And the same holds true of all the fundamental propositions of the philosophy of nature. (69)

This brings us to the consideration of an objection that has frequently been brought to bear against the view we have been upholding in relation to the question of philosophy and science. It has been formulated by Professor Alexander in the following terms:

> Mr. Adler defines philosophy as a body of logical conclusions drawn from common sense observations, and science as a body of conclusions drawn from specific observations obtained by specific investigative methods. I agree with Mr. Adler's definition of science but not with his definition of philosophy. Mr. Adler reduces philosophy to reasoning about inadequate (common sense) observations, science representing at the same time reasoning about more adequate observations obtained by refined and improved methods of investigation. And yet, in order to save the medieval hegemony of philosophy, with a peculiar twist of reasoning, Mr. Adler tries to subordinate science - - that is to say conclusions drawn from improved observations - - to philosophy, which according to his own definition consists of conclusions from inadequate observations. If Adler's definition of philosophy is correct philosophy should be discarded in the proportion to which scientific knowledge progresses by the use of steadily improving special techniques of investigation. With this definition Adler himself speaks the death sentence of philosophy. (70)

Let us suppose that the term "philosophy" here refers to what Thomists understand by philosophy of nature, and that the expression "common sense observations" means the simple, ordinary observation that is the point of departure of the

first speculations of the mind about nature. It may readily be admitted that this common observation is completely inadequate for the solution of specific problems. But no one has ever claimed that it is adequate for such a purpose. Our position is that common observation is adequate for common, generic problems, and that only highly specialized observation is adequate for specific questions. The common observation from which is derived the generic notion of motion is completely inadequate for the solution of a very special problem concerning the respiratory tubes of a certain species of animal, let us say. But at the same time knowledge of the exact kind of motion found in a particular type of respiratory tubes is wholly unnecessary for a determination of the generic nature of motion.

Doctor Alexander's objection with regard to the subordination of the experimental sciences to philosophy recalls what was said in Chapter II in connection with our analysis of the twenty fifth lectio of St. Thomas' Commentary on the Posterior Analytics. This subordination does not mean subalternation in the strict sense of the word. From this point of view the experimental sciences are completely independent of philosophy. It can only mean a subordination arising from an order in which one moves from the more generic to the more specific, that is to say a dependence of the more particular upon the more general. We feel that enough has already been said to make it clear that this dependence does not mean that the more general knowledge acquired in the philosophy of nature predetermines the solution of the more particular problems of the experimental sciences. Nevertheless, the anterior parts of natural doctrine have a definite influence upon the posterior parts. For the definitions arrived at in the philosophy of nature become methodological principles to guide the construction of hypotheses in the experimental sciences, to impose limits upon them, and to serve as criteria by which they may be criticised. Thus, for example, the definition of intellect in the De Anima becomes a methodological principle for experimental psychology. This role of philosophy of nature is not a restriction upon the experimental sciences. Rather it frees them from becoming enmeshed in false and useless hypotheses.

This discussion of the subordination of the experimental sciences to philosophy of nature suggests an important question: is it necessary, or at least helpful for experimental scientists to be acquainted with philosophy of nature. We know of no better answer to this

question than the one found in the following passage of Professor DeKoninck:

> N'est-il pas vrai que les meilleurs physiciens modernes ignorent à peu près le tout des questions étudiées dans les premières parties de la philosophie de la nature? Seraient-ils meilleurs physiciens s'ils savaient la définition du mouvement, ou que la comparaison de mouvements d'espèce différente suppose une prédication d'identité et un mouvement dialectique de la pensée? A cela on peut répondre par la question: Le maçon serait-il meilleur maçon s'il était architecte? Les ouvrages des savants modernes sur les aspects 'plus philosophiques' de leur science, montrent suffisamment les désastres du maçon qui veut faire l'architecte en tant que maçon. Ils font violence à l'ordre qu'il nous faut suivre dans la connaissance si nous voulons en arriver à voir la partie dans son ordre au tout. Ils ont négligé les considérations logiquement antérieures à celle de leur propre sujet, négligence qui se fait sentir quand ils veulent sortir de celui-ci. Faire violence à l'ordre, ne fût-ce qu'à celui qui nous est imposé par la nature même de l'intelligence humaine, c'est faire violence à la sagesse, à la science de la nature en tant qu'elle est philosophique."(71)

The greatest mistake of the modern students of nature is that they have insisted on starting in midstream. The most fundamental and most basic questions have been ignored. Having started midway, and pursuing their progress into deeper concretion, they have thought that they could ultimately find the solution of the fundamental questions. But the progress of the study of nature does not move in a circle; it moves in a straight line. And one has only to consider the answers that scientists have brought forward to such fundamental questions as: "what is life", to be convinced of this. Because the simple basic questions have been ignored, modern text books are filled with phrases and expressions which are utterly devoid of any definite meaning. They have much to say, for example, about "animal behavior" without ever having raised or solved the simple question: what is an animal in general. And all this brings home to us once again the utter futility of the efforts of modern scholastics to prove or disprove the doctrine of hylemorphism by means of chemistry and physics. The substantial composition of mobile being is a fundamental question that is anterior to, and therefore independent of all of the findings of modern experimental science.

The experimental sciences are, then, dependent in some way upon philosophy of nature. But from another point of view we may say that philosophy of nature is subordinated to the experimental sciences. For that which is less knowable *in se* is by nature subordinated to that which is more knowable *in se*. In other words the more abstract parts of natural doctrine are subordinated to the more concrete parts as potency is subordinated to act. In the concrete parts the abstract parts find their fulfillment. That is why the true philosopher of nature can never rest satisfied with the common,

general truths about nature, in spite of the fact that they alone provide him with scientific certitude. Such truths constitute only an introduction to the study of nature and are consequently completely orientated towards the more concrete parts which follow. The true philosopher of nature will never lose sight of that orientation, and he will be carried across the frontiers into the realm of experimental science. In so doing he will not be guilty of a naive optimism, or of a kind of "imperalism"; he will simply be obedient to the impetus of the dynamism that is intrinsic to the very study of nature. For the end towards which all the experimental sciences strive is at the same time the end towards which the philosophy of nature strives. And here we are touching upon the profound wisdom contained in the two texts from the *De Partibus Animalium* which seemed at first sight to constitute a paradox. On the one hand, the concrete parts of natural doctrine are distinguished from the more abstract parts by the fact that the latter are philosophical, that is to say truly scientific. But at the same time the philosopher of nature must study the concrete parts as well as the abstract parts, since the latter are a prolongation and a necessary fulfillment of the former. The following lines of Sir Arthur Eddington are relevant here:

> Not so very long ago the subject now called physics was known as 'natural philosophy'. The physicist is by origin a philosopher who has specialized in a particular direction. But he is not the only victim of specialization. By the breaking away of physics the main body of philosophy suffered an amputation.(72)

Perhaps we can sum up this discussion of the relation between philosophy of nature and the experimental sciences by drawing the following contrast between them. The former is of greater intrinsic importance than the latter for three reasons. First it provides us with the knowledge of nature that is most in conformity with the human intellect. It is significant that in modern times the mind in its dealings with nature has almost universally rejected the object that is most proportionate to it . But perhaps one might be tempted to object that experience shows that the experimental sciences are more easily accessible to a greater number than philosophy of nature. The answer to this objection has already been suggested earlier in this Chapter. In speaking of the relative "knowability" of the different parts of natural doctrine we have in mind only intellectual knowledge. In the measure in which sense knowledge enters into the discussion, it is evident that concrete singular sensible objects are the most easily

knowable. And in so far as the experimental sciences enjoy a close proximity to sensible singular objects they possess a facility that is not found in philosophy of nature. It must be noted, however, that in the measure in which physics is mathematicized it participates in the science that is the most proportionate to the human mind. We believe that these two facts explain the comparative accessibility of physics and the extreme attraction which it exercizes over the mind.

