to round out our consideration of the hierarchy of speculative science it is important to see the connection this hierarchy has with both an objective stratification in the structure of physical reality, (61) and a subjective stratification in the cognitive powers. (62) Physical reality is constructed in such a way that in it substance has a natural priority over the accidents which inhere in it and determine it. But even among the accidents quantity has a natural priority over the sensible qualities. Quantity is, in fact, the first accident; of all the accidents it is the closest to substance, for it is quantity which orders the parts of material substance and gives it actual extension. It is only because of this

extension that the other accidents can inhere in the substance. For example, a body can be determined by a certain color only because there is an extended surface that can receive this color. Hence sensible qualities are not rooted directly in the substance, but in the quantity. Only through it are they rooted in the substance. Because of its closeness to the substance, quantity possesses a source of intelligibility which the other accidents do not have. But at the same time it must be pointed out that from another point of view it has less intelligibility than the sensible qualities, for these latter follow upon the substantial form, whereas quantity follows upon the matter. We shall return to this paradox later, for it has an important part to play in the solution of our problem.

We find, then, in the structure of physical reality a definite stratification: substance, quantity, sensible qualities. It is possible for the mind to consider the essential determinations of reality independently of any relation to its quantitative and qualitative determination ~~determinations~~. It is likewise possible for the mind to consider reality in terms of its quantitative determinations without any relation to its qualitative relations. But the reverse of this process is not possible. It is impossible, (materia signata, in Thomistic terminology) but not of common sensible matter (materia non signata). Hence as we have already pointed out, it is not a question of abstracting a substantial form from its corresponding matter, for a form thus abstracted would have no meaning. Now as St. Thomas points out, (59) these abstract essences can be considered in two ways: first, in their abstract state in which they exist in the mind alone, and in this way they are without matter (individual) and motion; secondly, in relation to the mobile material things outside the mind from which they have been abstracted, and in this way they are the medium by which physical reality is known, for things are known by means of their form. Thus it is possible to have a science of mobile being.

Nevertheless, it is important to point out that the mobility of the things which form the object of the science of nature has profound repercussions upon the necessity of that science itself. It s only at the price of retreating into broad generalities that the study of nature can enjoy true necessity. Once it begins to press its object more closely as it is bound to attempt to do, the necessity starts to fade. That is why as the study of nature follows its natural course towards greater

concretion, true scientific knowledge (episteme) peters out into opinionative knowledge (doxa). We shall call this opinionative knowledge dialectical knowledge, for reasons which will become apparent later.

In connection with the type of necessity found in the study of nature the following lines of St. Thomas are significant:

> Modus autem demonstrationis est diversus; quia quaedam demonstrant magis necessarie, sicut mathematicae scientiae, quaedam 'vero infirmius', idest non de necessitate; sicut scientiae naturales, in quibus multae demonstrationes sumuntur ex his quae non semper insunt, sed frequenter. (60)

Almost instinctively the "doxa" will attempt to erect itself into an "episteme"; the "modus infirmior" demonstrandi will reach out for support to a more sure type of demonstration, the science of nature will seek to rid itself of the mobility to which it is a prey. And that is why physics will inevitably become mathematical.

And now we are in a position to see how the degrees of formal abstraction give us three levels of immobility as well as three levels of immateriality. The science of nature has to do with objects which in their for example, to conceive of quantity without substance, for quantity is precisely the order of the parts of the substance, and order cannot be conceived of without the parts. At first glance this point may seem to be in conflict with what was said above about the nature of formal abstraction. It was pointed out that total and formal abstraction differ in that the latter results in two independent concepts. And we added by way of example that just as the concept of quantity is independent of sensible matter, so the concept of sensible matter is independent of quantity. But from what has just been said it would seem that the concept of sensible matter cannot be independent of the concept of quantity. The solution of this apparent conflict lies in this that there are two kinds of quantity: abstract, mathematical quantity, and concrete quantity. The notion of sensible matter is independent of the former, though not of the latter.