Secondly, the philosophy of nature provides us with truly scientific knowledge. St. Thomas writes:

> Illi qui sciunt causam et propter quid, scientiores sunt et sapientiores illis qui ignorant causam, sed solum sciunt quia. Experti autem sciunt quia, sed nesciunt propter quid. (73)

It remains possible to have scientific certitude as long as the mind remains in generalities. That is why the wiseman in the realm of nature must be humble. To reject certitude in these things is a kind of pride. Thirdly, the philosophy of nature has as its object the most noble thing existing in nature, the focal point of the whole of material creation - the spiritual soul of man.

On the other hand, the experimental sciences are more important than philosophy of nature in the sense that they come closer to the realization of the goal of the whole study of nature - - the knowledge of things in their proper causes. (74) From this point of view they provide, as we noted in Chapter II, a type of knowledge that is closer to the knowledge that God has of the Cosmos than the knowledge found in philosophy of nature.

5. The Interrogation of Nature.

We have seen that nature may be defined in terms of a ratio indita rebus. It is this intelligence, this logos realized in material things that makes the science of the cosmos possible. And the goal of this science is to capture this ratio in some partial way at least, to bring into contact with the ratio of man. We have seen that this becomes increasingly difficult as experience carries the mind forward into deeper concretion. Nature appears less and less rational, less and less homogeneous with the intellect. It continually throws up greater obstacles to the mind's attempt to disengage the objective logos from the materiality in which it is concretized. And there ultimately remains only one thing for the mind to do if it is to continue its task: to impose upon nature the rationality which it lacks, to extract the

objective logos of the cosmos by injecting its own subjective logos into it. This process of rationalization eventually terminates in the mathematization of nature, in which the most irrational of all the speculative sciences become subalternated to the most rational. The intellect finds, for example, that the visual line is not rational enough for it, so it substitutes the mathematical line. But even prior to the introduction of mathematics an extensive process of rationalization takes place. We must now try to analyze this process.

In the first place, it is important to recall that experimental knowledge is essentially imperfect, for it implies physical passivity. To have an experience means to become subject to something, and in the case of sense experience is always a question of becoming entitatively subject to material things which physically affect the sense organs. That is why man cannot be satisfied with purely experiential knowledge. By the very fact that knowledge is vital it is opposed to passivity, and by the fact that it is intentional it is opposed to the purely physical. (75) That is why the mind is impelled to go beyond experience, to anticipate it by searching for the reason of what is presented in experience. The more the science of nature approaches concretion the more experience gets the upper hand, so to speak. The intellect cannot accept this state of affairs. It must try to rationalize experience and thus get the upper hand itself. For the intellect can never rest in pure givenness; it has, as Meyerson says, "une répugnance irrémédiable ... devant tout *donné*."[78] It cannot be content with a mere *quia*; it must search for the *propter quid*. It cannot remain imprisoned within singularity; it must strive to achieve universality. It cannot rest satisfied with purely synthetic judgments; it must find a way of making them *a priori*. And when nature does not provide what it seeks, it will reconstruct nature in such a way as to make it render what it wants, or at least in such a way as to allow the mind to give itself what it wants. All this explains why as soon as the propositions of the study of nature start to be purely experimental there begins a gigantic task of reconstruction of nature. And the greater the part that experience plays in this study, the greater must be the part that the mind plays. Science becomes a mixture of fact and fiction, and as fact increases so does fiction. As Duhem has remarked: "Le développement de la Physique provoque une lutte continuelle entre 'la nature qui ne se lasse pas de fournir' et la raison qui ne veut

pas 'se lasser de concevoir.'"[77] We must now try to point out the most salient features of this rationalization of experience.

This is far from being an easy task. For not only do the objective and subjective logos ultimately become so inextricably fused that it is impossible to draw the line between them, but it is also impossible to find an absolute starting point for the introduction of the subjective logos, since the whole process is essentially circular. It might be suggested that the first step in the rationalization of experience consists in this that at the beginning of a scientific experiment the scientist makes a selection of the elements that are to enter into the experiment and places them in especially chosen conditions in such a way that the whole experiment is an artificially constructed process. It might further be suggested that the second step consists in an intellectual filtration and purification of the elements entering into the experiment in such a way that they become idealizations which have no exact counterparts in experience. There can be no doubt that experimental science deals with idealized entities of this kind, such as perfect gases movement without friction, absolutely rigid bodies, perfect levers, perfectly geometrical cyrstals, absolutely pure metals, perfect fluidity, perfect elasticity, etc. (78) And all this represents a projection of thought into the cosmos. But the nature of this projection must be rightly understood. For at first glance it might seem that all that is involved here is the substitution of limiting cases for the brute phenomena that are directly perceptible. If this were true, we could, as Cassirer has pointed out, "attempt to do justice to this method by a simple extension of the positivistic schema." (79) As a matter of fact, however, the problem is much more complicated than that. And an attempt to unravel it will immediately show that in the process of rationalization there is a good deal prior to the steps mentioned a moment ago.

This brings us to the central point of our present discussion. And we know of no better way of coming to grips with it than by considering a passage from Kant's Critique of Pure Reason: (80)

> Mathematics and physics are two types of theoretical knowledge which must determine a priori their object; the first in an absolute way; the second at least in part, and to the extent to which the other sources of knowledge besides the reason allow it to do so.

After attempting to show that mathematics is a completely a priori science and that it has made true progress only since mathematicians have come to realize this, he goes on to consider the a priori character of physics:

> When Galileo rolled balls down an inclined plane with an acceleration determined and chosen by himself, when Torricelli attributed to the air a weight which he computed as equal to the weight of a known column of water, or when later Stahl transformed metals into lime, and the latter in turn into a metal, by separating and adding certain elements, then a new light dawned for all physicists. They understood that reason discovers only what it produces itself according to its own designs; it must take the lead with principles which determine its judgments according to constant laws, and force nature to respond to its questions, instead of leaving itself be conducted by nature as though by a string: for otherwise our observations made at random and without any plan traced beforehand would never lead to a necessary law, which the reason nevertheless looks for and demands. The reason must present itself before nature, holding in one hand its principles which alone are able to give the concordant phenomena the authority of law, and in the other hand it must hold the experiment such as it has planned according to the same principles. Reason demands to be informed not as a school boy, who is bound to speak only what pleases the teacher, but as a judge on his bench, who constrains the witnesses to answer the questions put to them. Physics, therefore, is indebted to the happy revolution which has been introduced into its method by this simple notion that it must seek for (and not imagine) in nature, in accordance with the ideas which the reason itself brings to it, what the reason ought to learn of nature, about which it can never learn anything simply by itself. It is thus that physics has been able to enter for the first time upon the sure road of science, after groping along for so many centuries.

The gist of this passage may be summed up by saying that according to Kant experimental physics owes its emancipation and its progress to the fact that it proceeds to a certain extent in an a priori fashion by posing questions which anticipate experience and predetermine it.

This doctrine has in recent times been applied to biology by an ardent disciple of Kant, J. von Uexkull:

> Natural science falls into two parts, doctrine and research. The doctrine consists of dogmatic assertions, which contain a definite statement concerning Nature. The forms these assertions take often suggest that they are based on the authority of Nature herself. This is a mistake, for Nature imparts no doctrines: She merely exhibits changes in her phenomena. We may so employ these changes that they appear as answers to our questions. If we are to get a right understanding of the position of science vis-a-vis of Nature, we must transform each of the statements into a question, and account to ourselves for the changes in natural phenomena which men of science have used for evidence for their answer. Investigation cannot proceed otherwise than by making a supposition (hypothesis) in its questions, a supposition in which the answer (thesis) is already implicit. The ultimate recognition of the answer and the setting up of a doctrine follow as soon as the investigator has discovered in Nature what he considers a sufficient number of phenomena that he can interpret as positive or negative on the lines of this hypothesis.
> The sole authority for a doctrine is not Nature, but the investigator, who has himself answered his own question. (81)

We do not subscribe to all of the implications of the doctrine found in these two passages. Nevertheless, we believe that the central idea running through them is essentially correct. Kant was right in holding that if

experimental science is to have any significance it cannot rest satisfied with the purely synthetic character of experimental propositions. The mind must introduce an a priori element into them. And this introduction does not take place only after the process of experimentation has been accomplished. It is something that is effected during the process itself. The mind must anticipate experience and by this anticipation predetermine the experimental process. Kant was wrong in believing that Newtonian physics was definitive, and that as a consequence the a priori element introduced by the mind was something absolute and necessary. (82) Let us examine each of these two points in turn.