This distinction between abstract and concrete quantity is of great importance for the question of mathematical physics. Since it is possible to lay hold of the concrete quantitative determinations existing in nature by a kind of negative abstraction the road is open to a confusion between this way of considering quantity and

the way it is considered in mathematics which is the fruit of a special type of formal abstraction. As a matter of fact, some authors have fallen into this confusion, as we shall point out in a few moments. The consequences of this confusion are disastrous. For if mathematical physics consisted merely in a study of the concrete quantitative determinations existing in nature by means of negative abstraction, it would not be a hybrid science, but a pure physical science. The mind would not travel out beyond the physical world to a realm apart, to return to the physical world later with a rationality fundamentally alien to it, yet in a mysterious way capable of being applied to it. The mind would remain enclosed within the physical world. This would change the whole epistemological character of mathematical physics.

Now the relation between this stratification and the hierarchy of speculative science does not consist in this that natural science studies the sensible qualities alone, mathematics the concrete quantity as it is found in nature, and metaphysics the substance of reality without any consideration of the accidents. All three of these statements are false. Rather, the connection between the two hierarchies must be expressed in this way: because of the logical priority existing in the objective structure of the universe, it is possible for the mind in its attempt to lay hold of reality scientifically to take three specifically distinct steps: first to prescind only from the individual characteristics and to consider reality in terms of all its concrete determinations, including the qualitative determinations of sensible matter; secondly to prescind from all sensible qualities and to consider reality in terms of its quantitative determinations alone (but here it must be noted again that it is not concrete quantity that is being considered, but abstract quantity, for concrete quantity is precisely quantity concretized in sensible matter — here we have a key to the paradox just mentioned about the greater and lesser degree of intelligibility possessed by quantity); thirdly, to prescind from all matter and to consider being as such.

The hierarchy of speculative science also has an essential connection with a hierarchy of cognitive powers. All knowledge begins in the external senses, but not all knowledge terminates there. Likewise all the sciences considered from the point of view of their origin have some kind of relation to the external senses, but considered from the point of view of their term, some

sciences are independent of the external senses, and bear an essential relation to some other cognitive power. For example, our knowledge of God depends upon the external senses for its origin, since the only way we can get to know God is through the material things in the world about us. But it does not terminate there, that is to say, in our conclusions about the nature of God we do not judge that He is like the sensible things in the material cosmos.

This is the basis of St. Thomas' doctrine that natural science terminates in the external senses, mathematical science in the imagination, and metaphysical science in the intellect alone. The reason why natural science must terminate as well as originate in the external senses is clear: all of its conceptions and definitions are necessarily in terms of sensible matter. As St. Thomas puts it, "qui sensum negligit in naturalibus incidit in errorem."(63) Hence all of its judgements must be verifiable in sensible experience. It is to be noted that we say "verifiable" and not "verified" in sensible experience, for as we shall see later, it is only that part of natural doctrine which is purely dialectical that must necessarily be verified in sensible experience. We shall discuss this question of the relation between the study of nature and

This point of Thomistic doctrine must be rightly understood if confusion is to be avoided. Even though only the study of nature terminates in the senses in the way in which we have explained, all science of reality must retain an essential connection with the deliverances of the senses if it is to have any validity. That is to say, it must b e able to be resolved back to the sense experiences from which it took its rise. For abstraction does not consist in burning bridges behind one. And this is true even of metaphysics, as St. Thomas explains in the following lines:

> Sed quia primum principium nostrae cognitionis est sensus oportet ad sensum quodamodo resolvere omnia de quibus judicamus; unde Philosophus dicit in III Caeli et Mundi quod complimentum artis et naturae est res sensibilis visibilis ex qua debemus de aliis judicare; et similiter dicit in VI Ethicorum (cap.VIII in fin.) quod sensus sunt extremi sicut intellectus principiorum; extrema appellans illa in quae fit resolutio judicantis.(65)

Taken in this sense, the principle of logical positivism that nothing has meaning except in the measure in which it is capable of verification in sense experience is quite acceptable, and is actually realized fully in metaphysics, in spite of the violent opposition to metaphysics on the part of the logical positivists.

Our discussion of the specification of the sciences would not be adequate if we did not touch at least briefly upon another point which emerges from a reading of the passages in which St. Thomas treats the problem. John of St. Thomas (66) calls our attention to the fact that there are a number of texts in which Aquinas seems to use other criteria for the distinction and specification of speculative science than the one upon which we have based our entire discussion, namely the three degrees of formal abstraction. Sometimes he founds the distinction upon a difference in the type of medium used by a science in its demonstrations. (67) In other places he appeals to a difference in the type of definition employed in the science for definitions are the principles of scientific demonstration. (68) With admirable clarity John of St. Thomas goes on to show how all of these different points of view are reducible to the same thing. In doing so he is merely making explicit what is found in St. Thomas himself, for in his Commentary on the Metaphysics (69) the coincidence of the three points of view is already clearly indicated. Since scientific knowledge is precisely knowledge arrived at by demonstration, it is clear if there are specifically different sciences there will be specifically different types of media used in the demonstrations by which sense experience in Chapter IV.

The connection between mathematics and the imagination is not so immediately evident perhaps. Since we have the intention of considering this problem in some detail in Chapter VI we shall content ourselves here with merely indicating the basis of the connection. In the first place it is fairly clear that mathematical science does not terminate in the external senses. It is independent of sensible matter in its conceptions and definitions. No mathematician has ever seen in the world of sense a straight line, a perfect circle, or a line touching a sphere at only one point. (64) But that does not affect his science in any way. Yet, while independent of sensible matter, the mathematician still retains intelligible matter, and it is because of this intelligible matter that his science must terminate in the imagination. For intelligible matter signifies homogeneous exteriority, that is to say, the multiplication of the same form through either continuous or discrete homogeneity. This exteriority and multiplicity demands some kind of individuation, and it is precisely the imagination that provides this individuation which in physical things is provided by the matter. Of itself, the intellect has to do with pure form, separated from matter.

Hence if the intellect alone functioned in mathematics we could not have the notion of homogeneous multiplicity. At first glance this may seem to give rise to a serious difficulty. For it is certain that God knows mathematics, and yet He is without imagination. The difficulty vanishes, however, when we take into account the vast difference between the human and the divine intellects. Man's knowledge is posterior to things and his intellect is dependent upon them and measured by them. All of his mathematical notions are drawn from concrete material things. Consequently, when they are lifted out of concrete matter, there must be something to substitute for the individuation which this matter provides. But God's knowledge is prior to things, and His intellect is not measured by them. That is why He does not have need of imagination to provide for individuation.

The connection between metaphysical science and intellect is quite clear. We may arrive at the notion of immaterial things by means of material things presented to us by the external senses and the imagination. But in the end we do not judge that immaterial things are like material things.

they arrive at their conclusions. Now these media are the premises employed in the scientific syllogism. These premises in turn are necessarily definitions, and hence a specific difference of media reduces itself to a specific difference of definition. But a specifically different type of definition can be had only be means of a specifically different type of formal abstraction. Since immateriality is the source of intelligibility, a specifically distinct level of immateriality is at the root of the specifically distinct ways the mind has of rendering reality intelligible, i.e. of laying hold of its essence, of setting forth its "quod quid est." But to set forth the quod quid est of a things is to define. Hence the source of the unity and distinction of the sciences is the specific types of immateriality. These types of immateriality result in different types of definition. And this difference in definition gives rise to a specific difference in the principles and media used in scientific demonstrations. The difference in immateriality or intelligible light found in the principles are communicated by means of the demonstration to the scientific conclusions.

In introducing this question of the distinction of the speculative sciences, we said that we would adopt as

our guide the treatment of the problem given by John of St. Thomas. At the same time we noted that this treatment is merely a summary of the doctrine of St. Thomas and Aristotle, and that it in no way adds to it or modifies it in any respect. Perhaps the numerous references of St. Thomas and Aristotle adduced in our discussion of the question suffice to establish the truth of this assertion. But because the issue is of some moment for our study, and because some contemporary Thomists have thrown doubt upon it, we consider it worth while to stop for a moment to consider the problem explicitly.(70)