We have already suggested that modern science is far from being an outgrowth of the naive empiricism of Francis Bacon whose ideal it was to have experimentation carried on without any preconceived ideas. In this connection Poincaré writes:

> On dit souvent qu'il faut expérimenter sans idée préconçue. Cela n'est pas possible: non seulement ce serait rendre toute expérience stérile, mais on le voudrait qu'on ne le pourrait pas. Chacun porte en soi sa conception du monde dont il ne peut se défaire si aisément. (83)

Perhaps the first author in modern times to bring out with great clarity and emphasis the importance of preconceived ideas in scientific experimentation was Claude Bernard. In his classic work, Introduction à l'étude de la médecine expérimentale, he says:

> Il n'est pas possible d'instituer une expérience sans une idée préconçue; instituer une expérience... c'est poser une question; on ne conçoit jamais une question sans l'idée qui sollicite une réponse. Je considère donc, en principe absolu, que l'expérience doit toujours être instituée en vue d'une idée préconçue, peu importe que cette idée soit plus ou moins vague, plus ou moins bien définie...(C'est) l'idée qui constitue... le point de départ ou le primum movens de tout raisonnement scientifique, et c'est elle qui en est généralement le but, dans l'aspiration de l'esprit vers l'inconnu... Sans cela on ne pourrait qu'entasser des observations stériles. (84)

This opinion of Claude Bernard has become universally accepted among the best modern scientists and philosophers of science. Innumerable authorities besides the ones already cited could be brought forward to attest to this universal acceptance. (85) It has become increasingly clear that, as Meyerson says, " toute expérience n'est et ne peut être qu'une expérience de pensée." (86) And these authorities are unanimous in attributing the whole fecundity of experimental science to the projection of an a priori idea into experimentation. Without this projection experimentation could render only pure data without any unified significance. And these data could lead to nothing

beyond themselves. They would be utterly sterile, unable to carry the mind forward in any definite direction. It is from the a priori idea that science derives its essential dynamism. (87)

But it is important to see in what precise way this projection of the a priori into experimentation is effected. The texts cited above have already suggested that it is brought about essentially by the way in which the experimenter interrogates nature. Every experiment is in fact a very definite question which the experimenter puts to nature. And the results of the experiment have no meaning except in so far as they are the answer to this definite question. That is why these results are already predetermined by the experimenter. The whole pattern of the experiment, the selection of the elements that are to enter into it, the structure of the instruments that are to be employed, the precise character of every action that carries the experiment forward — all these are predetermined by the precise question that is in the mind of the experimenter. And this question has no meaning in relation to the very complicated theoretical background which forms its context. Max Planck has brought out this point with his usual clarity:

> Therefore from the results that are given by experi-

mental measurement we must choose those which will have a practical bearing on the object of our inquiry, because each particular attempt at discovering reality in the physical universe represents a special form of a certain question which we put to nature. Now you cannot put a reasonable question unless you have a reasonable theory in the light of which it is asked. In other words, one must have some sort of theoretical hypothesis in one's mind and one must put it to the test of research measurements. This is why it often happens that a certain line of research has a meaning in the light of one theory but not in that of another. And very often the significance of a question changes when the theory in the light of which it is asked has already changed.(88)

But it is necessary to try and analyze more accurately the character of the questions that it is possible to put to nature in experimental sciences. There are in fact two conceivable ways in which a question may be posed. In the first place it is possible to ask a question which demands in an absolute fashion what the nature of a thing is, for example: "what is man?" Such a question can never be answered by either "yes" or "no". The answer must be "rational animal" or "featherless biped" or something similar. And the reason is that such a question does not contain an hypothesis. But there is another type of question which does contain an hypothesis, for example: "is the definition of man: featherless biped?" In this

case the hypothesis involved constitutes a suggestion to which one is forced to answer by either "yes" or "no". This suggestion is already in some sense a predetermination of the answer. And it is clear that in posing a question of this second type the mind is taking the initiative and anticipating nature.

Now it is only questions containing an implicit hypothesis that are used in experimental science. As Meyerson has remarked, "il est parfaitment impossible d'arracher à la nature ses secrets en l'interrogeant directement."(89) And because it becomes increasingly difficult to induce nature to yield up its secrets as progress is made towards fuller concretion, it is necessary that the questions posed by the scientist become increasingly artificial and hypothetical. Scientific method has often been compared to the methods employed in tracking down criminals. Now the criminal which is nature will never answer a direct question. And as a result the scientific detective never succeeds in pinning this criminal down in an absolute and definitive fashion. For there is this difference between nature and ordinary criminals that when for former answers "yes" it does not necessarily mean "yes" in an absolute way. That is to say, when the hypotheses of the scientist's question is verified in experience, this does not mean that the hypothesis is necessarily true - "quia forte secundum aliquem alium modum apparentia salvantur". It does not follow from this, however, that von Uexkull is completely correct in maintaining that "the sole authority for a doctrine is not nature, but the investigator, who has himself answered his own question." For though it be true that nature's answers are to some extent predetermined by the questions formulated by the investigator, they are not completely determined thereby. It cannot be denied that nature has something to do with the answer, and that throughout the whole dialectical process of interrogation it remains the measure to which the scientist must ever seek to conform himself.

Even among those who readily admit that hypothesis plays a major role in experimental science the notion is often current that hypothesis is always something posterior to experimentation and merely superimposed upon it, in such a way that it remains a comparatively easy task to distinguish the factual elements deriving from experience from the hypothetical elements contributed by the mind. We feel that enough has already been said to show that this is false. Hypothesis must anticipate experience and pre-

determine it. And this predetermination is such that, in the more complicated experimental processes at least, it is impossible to distinguish sharply between the subjective and objective logos. The analysis which is to follow will serve to bring out this truth with greater evidence.

6. Operationalism

In order to come to understand more fully the way in which the subjective logos is projected into nature in the procedure of experimental science, it is necessary to examine closely the precise character of a scientific experiment.[90] During the reign of classical physics, it was generally believed that a scientific experiment was essentially a revelation of a property that existed as such in objective reality. It was taken for granted that the whole experimental procedure was merely a means by which the scientist was able to disengage a definite feature that was embedded in the absolute world condition. Contemporary physics has shown how naive this view was. In fact, we are touching here the very heart of the profound difference between Newtonian physics and Relativity and Quantum physics.

We have already laid considerable insistence upon the purely experimental character of the definitions that form the structure of experimental science. We have seen that experimental science never really succeeds in disengaging an essence, that it never really rises above the realm of singularity. As a consequence, the definitions of experimental science are merely formulations of what is presented by sense experience. All this is true even of propositions which derive from ordinary observation, that is to say, observation into which no element of control or artificial construction has been introduced.

But the true well spring of science, and particularly of physics, is not this ordinary observation. By the very fact that the scientist is unable to really disengage essences from it and thus rise to true universality and necessity, it appears as a frustration to the mind. For this reason the student of nature cannot rest satisfied with it. If nature will not yield up its secrets of its own accord, it must be forced to do so. That is why he finds it necessary to operate upon nature, to bring it under his guidance and control, to manipulate it in ways dictated by his preconceived ideas. All this is known as a scientific experiment.