It has been maintai ed that the doctrine of the three degrees of formal abstraction taught by Cajetan and John of St. Thomas is not found in St. Thomas himself. Aquinas, we are told, taught that only mathematical abstraction is formal abstraction and that the study of nature employs nothing but total abstraction. Certain texts of the Angelic Doctor seem at first sight to give substance to this claim. In the second article of the fifth question of his Commentary on the De Trinitate of Boethius, he seems to say that only in mathematical science do we have the abstraction of a form from matter. And speaking of the kind of abstraction found in the study of nature, he adds:

"Ideo praedicta abstractio non dicitur formae a materia absolute, sed universalis a particulari." In the next article, he explains that there are three different kinds of intellectual operation found in the three speculative sciences and that the one that is proper to the study of nature is had "secundum oppositionem universalis a particulari, et haec competit etiam physicae, et est communis omnibus scientiis, quia in omni scientia praetermittitur quod est per accidens, et accipitur quod est per se."

It is obvious that these texts must be interpreted in the light of St. Thomas' general doctrine. And in the first place it must be noted that if there is no formal abstraction of any kind in the study of nature, it cannot be a science, for without formal abstraction it cannot have a <u>ratio formalis</u>. Consequently, to hold that St. Thomas and Aristotle in no way associated formal abstraction with the study of nature is equivalent to saying that for them natural doctrine was not a true science — which is patently absurd. Moreover, there is a special reason why St.Thomas associates total abstraction with the study of nature, for it is only in the things of nature that there are individuals which are not species, and consequently it is only in natural

doctrine that it is necessary to begin by abstracting from individuals in order to get at the object of science.

Many of those who deny formal abstraction to the study of nature admit it for metaphysics. This admission should lead them to recognize the fact that when St.Thomas says that formal abstraction is found only in mathematical science he is taking the term in a very special sense. As a matter of fact it is only mathematics which considers forms that are separated from the sensible matter to which they must be united if they are to exist. In other words, there is formal abstraction in all of the three species of speculative science, but over and above this there is in mathematics a particular kind of formal abstraction. The proper nature of this type of abstraction will be analyzed in detail in Chapter VI. When St. Thomas seems to restrict formal abstraction to mathematics he warns us how this should be interpreted for he says: "...praedicta abstractio non dicitur formae a materia <u>absolute</u>." It is true that in the essences which constitute the object of the study of nature there is common matter as well as form, but it is illegitimate to use this as a foundation for a denial of formal abstraction in natural doctrine, for St. Thomas points out in innumerable places that even material essences can be considered as forms in relation to the individual matter from which they have been abstracted.

And now we feel that enough has been said to bring out the central point about which this whole chapter revolves: the basic principles which govern the distinction between physics and mathematics. Subsequent chapters will provide an elaboration of these principles. But perhaps it would be well at this point, in order to give sharper outline to the distinction, to consider briefly some observations made by a contemporary Scholastic on the Aristotelian doctrine of physical and mathematical abstraction in so far as it applies to the problem of mathematical physics. In an article to which we have already made reference Professor Mansion of Louvain has this to say:

> Notons enfin que les déterminations quantitatives ne sont pas plus indépendantes de l'expérience concrète et de la réalité existante que les autres attributs,— d'ordre qualitatif — appartenant au monde des corps. Elles présentent seulement cet avantage que, isolées par l'abstraction, elles se prêtent mieux, — merveilleusement mieux, à une élaboration conceptuelle ultérieure: cette élaboration, oeuvre de raison tout à fait remarquable, a donné naissance, en effet, à des disciplines indépendantes, construites suivant une rigueur logique inégale. Si l'on voulait soumettre à

> un traitement semblable un concept tel que celui de chaleur, j'entends le concept répondant de façon immédiate dans l'abstrait à notre sensation de chaud, nos spéculations s'arrêteraient court avant d'être arrivées fort loin. Cette notion, en effet, paraît réfractaire à toute analyse un peu poussée; elle est inapte à entrer telle quelle dans une systématization plus développée, où seraient déterminés ses rapports avec des objets connexes, tels que le froid, etc. Ce n'est pourtant pas que nous ayons affaire ici à un concept abstrait à un moindre dégré, que la notion de nombre par exemple; mais simplement que nous sommes en présence d'un concept de contenu différent, moins accessible à notre intélligence humaine dans ses conditions actuelles.(71)

This passage is filled with ambiguities and contradictions. In the first place, it must be admitted, of course, that there is a sense in which the initial statement (that the quantitative determinations of nature are no more independent of concrete experience and of existing reality than the sensible qualities) is true. It is obvious that we get to know these quantitative determinations only by grasping them in their state of concretion through concrete experience. It is likewise obvious that they are directly given in existing reality along with the qualitative determinations.