An experiment has often been defined as controlled sense perception. But it should be clear from what has just been said that it is a good deal more than that. It is, in fact, a reconstruction of nature. Because the routes provided by nature are not sufficient to enable the scientist to arrive at his goal, it is necessary for him to construct an artificial detour. This detour carries him closer to his goal than he would have been able to get without it, but it does not do so in the way conceived by the classical physicists. For the detour is inseparable from the goal. And this brings us to an extremely significant paradox to which we shall return more than once in this study: scientific method carries us closer to nature only at the expense of carrying us farther away from it. (91)

And what happens to the scientific definitions in this process? The reconstruction of nature effected by the scientist enables the mind to penetrate more deeply into meaning, but this penetration never arrives at a point at which the mind is able to rise above purely experimental propositions which are of the very essence of experimental science. In fact, as we have just suggested, from one point of view the very reconstruction makes it even less possible to escape from them. The mind remains bound down to experience, bound down to a mere formulation of what is presented by experience. But now what is presented by experience has become something different. It is no longer something produced by nature, but rather something produced by the scientist himself in his operation upon nature. That is why the results of experiments have no meaning except in terms of the precise operations by which they are produced. They depend upon every element which enters into the experiment: upon what he does, the way in which he does it, all the concrete circumstances in which he operates, etc. And because it is impossible for him to know exactly what he is doing and all the circumstances of the operation, he is never able to rise above the sensible individual operation except by means of provisional and dialectical generalizations. All this amounts to saying that the definitions of experimental science derive their significance from the series of operations employed in the experiments which led to their formulation. That is to say, the only way to define physical quantities is by an enumeration of all the concrete operations by which these physical quantities have come to be known. And every attempt to analyze the meaning of the definitions of experimental science must necessarily end in the mere designation of a concrete series of operations performed with a concrete

(92)

set of instruments. There must be a <u>reductio ad materiam sensibilem individualem</u>. The more experimental science attempts to achieve the natural desire of the intellect to rise above the senses and the pure givenness of experience, the more is it obliged to fall back upon them.

In order to be convinced that all the definitions with which physical science deals are essentially operational one has only to open a book of physics and read the definitions of the fundamental quantities which constitute the science. Mass, force, temperature, electricity, magnetism, light, sound, energy, entropy, atomic and molecular properties, etc. -- all without exception are defined in terms of definite physical operations performed with definite physical instruments. And we must be constantly on guard against the natural tendency to hypostatize terms which designate no more than experimental processes. The way in which scientific progress forces physics to introduce progressive modifications into the definitions of its fundamental quantities should be a constant warning that these quantities are not real, ontological properties.

As we have suggested, the realization of the operational character of the definitions of experimental

> that that distance x is something already existing in the picture of the world - - a gulf which would be apprehended by a superior intelligence as existing in itself without reference to the notion of operations with measuring rods...
> Having regard to this distinction between physical quantities and world-conditions, we shall not define a physical quantity as though it were a feature in the world-picture which had to be sought out. A physical quantity is defined by the series of operations and calculations of which it is the result ...
> We do not need to ask the physicist what conception he attaches to 'length', we watch him measuring length, and form our definition according to the operations he performs.(95)

The epistemological implications of this principle of operationalism are far reaching. They may, perhaps, be summed up by saying that the physicist is never confronted with a pure object. The fundamental quantities, such as length, mass, energy, potential, etc. out of which the whole structure of physics is erected are not things or natures or properties or features of the absolute world condition. They are articles manufactured by the subject. They are synthetic products. They are not things of nature, but things fabricated in order to explain nature. As Professor Petit has remarked, "le faire est au coeur du connaître expérimental."(96) In other words, in the experimental sciences, speculative knowledge can reach out towards its object only by giving way in some measure to

practical knowledge.

All this, however, does not favor the idealistic position. For the operations which constitute a scientific experiment are physical, and they are performed upon objective physical nature. As a consequence, the results, while not purely objective, are not purely subjective. They are a composite of the objective and the subjective. But it is extremely important to recognize the part played by the subjective element. As we shall have occasion to point out in a future Chapter, it is only by acknowledging the role of the subjective in experimental science that we can become truly objective.

It should be clear from what has been said thus far about operational character of experimentation that the subjective enters into science in two ways. In the first place there is a mental intrusion through hypothesis and theory in the sense that all of the operations and the whole structure of the instruments employed are determined by some preconceived theory. Instruments are in fact nothing but materialized theories. This point has been developed in the last section of this Chapter. In the second place there is a physical intrusion in the sense that the subject operates physically upon nature through physical operations carried on by physical instruments constructed of copper, and glass, and aluminum and silk, etc. This obviously results in a physical interaction between the object and the subject, which makes it impossible for the subject to get at the object in its pure state of objectivity. We intend to return to this question in our discussion of the limitations of measurement in Chapter VIII, but perhaps at this point it will be worth while to quote the following lines from Heisenberg, who has done so much to bring out the significance of this interaction:

> Particularly characteristic of the discussions to follow is the interaction between observer and object; in classical physical theories it has always been assumed either that this interaction is negligibly small, or else that its effect can be eliminated from the result by calculations based on 'control' experiments. This assumption is not permissible in atomic physics; the interaction between observer and object causes uncontrollable and large changes in the system being observed, because of the discontinuous changes characteristic of atomic processes. The immediate consequence of this circumstance is that in general every experiment performed to determine some numerical quantity renders the knowledge of others illusory, since the uncontrollable perturbation of the observed system alters the values of previously determined quantities. If this perturbation be followed in its quantitative details, it appears that in many cases it is impossible to obtain an exact determination of the simultaneous values of two variables, but

> rather that there is a lower limit to the accuracy with which they can be known.(97)

Until rather recently it was customary to contrast the method of introspection employed in experimental psychology with the methods used in the other experimental sciences by pointing out that in the case of introspection the intrusion of the subject makes it impossible to arrive at the object in its pure state of objectivity. And it was more or less taken for granted that this pure objectivity was attained in the other experimental sciences. Niels Bohr, however, has shown that this pure objectivity is a mere illusion and that throughout physics there is an intrusion of the subject comparable to that found in the method of introspection. One of the reasons why scientists become easily susceptible to this illusion is that, as Duhem has brought out so fully and so accurately,(98) they tend to substitute in their mind an idealized instrument, a kind of mathematical model for the actual physical instrument employed. For a copper wire of a certain breadth, for example, is substituted a geometrical circle without breadth; for a steel magnetic needle which has a definite magnitude and which is unable to move without friction is substituted an infinitely small horizontal magnetic axis which moves around a vertical axis without friction, etc. In fact there is a tendency to go even beyond this; to dematerialize the instrument completely, to attribute to it the properties of a transsubjective cognitive faculty. And the reason for all this is clear: it pertains to the nature of the intellect qua intellect to know things independently of physical means.

Perhaps the most significant conclusion that can be drawn from this discussion of operationalism is that irrationality enters into experimental science in a way in which it does not enter into any other science. It is true that irrational elements enter into all the sciences in one way or another, but in all the other sciences these elements remain extrinsic to the formality of the concepts that are proper to these sciences. But because the very notions out of which experimental science is constructed remain inseparable from the physical, material operations by which they are formed, that is to say, because a mere series of physical operations plays the role that essences play in philosophical knowledge, there is a profound element of irrationality intrinsic to these notions. And it is all too easy to lose sight of this fact simply because of the operational clarity that these notions possess.

7. Laws and Theories.

But science is not made up merely of isolated notions. It is a highly coordinated and unified system. And this coordination and unification is brought about chiefly through the formulation of laws and theories. To this formulation we must now turn our attention. Since we shall have to return to this question later when we come to consider the mathematical transformation of physical science, we shall content ourselves here with a brief outline of the structure of physical laws and theories and with a summary discussion of their epistemological significance, in such a way that the central thought we have been pursuing, namely the projection of the subjective logos into nature, will be rounded out and fully crystallized.

Unity is a condition of intelligibility, for pure diversity is essentially irrational.(99) That is why the mind in its efforst to rationalize nature cannot rest content with a mere collection or tabulation of phenomena. As we shall see in Chapter VIII, the process of measurement in physical experiment is already a unification, for measurement consists essentially in reducing a multiplicity to the unity of a standard. But this initial unification is not sufficient to satisfy the minds desire for rationality. It has an innate aspiration to approach as closely as possible to the higher forms of intellect which grasp an increasing plurality of things in a diminishing plurality of species. It instinctively tends to rise to a higher unity by establishing definite relations between the multiplicity of events which reveal themselves in experiment. That is why the development of science manifests two paradoxical tendencies. For on the one hand, we have seen that the movement towards concretion is a movement towards greater multiplicity, since it approaches things in their proper specific nature. This is a tendency towards a pluralistic universe. On the other hand, the mind instinctively seeks to reduce thus multiplicity to an ever more perfect unity, and the terminus of this movement is a completely monistic universe. The amazing thing is that these two contrary movements, far from being irreconcilable, are actually cooperative. (100) The early part of this Chapter was devoted to a consideration of the movement towards pluralism. Now, before bringing this Chapter to a close we must discuss the tendency towards monism. This tendency is carried forward principally by means of laws and theories.

Now nature lends itself admirably to this tendency of the mind. For the events which present themselves in experience are not mere desperate phenomena. They reveal

themselves as belonging to a pattern. For nature is defined precisely in terms of those things which happen, "ut in pluribus." (101) This natural order and regularity makes it possible for the mind to establish legality among phenomena, and this is the first step in the movement of the mind towards a more perfect unification than that found in the reduction of phenomena to a standard.

But are physical laws a mere reflection of the order and regularity of nature? Classical physicists seem to have been persuaded that they are. All the best modern epistemologists, however, are agreed that this is very far from being the case. And we feel that enough has already been said to show why this is so.

For in the first place, it is clear from our discussion of the nature of the propositions of experimental science that the universality and necessity which are found in physical laws, and which are of the very essence of all law, can be nothing but a gift of mind to nature. Nor is this gift gratuitous. The mind bestows it only that it may be carried nearer to the goal towards which it is striving. That is why physical laws are essentially functional. That is why they must not be looked upon as something fixed and static, as a finished reflection of an absolute order existing in nature.

But there is much more to the case than all that. For, as we have just seen, the quantities which form the stuff out of which physical laws are formulated are not objective entities. They are articles manufactured by the subject in his operations upon nature.[102] Into this manufacture has gone both hypotheses and physical action. That is why the resultant laws have no meaning except in terms of the projection of subjective logos that all this entails. Moreover, in the highly complex structure that is physical science, laws do not have a completely independent and absolute meaning in their own right. Their meaning is derived from their context, which is a closely woven pattern of mutually interdependent laws and theories. In this connection, Professor Campbell writes: "Nous remarquons d'abord que les termes ne sont pas habituellement des jugements simples et immédiats sur les sensations, mais des collections complexes de tels jugements. Dans la plupart des lois, ces collections sont telles que les lois ne sont vraies que si d'autres lois le sont. Elles en dépendent à la fois pour leur sens et pour leur vérité. Ce caractère de dépendance mutuelle est très important pour

nos recherches."[103] The significance of laws also depends upon the particular theory into whose structure they are fitted, in such a way that if the theory changes the significance of the laws changes. Duhem writes: "selon que l'on adopte une théorie ou une autre, les mots même qui figurent dans l'énoncé d'une loi de physique changent de sens, en sorte que la loi peut être acceptée par un physicien qui admet telle théorie et rejetée par un autre physicien qui admet telle autre théorie."[104] The difference of meaning attached to the law of gravitation in Newtonian and in Einsteinian physics is a case in point.

It is evident, then, that there is a vast difference between the objective laws of reality and the laws of physical science. Eddington has brought out this difference in the following terms:

> We are in danger of falling into a confusion regarding laws of nature - - a confusion between what they are and what we originally intended them to be. To avoid ambiguity I will discriminate (temporarily) between 'laws of nature' and 'laws of Nature'. Law of Nature will have the meaning that the term was originally intended to bear - - a law emanating from the world-principle outside us, which we often personify as Nature. Law of nature will mean as heretofore a regularity which we have found in our observational knowledge, irrespective of its source. In short a law of nature is whatever would be designated by that name in current physical practice.

It will be seen that a law of Nature is a law of the objective universe. But all recognized laws of nature are subjective. We have thus reached the verbal paradox that no known law of nature is a law of Nature. Effectively the terms have become mutually exclusive.
It is true that we have left an opening. A law of Nature is a law of nature if it would be (not necessarily if it already is) accepted as such in physics. This brings me to a further question. Have we any reason to believe that if a law of Nature - - a generalization about the objective world - - were to become known to us, it would be accepted by current physics as a law of nature? I think it would only be accepted if it conformed to the pattern of physical law that we are accustomed to. But this pattern is the pattern of subjective law. We shall try later to show by epistemological study how the pattern has grown out of the subjective aspect of physical knowledge. The pattern is the very hall-mark of subjectivity. Any expectation we may have formed that the objective laws of Nature, when they are discovered, will conform to the same pattern is quite unreasonable.(105)

In order to be convinced that physical laws are ideal constructions of the mind it is sufficient to analyse any one of them accurately. This analysis will reveal the utter impossibility of their being realized as such in nature. And this is true of even the most fundamental laws which have come to be considered as the principles of the whole structure of physical science. The principle of inertia is a case in point.(106) The verification of this law in nature would involve a contradiction. For in order to show that a moving body preserves its rectilinear and uniform motion unless influenced by another body, it would

be necessary to have only one body in existence - - and then all motion would be impossible, since bodies can move only in relation to one another. Moreover the exact verification of the principle would demand that the volume of the body be reduced to zero.(107) It is important to note that laws of this kind become conventions which serve to define the very concepts which are involved in them, in such a way that it becomes impossible for experience not to conform to them. If a moving body were to fail to preserve its rectilinear and uniform motion a scientist would never conclude that the law of inertia had been violated, but rather that some secret influence of which he was ignorant was being exercised upon the moving body. In like manner, the law which formulates the functional relation between the length of a piece of metal and its temperature is transformed into the definition of coefficient of linear expansion; the law which states the dependence of the stress in an elastic body upon the strain is transformed into a definition of elastic constant. First the law is established that light travels in a straight line, and then the path of light becomes the definition and the norm of a straight line. That is why Le Roy could write: "Les lois sont invérifiables, à prendre les choses en toute rigueur . . ., parce qu'elles constituent le critère même auquel on juge les apparences et les méthodes qu'il faudrait utiliser pour les soumettre à un examen dont la précision soit susceptible de dépasser toute limite assignable." (108)

It is necessary to conclude, then, that physical laws are not found - - they are made. They do not exist before they are formulated by the mind. This does not mean that they are purely fictitious. They have a basis in reality in the sense that they are suggested by experience. The law of inertia, for example, was formulated only after it had been suggested in countless ways by nature. Moreover, the term of the process which constructs physical laws is always the true, objective laws of nature. And that is something which those who insist upon the subjective character of scientific laws usually forget. Nevertheless, it remains true that only a suggestion of these laws is actually found in reality. That is why there is something essentially Platonic about them. That is why Kant was in this respect correct in making the mind the lawgiver of nature. For scientific laws come from reality only materially; formally they are from the mind. The essence of scientific knowledge is made up of a kind of noetic hylemorphism in which the matter presented by reality is formalized by the mind. In all of the laws of experi-

mental science, as Eddington writes: "the mind has by its selective power fitted the processes of Nature into a frame of law or a pattern largely of its own choosing; and in the discovery of this system of law the mind may be regarded as regaining from Nature that which the mind has put into Nature." (109)

The establishment of legality among phenomena was for Comte the ultimate terminus of the scientific movement. But in this respect as in many others Comte failed to seize upon the true spirit that animates scientific endeavor. As Einstein and Infeld have pointed out, "la science n'est pas une collection de lois . . . Elle est une création de l'esprit humain au moyen d'idées et de concepts librement inventés. Les théories physiques essaient de former une image de la réalité et de la rattacher au vaste monde des impressions sensibles. Ainsi, nos constructions mentales se justifient seulement si, et de quelle façon, nos théories forment un tel lien." (110) Just as the mind's desire for rationality impells it to rise above the initial unification achieved in measurement to the higher unity of law which establishes a definite relation in the multiplicity of phenomena, so it likewise impells it to go further and arrive at a higher synthesis which establishes relations in the multiplicity of laws. This higher synthesis is achieved by means of a physical theory. The kinetic theory of gases, for example, makes it possible to synthetize the laws of Mariotte, of Gay-Lussac, and of Avogadro. By means of this principle of gravitation Newton was able to synthetize the laws arrived at by Kepler and Galileo and the laws governing the tides.