In this sense Mansion is justified in remarking:

> Toutes (les notes caractéristiques de l'objet physique et celles de l'objet mathématique) font partie originairement d'un même complexe sensible, objet d'une perception globale, et dans lequel on retrouve les déterminations quantitatives au même titre que les autres."(72)

But at the same time there is a sense in which it is true to say that they are more independent of concrete experience and existing reality than the qualitative determinations. (73) Because of the hierarchical structure of physical reality, we get to know the quantitative determinations only by means of the qualitative determinations. This does not involve a process of illation, of course. It merely means that all the proper objects of the senses are qualitative determinations, and that it is only through them that this quantitative determinations can be grasped at all. Moreover, even though these quantitative determinations never exist objectively except in the state of concretion with sensible matter, they are, as we have seen, conceptually independent of this sensible matter in the sense that quantity is the first accident and the subject of all the other accident. That is why they can be lifted out of it and elaborated into a world apart — a world of knowledge which does not have to terminate in the world of existing reality as presented by concrete experience, but merely in the intuitive imagination.

Does not all this involve an independence of both concrete experience and existing reality in which the qualitative determinations have no share? Does not Mansion himself admit this independence when he states that once isolated by abstraction these quantitative determinations can be elaborated into "des disciplines independantes"? Nor is there any force in Mansion's argument when he claims that Aristotle contradicts himself by postulating a special degree of abstraction for mathematics and at the same time admitting that mathematical beings are τὰ ἐξ ἀφαιρέσεως, that is to say, extracted from the ensemble perceptible to the senses, which constitutes the physical object. (74) How else could mathematical beings have a special degree of abstraction except by being abstracted from the physical objects presented by the senses?

After pointing out that the quantitative determinations in their state of abstractive isolation lend themselves readily to a remarkable conceptual elaboration, Mansion goes on to say that this does not argue to a higher degree of abstraction. This statement seems to ignore completely the nature of formal abstraction which, as we have pointed out, is based precisely upon greater objective intelligibility. Moreover, to attempt to establish a parallel between the way the concept of heat is abstracted from the other sensible qualities, and the way the concept of straight line is abstracted from sensible matter is to vitiate the whole Thomistic doctrine of abstraction. For the process of singling out the quality of heat from among the other sensible qualities is not necessarily positive abstraction at all, to say nothing of formal abstraction. Actually it is merely a kind of negative abstraction in which the mind fixes its attention on one point while neglecting everything else that is connected with it. And even if it were positive abstraction, there would still be a vast difference between it and the type of abstraction proper to mathematics. Enough has been said to show that quantity is in no more "abstractable" than the sensible qualities. The former can be conceived without the latter, but not vice versa. We can get at the quod quid est of a straight line, for example, and define it, but it is impossible to give a proper deinition of heat or any of the sensible qualities. Perhaps we should mention here something that will be discussed in a later context: it is possible for the student of nature to consider quantitative determinations of the cosmos, but in his consideration they will always be united with sensible qualities and

connected with mobility; it also pertains to the metaphysician to study quantity, but only in so far as it is a principle of being. Both of these ways of considering the quantity of nature are vastly different from the way it is considered by the mathematician in the second degree of abstraction. The central error of this whole section of Mansion's essay seems to be a confusion between the way of grasping quantity that is proper to the mathematician and the other ways in which it may be laid hold of by the mind. This is evident in the following lines:

> En s'en tenant à ce point de vue, on serait donc autorisé à affirmer qu'il y a moyen d'abstraire et d'isoler — par la pensée seule, bien entendu, — tel groupe particulier de qualités sensibles, appartenant à l'objet physique global, — le chaud et le froid, par exemple, — aussi bien que l'ensemble des déterminations quantitatives. On aurait ainsi un objet plus abstrait, parce que plus simple, que si l'on retenait tous les groupes de qualités sensibles analogues: on n'aurait pas pour autant un degré d'abstraction caractéristique, mais une même abstraction poussée un peu plus loin, dans un certain sens, choisi d'ailleurs de façon arbitraire.(75)

Arising out of this initial confusion is the confusion between the concrete quantitative determinations as they exist in nature and the abstract quantity that is the object of mathematics. Professor Mansion seems to hold that the object of mathematics is what is known in Thomistic terminology as the common sensibles, and in modern terminology as the primary qualities. That is why he objects so strongly to Aristotle's distinction between sensible and intelligible matter:

> Or on est forcé de constater ici dans cet emploi des mots 'intelligible' et 'sensible', un abus de langage d'autant plus grave, qu'il paraît couvrir une confusion dans la pensée et constituer ainsi le point de départ d'une erreur formelle ... Cet objet (mathématique) est, à l'origine et fondamentalement, perceptible par les sens, tout autant que l'objet physique, et de manière aussi directe.(76)

It is this same reason that leads him to write:

> Il y a plus, et cette particularité ne manque pas de saveur: le mouvement d'après lui est caractéristique de l'objet physique: l'objet mathématique en fait abstraction. Or le mouvement est aussi rangé parmi les sensibles communs, mais, <u>en outre</u>, c'est par la perception du mouvement, que nous avons celle de tous les autres, notamment les déterminations quantitatives, que retient seules le mathématicien. (De Anima T, 1,425,a19 -19)(77)

The basis of these difficulties vanishes when one points out that Aristotle never held that the common sensibles constitute the object of mathematics. As for the question of movement, it is sufficient to remark that it falls under the common sensibles only indirectly, because of the

extension of space covered by the movement. Movement in itself, i.e. the act of being in potency in so far as it is in potency, is not a common sensible. The student of nature considers it, not as a common sensible, but in its intrinsic nature.

And thus St. Thomas writes:

> Motus secundum naturam suam non pertinet ad genus quantitatis, sed participat aliquid de natura quantitatis aliunde, secundum quod divisio motus sumitur ex divisione spatii vel ex divisione mobilis: et ideo considerare motum non pertinet ad mathematicum, sed tamen principia mathematica ad motum applicare possunt: et ideo secundum hoc quod principia quantitatis ad motum applicantur, naturalis considerare debet de divisione, et continui, et motus, ut patet in VI Physicorum. Et in scientiis mediis inter mathematicam et naturalem tractatur de mensuris motuum, sicut in scientiis de sphaera mota, et in astrologia.(78)

The last remark of Mansion quoted above has no particular relevance, for in the place indicated in the <u>De Anima</u> Aristotle merely states that sensibles are perceived only through an immutation of the sense.

We have devoted considerable attention to these difficulties proposed by Professor Mansion not only because they serve as an excellent back-drop against which to bring out in clearer focus the fundamental notions we have been laboring to formulate in this chapter, but also because if left unsolved they inevitably give rise to an entirely faulty view of Thomistic philosophy of science in general, and of mathematical physics in particular. As a matter of fact, they have led Professor Mansion to the fundamentally erroneous view of mathematical physics already pointed out earlier in this chapter - - that of considering it not as an interpretation of physical nature in terms of higher science, but merely as a study of the concrete quantitative determinations existing in the cosmos. He writes:

> Car, remarquons-le bien, s'il est question ici de science ou de physique mathématisée, ce n'est pas qu'on ait substitué, dans l'objet d'expérience brut, à des attributs qualitatifs, apparaissant comme tels dans la sensation, des entités géométriques ou purement mathématiques; ces sciences ne sont encore mathématisées que parce qu'on à fait entrer dans la construction scientifique du phénomène la mesure exacte de ce qui est déjà donné comme quantitatif ou quantifié dans l'objet d'expérience lui-même. La part d'hypothèses géométriques qui s'y ajoute, par exemple en astronomie, pour importante qu'elle soit dans la construction systématique de la science, n'a qu'un rôle secondaire et simplement instrumental dans la détermination des lois quantitatives - - de forme mathématique - - régissant les phénomènes étudiés. Et de plus, à ce stade de l'évolution des sciences, les hypothèses utilisées ne sont, par ailleurs, pas encore hétérogènes au donné empirique, dont on cherche à formuler les lois. (79)

We shall analyse the falsity of this position later.

In the difficulties enumerated above Professor Mansion finds the reason why, according to him, Aristotle cut himself off from the study of mathematics and of mathematical physics. From them he draws his conclusion that in Aristotelianism no true science of mathematical physics is even theoretically possible. We have referred to this conclusion in Chapter I and perhaps enough has already been said to call its validity into question.

4. Ultimate Specification

The above sketch of the hierarchy of speculative science will serve to draw a clear cut line of demarcation between physics and mathematics and at the same time to localize both of these sciences in the general field of knowledge. But it is extremely important for a true understanding of the nature of mathematical physics to press this question of epistemological pluralism a bit further. The three degrees of formal abstraction provide us with the basic structure of speculative science. But it may be asked whether they give us the absolutely ultimate specification of the sciences. Is it not conceivable that in the general framework provided by a certain degree of abstraction a plurality of more specific formalities might be discovered which would serve as the basis for a sharper and more ultimate specification of the sciences? In this case, the degrees of abstraction would be a genus containing within it a number of scientific species. To the question posed in this general fashion the Thomists have traditionally given an affirmative answer. And John of St. Thomas provides us with the reason.(80) Because abstraction is a kind of process or movement, there are in it two points to be considered: the point of departure and the terminal point. This point of departure is the materiality that is sloughed off; and corresponding to the three types of matter there are three levels of abstraction. The terminal point is the particular grade of immateriality, the specific spiritual mode, the special type of intelligibility that an object is brought to when it is once cut free of a certain level of materiality. It is not the mere leaving behind of a certain general type of materiality that gives us the ultimate specific difference of the sciences, but the particular mode of intelligibility that is arrived at. For it is possible within one and the same degree of abstraction to have an intrinsic differentiation consisting in a greater or lesser

approach to immateriality. In other words, once the mind has performed the initial abstraction which gets rid of a certain general level of materiality, it may have the freedom to move to different points of terminal abstraction. Thus all of mathematics has the same general degree of abstraction: the leaving behind of sensible matter. Yet Thomists agree that within this degree of abstraction two specifically distinct sciences are found: geometry, which deals with continuous quantity, and arithmetic which deals with discrete quantity. All of the other branches of mathematics are either further elaborations, or appendages, or combinations or dialectical superstructures of these two fundamental sciences. The reason why they are specifically distinct is that arithmetic achieves a closer approach to immateriality than geometry. This can be brought out both by a proof and by a sign. The proof consists in this that continuous quantity has more subjectivity and more potentiality than discrete quantity. Continuous quantity is, in fact, principally matter, whereas number is principally form. In continuous quantity there is a subject which has infinite potentiality for division. It is true that number can be added to <u>ad infinitum</u>, but this potential infinity lies outside the number that is being added to, whereas in the case of continuous quantity the infinite potentiality is within. Number is something definite and determined. Continuous quantity is something intrinsically indetermined.

Aristotle brings out the distinction between arithmetic and geometry in the <u>Posterior Analytics</u>:

> A science such as arithmetic, which is not a science of properties qua inhering in a substratum, is more exact than and prior to a science like harmonics, which is a science of properties inhering in a substratum; and similarly a science like arithmetic, which is constituted of fewer basic elements, is more exact than and prior to geometry, which requires additional elements. What I mean by 'additional elements' is this: a unit is substance without position, while a point is substance with position; the latter contains an additional element. (81)

It is clear that the distinction laid down here by Aristotle is based upon the greater immateriality of arithmetic. In fact, as St. Thomas explains in his commentary on this passage, the contrast brought out by Aristotle between geometry and arithmetic is a contrast between matter and form: "alii autem duo modi accipiuntur secundum quod forma est certior materia, utpote quia forma est principium cognoscendi materiam." (82)

A sign of the more abstract character of arithmetic is found in the fact that it is far less dependent

upon the imagination than geometry. We can imagine any kind of a thing as a phantasm for number, as long as there is homogeneous plurality; but not any kind of thing represents a circle, for example. Another sign consists in this that by extension number can be applied to spiritual beings, whereas continuous quantity cannot.