Without theory the movement of the scientific mind would be essentially frustrated. For the two essentially properties of science are universality and necessity. By means of laws the mind is able to rise above the singularity of phenomena and arrive at a kind of universality. But this universality is lacking in necessity. That is to say, even when laws have been formulated there is nothing intrinsic to them which shows that they could not have been otherwise. In other words, propositions which merely state an association between the values of one variable and the values of another variable are not logically necessary. For example, an increasing temperature is associated in a determined way with increasing volume but there is nothing in this law which shows that the reverse might not have been the case. The mind cannot rest satisfied with this contingency; it must strive to reduce

it to some kind of necessity by finding a reason which explains why increasing temperature is associated with increasing volume. This is accomplished by the construction of a theory which postulates the existence of unobserved entities whose hypothetical behavior will explain the observed phenomena. Thus physical theory becomes a substitute for the analytical character that the propositions of experimental science lack.

In other words, science, as we saw in Chapter II, is a knowledge of things in their causes arrived at by demonstration. But without theories experimental science is unable to discover the causes of the laws it formulates, nor can it deduce these laws. That is why it is only by having recourse to theories that the scientific mind can realize its ideal of rationalizing nature by making it deducible. We are touching here upon the central theme which runs through the works of Meyerson.(111) He has shown that the ultimate terminus towards which all science moves is the perfect rationalization of reality through deduction. The realization of this ideal would mean that the whole of nature could be deduced from one simple theory. And that would mean the destruction of nature, since it would involve the destruction of all heterogeneity. Thus the full realization of the ideal of science of nature would mean its complete destruction. And this is just another example of a phenomenon which has already been noted and to which more attention will be given later: experimental science tends towards a contradiction. The realization of its ideal will ever remain a mere dialectical limit, for nature will never fail to reveal irrational elements to prevent its perfect deductibility.

To say that science tends towards monism while it moves towards pluralism is to say that it tends towards universality while it moves towards specific concretion. But it is important to note that the universality towards which it tends is not the same kind of universality from which it is escaping by its movement towards specific concretion. For as we have already pointed out, this latter universality is a mere <u>universalitas in praedicando</u>, which in no way lends itself to deduction. What science seeks to achieve in its construction of theories is a universality which will permit deduction. And that is why it instinctively reaches out to mathematics whose principles are not only universal <u>in praedicando</u>, but also <u>in causando</u>. And this explains why Descartes' attempt at the global deduction of nature by means of mathematics

was much more intelligent than Hegel's attempt to arrive at the same goal by means of logical categories.

It is in the construction of theories that the mind finds the fullest scope for the projection of its subjective logos into nature. For to a far greater extent than in the case of laws, physical theories are not so much a gift of nature to the mind as a gift of mind to nature. They are fictitious constructions freely chosen by the subject.[112] It is true that these constructions must be made to conform with reality. Nevertheless, this conformity is not a logical proof of the objective truth of the theory concerned, for : ex falso quodlibet. In other words, one cannot conclude to the truth of a theory from its perfect and constant verification in reality without falling into the logical fallacy of affirming the consequent.[113]

It is true that deduction from a theory can lead to the experimental discovery of a fact. For example, the law of gravitation as conceived by the theory of Relativity led to the discovery of the fact that in the neighborhood of ponderable bodies a path of light undergoes considerable deviation. This fact is true, but the truth does not derive from the logical discourse which first suggested it. Rather, it derives formally from the experience by which it was actually discovered. And this brings us once again to the essential reason why experimental sciences are experimental; their truth is in experience only; the logical discourse is only an instrument, and even the conclusion of this discourse is only instrumental in the sense that it leads to or suggests an observation or experiment to be made. Consequently, hypotheses can be said to be "verified" only by extrinsic denomination. An infinity of theories can lead to the same conclusions. The laws of electrostatics, for example, can be "explained" successfully by a number of different theories, such as the theory of two electric fluids, or the theory of a single fluid, or the theory of discrete smallest charges, namely, electrons and protons. The corpuscular and undulatory theories of light, both of which have been successively "verified", are a classical example of the same thing.

The impression is fairly prevalent that physical theories are founded directly upon facts. This is, however, an inexact way of representing them. They are not founded directly upon experience, rather they seek to posit a point of departure from which experience may be arrived at, that

is to say, from which relations may be logically deduced which will be equivalent to those derived from experience.

It must not be thought, however, that theories may be constructed in a purely arbitrary fashion. There are certain criteria which must guide the mind in this construction. And the three most important of these criteria may be deduced from the foregoing analysis of the nature of physical theory. First, because every theory is an attempt to arrive at the most perfect unity possible, the one which has the greatest logical simplicity will be preferred to all others. Secondly, because every theory is an attempt to make nature deducible, the one which has the most perfect conformity with reality must be chosen. Thirdly, because the ideal of science is a merely dialectical limit towards which it must ever tend, that theory will be preferred which has the greatest fecundity, that is to say, which is most significantly suggestive of new experience. This last point means that a good theory is one which reaches beyond itself; if it does not give rise to new problems which it cannot adequately solve, it is not truly scientific. A good theory must not only solve problems; it must create them, for otherwise science will become static and sterile.(114) The new experiments suggested by a theory will at once increase the multiplicity of data and prepare for a higher synthetic unity, that is to say, for a more perfect theory.[115] That is why a good theory must contain the seed of its own destruction within its bosom. For a theory that explains everything explains nothing. Newton's theory was good, not only because it explained many things, but because it brought to light things which it was unable to explain. "Crises" are essential for the development of science, and if contradictions did not continually arise it would become stagnant.[116] But it is significant that no matter how many contradictions may arise in the face of one theory, it is not abandoned until another theory is ready to take its place. The mind will not descend from its plane of rationality. All this amounts to saying that experimental science develops through a constant interaction of objective and subjective logos, and it is this interaction that we must now attempt to analyze before bringing this Chapter to a close.

8. Objective and Subjective Logos.

If there is any conclusion which emerges from the preceding discussions it is that the evolution of science is essentially a creative evolution. The mind does not merely

discover nature; it constructs it to its own image and likeness. And it is only by so doing that it is able to discover it.[117] But because this construction is never free from its relation to discovery, it is not a pure creation, but a re-creation. The mind can progress in production only by becoming increasingly dependent upon induction; it can perfect its construction, only by perfecting the instruction it receives from nature.[118] It can advance only by keeping up an incessant dialogue with reality. It cannot reason without experimenting, nor can it experiment without reasoning. This is not, however, a circle without any definite direction. For the reasoning is always orientated towards reality,

In other words, experimental science must be at once synthetic and *a priori*. And it is only by maintaining a proper balance between these two elements that the extremes of idealism or empiricism can be avoided. All this may be summed up by saying that experimental science is a mixture of science and art, and for this reason it is neither a science nor an art in the full sense of the word. And there is perhaps no better way of getting at its nature as a quasi science than by analyzing the way in which art enters into it.

Rousselot is correct in maintaining that in the epistemology of St. Thomas the sciences in the modern sense of the term are rather arts than sciences.[119] And it is highly significant that as the science of that part of reality which, as we saw above, cannot be defined in a more profound way than as a work of art, tends towards its perfection, its nature is transformed in such a way that there is no more penetrating way of knowing it perhaps, than by viewing it as an art. There is, in fact, a remarkable parallel between the way in which art enters into nature, and the way in which it enters into the experimental sciences of nature. As we pointed out earlier in this Chapter, all created reality is a work of art, but nowhere does divine art penetrate so deeply into reality than in the material cosmos. In the same way, art enters into all the sciences if for no other reason than that they all employ logic, but in no science does it penetrate so deeply as in the experimental natural sciences. And it is extremely important to see why this is so.

Logic

Logic reaches further down into the structure of the sciences than might at first be supposed. It has to do even with the first operation of the mind. One might perhaps be tempted to doubt this statement. For logic has

to do with an ordering of thought, and since simple apprehension grasps things in an absolute fashion, it may be difficult to see how the mind can introduce order in relation to this first operation, as it does in the construction of propositions and syllogisms. Nevertheless, as John of St. Thomas says, "prima apprehensio absolute et per se pertinet ad logicam."[120] As is evident from The Categories of Aristotle, a certain distinguishing and ordering of terms is necessary prior to their construction into propositions. In this way, art surrounds the terms in all the sciences from the very beginning. But the vital point is that in all the other sciences besides the experimental sciences this art merely surrounds the terms -- it does not posit them. Only in the experimental sciences are the very terms themselves artefacta. The student of nature fabricates the very stuff out of which the whole universe of physical science is constructed. To use scholastic terminology, the objects are never a pure quod; they are always a mixture of a quod and a quo. The quod and the quo constitute an accidental unity and are considered ad modum unius.

This penetration of art into the very essence of experimental science is continued throughout its whole structure. As we saw in our discussion of laws and theories, the form of experimental science proceeds not only from the object, but also from the subject.[121] The philosophical sciences, are constructed by means of art, but this art remains a purely extrinsic tool. It does not become a part of the structure itself. That is why these sciences are sciences in their own right independently of the logic they employ. But in the experimental sciences the art employed becomes an essential part of the scientific structure. That is why they are not sciences in their own right independently of the dialectics they use. <u>They are dialectics.</u> This point will be clarified in the next Chapter when we come to analyze the relation between experimental science and dialectics.

Another way in which art penetrates into the very essence of physics is found in its subalternation to mathematics, which is at once a science and a speculative art. How deep this penetration is may be seen by considering the intimate union existing between subalternating and subalternated sciences. The mind, which finds it necessary to reconstruct nature, discovers great scope for its artistic impluse in the vast constructibility of mathematics. In this connection attention must be called to a significant

text in the <u>De Trinitate</u> in which St. Thomas says that logic, mathematics and mathematical physics "inter ceteras scientias artes dicuntur quia non solum habent cognitionem, sed opus aliquod, quod est immediate ipsius rationis, ut constructionem, syllogismum, et orationem formare, numerare, mensurare, melodias formare, cursus siderum computare."[122]

It is interesting and instructive to try to [illegible]ize the nature of the art which enters into experimental science. A moment's reflection will reveal the extreme complexity of its character. For, in the first place, it is at once both speculative and practical. In so far as it involves the use of dialectics and mathematics, it is speculative; in so far as it involves a physical operation performed upon nature, it is practical. In the second place, it has characteristics which are proper both to fine art and to useful art. The fine arts are essentially arts of imitation. But as St. Thomas points out,[123] an imitation is not a mere similitude, that is to say a materially exact copy. It is the expression of an original by an intellect, and this means that the original has passed through an intellect, and in passing has acquired something of the order and light that are proper to the intellect. And the purpose of all fine art, except religious art, is to make the original in some way better than it actually is. We believe that all this is true to some extent of experimental science. The physical universe constructed by the scientist is an imitation of the real world. It is not an exact copy or model of it. For the intellect has contributed much to this imitation. And in this imitation we make the world in some sense better than it really is. Our knowledge of material things is better than the things themselves; intelligence is the best thing in nature. The forms that are found in the mind are better than those found in reality.

But precisely because they are better they are worse. They are worse because experimental science is not a pure art but a science. That is why the whole purpose of these forms is to lead to the knowledge of the forms existing in nature. No matter how perfect the constructions of science may be, they are never anything more than mere scaffoldings. (124) That is to say, the art that is found in experimental science is purely functional, and from this point of view it is utilitarian. Scaffolds are to some extent an imitation of the building against which they are erected for they must take on some of its general out-

lines at least. Nevertheless their most fundamental aspect does not consist in this but rather in the fact that they are built in order to reach the house.

The medieval schoolmen made a further distinction in the arts - - the distinction between those which co-operate with nature and those which do not. In the latter case there is a projection into matter of a form which is independent of the natural form that is native to the matter. In the former case there is an extrinsic assistance brought to bear to enable the natural form to achieve its end more fully. It would seem that the art which enters into experimental sciences participates in both of these categories. For in so far as it is purely functional, in so far as its purpose is to induce nature to yield up its <u>logoi</u>, it is an art cooperating with nature. But in so far as the projection of the subjective <u>logoi</u> is not a purely extrinsic assistance, as is true, for example, of the use of logic in the sciences; in so far as this projection results in the construction of a physical universe that is in a sense distinct from the absolute world condition, it shares in some way in the second category.

A number of recent authors have insisted upon the fact that modern scientific progress has meant a gradual emancipation of science from the profound anthropomorphism that was characteristic of the views of nature current in past centuries. (125) And the truth of this can hardly be doubted. Yet if the foregoing discussion of the projection of the subjective logos into nature means anything at all, it must mean that from another point of view modern science is immeasurably more anthropomorphic than ancient science. For all art, as Bacon has remarked, is man added to nature. This is just another of the innumerable paradoxes that one constantly encounters in attempting to analyze the nature of experimental science: modern science is less anthropomorphic precisely because it is more anthropomorphic; in other words it is more objective precisely because it is more subjective. A specific example of this is found in the mathematization of nature. This mathematization is in a sense anthropomorphic for it consists in viewing nature in terms of the science that is most connatural to the human mind. And yet it is this mathematization that delivers us from the anthropomorphism which derives from the subjectivity of sense perceptions. Ernest Cassirer has brought out this paradox of modern science:

> Physical thought strives to determine and to express in pure objectivity merely the natural

> object, but thereby necessarily expresses itself, its own law and its own principle. Here is revealed again that 'anthropomorphism' of all our concepts of nature to which Goethe, in the wisdom of old age, loved to point. 'All philosophy of nature is still only anthropomorphism, i.e. . . man, at unity with himself, imparts to everything that he is not, this unity, draws it into his unity, makes it one with himself... We can observe, measure, calculate, weigh, etc., nature as much as we will, it is still only our measure and weight, as man is the measure of all things.' Only, after our preceding considerations, this 'anthropomorphism' itself is not to be understood in a limited psychological way but in a universal, critical and transcendental sense. Planck points out, as the characteristic of the evolution of the system of theoretical physics, a progressive emancipation from anthropomorphic elements, which has as its goal the greatest possible separation of the system of physics from the individual personality of the physicist. But into this 'objective' system, free from all the accidents of the individual standpoint and individual personality, there enter those universal conditions of system, on which depends the peculiarity of the physical way of formulating problems. The sensuous immediacy and particularity of the particular perceptual qualities are excluded, but this exclusion is possible only through the concepts of space and time, number and magnitude. In them physics determines the most general content of reality, since they specify the direction of physical thought as such, as it were the form of the original physical apperception. (126)

As Cassirer suggests, one of the fundamental differences between the anthropomorphism of past centuries and the anthropomorphism of modern science is that the former tended to be individualistic, whereas the latter tends to rise above the restrictions of individual sensuous perceptions and of the interpretations proper to particular groups. There is some truth in Claude Bernard's remark, "Si l'art c'est moi, la science c'est nous." Yet of the art of which we have been speaking it may be said: "c'est nous." And the reason is that this art is at the same time a science.

All this explains the spell that mathematical physics has succeeded in putting upon the human intellect in modern times. For in it man can be at once both the <u>homo sapiens</u> and the <u>homo faber</u>.(127) The mind is allowed to indulge in unlimited speculation in the realm that is most connatural with it - - that of mathematics, and this speculation is inseparable from construction in which the intellect posits its own object. At the same time this speculation brings it closer to the object that is most proper to it - - the essence of material things. And this intimate knowledge of material things reveals the plasticity and malleability that is native to them and thus gives to the mind the power to refashion nature according to its own designs.