Geometry still has something of the qualitative clinging to it, even if it be only a question of quantitative quality, such as figure. Speaking of this distinction between geometry and arithmetic, Duhem writes:

> Parmi les sciences, l'arithmétique seule, avec l'algèbre, son prolongement, est pure de toute notion empruntée à la catégorie de la qualité; seule, elle est conforme a l'idéal que Descartes propose à la science entière de la nature. Dès la géométrie, l'esprit se heurte à l'élément qualitatif, car cette science demeure 'si astreinte à la considération des figures qu' elle ne peut exercer l'entendement sans fatiguer beaucoup l'imagination.' - - 'Le scrupule que faisaient les anciens d'user des termes de l'arithmétique en la géométrie, qui ne pouvait procéder que de ce qu'ils ne voyaient pas assez clairement leur rapport, causait beaucoup d'obscurité et d'embarras dans la façon dont ils s'expliquaient.' Cette obscurité, cet embarras, disparaîtront si l'on chasse de la géométrie la notion qualitative de forme, de figure, pour n'y conserver que la notion quantitative de distance, que les équations qui relient les unes aux autres les distances mutuelles des divers points étudiés. (83)

John of St. Thomas makes the following clear cut distinction between the two:

> Sed Mathematica considerat proportiones et mensuras, quae in quantitate discreta et continua ita variantur, quod ad diversa principia reducuntur et ad diversam abstractionem et modum definiendi, quia mensuratio per magnitudinem nullo modo convenit cum modo mensurandi penes numerationem. Haec enim abstractiori modo procedit, quia magnitudo mensurat per modum continentis, ut locus, numerus per intellectum numerando.(84)

In the Ars Logica (85) he points out that geometry not only has greater dependence upon place but also upon time. It is not too clear just what this dependence upon time consists in, but in all probability he is referring to the generation of the figures in geometry.

A further indication of the greater materiality of geometry is found in the fact that some modern authors erroneously believe that, at least in certain aspects, it is more truly a physical science than a pure mathematical science. (86)

> Telle était déjà l'idée de Gauss. 'Nous devons admettre humblement, écrivait-il à l'astronome Bessel, que, le nombre est uniquement le produit de notre esprit, l'espace, même au point de vue de notre esprit, constitue une réalité à laquelle nous ne pouvons a priori dicter complètement ses lois'. Dedekind, dans la préface de son fameux opuscule sur la nature du nombre a vivement insisté sur cette idée de l'autonomie de l'arithmétique à l'égard du réel. Le nombre est 'une émanation immédiate des lois pures de la pensée' et 'entièrement

> indépendant des concepts de temps et d'espace'; les nombres sont 'des créations libres de l'esprit humain, ils servent de moyen pour saisir plus aisément et avec plus de précision la diversité des choses' (Was sind und was sollen die Zahlen? 52 ed., Brunswick 1923, p.111 ...
> "Mais Locke, déjà, jugeait que 'le nombre est la plus simple et la plus universelle de toutes nos idées" (Essai Philosophique, 11, Ch. XVI, no. 1), et Hume considérait la géométrie comme moins assurée que l'arithmétique et l'algèbre au point de vue de la valeur apodictique de ses affirmations. (Psychologie, tr. Renouvier et Pillon, Paris 1878, p.98).

Similarly within the general scientific framework which leaves all matter out of consideration, Thomists distinguish three specifically distinct sciences: metaphysics, logic, and supernatural theology, and once again the distinction is based upon different modes of immateriality. Supernatural theology is distinguished from the other two in that it enjoys the highest grade of immateriality that any speculative science can have — that provided by the light of revelation. Logic is distinguished from metaphysics in that its abstraction is purely negative, that is to say, since the object of logic is not anything real, it has only a negative immateriality.

Thus far all Thomists are in agreement. But when the question is raised about the possibility of a