But this spell constitutes a great intellectual danger. For not only will man fall a prey to a kind of scientism which will make mathematical physics absorb his whole attention, in such a way that in the speculative intellect wisdom will be dethroned by science, and not by

science in the full sense of the word but by mere dialectical prolongation of science; and in the practical intellect, prudence will be dethroned by art, and not by highest form of art but by technological art - - not only will he fall a prey to this form of intellectual suicide, but because by nature he is more a being of action than of contemplation, more an artisan than a philosopher, he will be tempted to make all science a kind of art. That is to say, he will become so fascinated by the projection of his own subjective logos into nature that he will sever this projection from its complete orientation to the objective logos and make it an end in itself. Bergson has characterized this tendency in the following terms:

> Nous ne dirions peut-être pas homo sapiens, mais homo faber. En définitive, l'intelligence envisagée dans ce qui en paraît être la démarche originelle, est la faculté de fabriquer des objets artificiels, en particulier des outils à faire des outils, et d'en varier indéfiniment la fabrication. "Son objet n'est pas . . . de nous révéler le fond des choses, mais de fournir le meilleur moyen d'agir sur elles." "Quel est l'objet essentiel de la science? C'est d'accroître notre influence sur les choses.(128)

We have seen that experimental science is more <u>a priori</u> than the disciplines that are sciences in the strict sense of the word precisely because it is less <u>a priori</u>. That is to say, in the latter case the connections of things are independent of the experience in which they are first recognized, and in this sense they are a priori. It is precisely because this is lacking in experimental science that a substitute a priori must be introduced. But this a priori of experimental science actually anticipates nature. The mind determines beforehand what is going to happen. And when experience confirms this anticipation the mind has in some way become the principle of experience. Experience does not manifest, it merely confirms the manifestation that the mind has made to itself. That is why the intellect in experimental science becomes the creator of the universe, as Professor Campbell has remarked:

> Un Newton, un Faraday, un Maxwell, conçoivent une théorie, et la vie s'adapte pour toujours aux lois qu'ils ont prédites. Par la puissance de leur imagination, ils créent la structure durable du monde. Ils ne sont pas des créatures chétives, enchaînées par les lois du temps et des sens; ils sont les créatures qui enfantent ces lois; les vents et les flots leur obéissent.(129)

When this creative element is made an end in itself, the mind becomes utterly free, and the measure of all things. In this connection the following lines of Abel Rey are extremely pertinent:

> The present era announces a new liberation, as profound perhaps as the two previous ones. It aims at those immutables, those mathematico-physical absolutes. There is no longer a tool

> that serves the intellect, except the intellect itself in its inventive omnipotence. The universalization of the hypothetico-deductive method, in its broadest signification, is the logical illustration of it . . It renews itself by changing, whenever necessary, even its very foundations. Logic, a collection of rational formulas, appears no longer as an architectural conception constructed once and for all into an unchangeable unity resting on an eternal foundation. Thought must constantly be ready to build on new foundations, or to modify the arrangement of the edifice, and consequently to complete, to adjust, and to renew its tools.(150)

This tendency has been extremely prolific and extremely virulent in recent years.(151) One of its results has been the instrumentalism of John Dewey. The following passage, which is typical of his thought shows how the creative element has been made the whole raison d'être of all scientific endeavor, how science has been transformed into art:

> If Greek philosophy was correct in thinking of knowledge as contemplation rather than as a productive art, and if modern philosophy accepts this conclusion, then the only logical course is relative disparagement of all forms of production, since they are modes of practice which is by conception inferior to contemplation. The artistic is then secondary to the esthetic: "creation", to "taste," and the scientific worker — as we significantly say - - is subordinate in rank and worth to the dilettante who enjoys the results of his labors. But if modern tendencies are justified in putting art and creation first, then the implications of this position should be avowed and carried through. It would then be seen that science is an art, that art is practice,

> and that the only distinction worth drawing is not between practice and theory, but between those modes of practice that are not intelligent, not inherently and immediately enjoyable, and those which are full of enjoyed meanings. When this perception dawns, it will be a commonplace that art - - the mode of activity that is charged with meanings capable of immediately enjoyed possession - is the complete culmination of nature, and that 'science' is properly a handmaiden that conducts natural events to this happy issue. Thus would disappear the separations that trouble present thinking: division of everything into nature <u>and</u> experience , or experience into practice <u>and</u> theory, art and science, of art into useful <u>and</u> fine, menial <u>and</u> free. (152)

Enough has been said to show that there is a sense in which the whole structure of experimental science is instrumental and functional, but as we shall point out in a few moments it is so primarily in relation to contemplation, to the apprehension of the objective logos of nature. Dewey segrates this instrumental and functional character and destroys its essential orientation.

But the tendency to exact the projection of the subjective logos has led man far beyond this form of instrumentalism. It has led him to conceive the mind as a kind of Platonic demiurge whose sole purpose is to work the world, to fashion it according to its own designs. Nature becomes merely a kind of matter for the art of man; it is viewed only in terms of its plasticity. Everything

in nature that does not yield itself up as malleable matter for the free play of human art is neglected or its existence is denied. All the proper distinctions which lift things out of pure plasticity and set them up as natures in their own right must be wiped out even at the expense of contradiction. Every determination in nature must give way before the constructive genius of man. Nature must no longer be defined as "ratio alicuius artis, scilicet divinae," but "ratio alicuius artis, scilicet humanae." Man substitutes himself for God.

We believe that this is the profound significance of the Marxist philosophy of nature and science, and in fact of the whole Marxist system. Marx writes: "La question de savoir si la pensée humaine peut comporter une vérité objective n'est pas une question théorique mais pratique. C'est dans la pratique que l'homme doit prouver la vérité de sa pensée, c'est-à-dire sa réalité, sa puissance, son en-de-ça." "Les philosophes n'ont fait qu'interpréter le monde de différentes manières. Or il s'agit de la transformer." (153)

Bertrand Russell touched the core of Marxist philosophy when he wrote: " Roughly speaking, all matter, according to Marx, is to be thought of as we naturally think of machinery: it has a raw material giving opportunity for action, but in its completed form it is a human product." (134) When man has succeeded in breaking down every determination which resists his creation of the cosmos, he will at last be able to "revolve about himself, his own true sun." (135) Never before has there been let loose upon the world a more frightful philosophy, nor one that is more pregnant with fearful consequences.

From many points of view this doctrine is but the logical outcome of the general trend that modern thought has taken since the time of the Renaissance. In every order there has been a tendency to construct rather than to accept. And in the last analysis this revolt against mere givenness is nothing but a revolt against the finiteness of the human mind. As great an authority as Ernst Cassirer assures us that at the time of the Renaissance all the properties that the Deity had formerly claimed for itself were made the attributes of the human soul.

In so far as all this affects the philosophy of science, it is clear that the error of the moderns has been to divorce the projection of the subjective logos into nature from its essential orientation to the objective logos. The subject becomes the measure of the object only

in order that the object may in a more perfect way become its measure. (136) Kant was correct in pointing out that in the construction of hypotheses we anticipate experience. But even before we give our assent to an hypothesis we have already admitted an objective criterion by which it is measured, namely objective truth. For an hypothesis must be likely, that is to say, have at least the appearance of truth. We are not the ones who create this likeness to truth. Moreover the only reason we posit an hypothesis is to help us to know objective truth, and we submit it to experience as to the determining measure of its worth. The moderns see in the power to construct hypotheses a manifestation of the supreme excellence of man. Undoubtedly, it is better to be able to construct hypotheses than to have to remain in the state of pure passivity. But in the last analysis the necessity of having recourse to hypothesis in order to know nature springs from the extreme imperfection of the human intellect.

Yet the modern exaltation of the constructive genius of man in experimental science is but the exploitation of a profound truth. For we have already noted that the advancement of science means that man's knowledge of the universe is becoming at the same time more objective and more subjective. And it is interesting to note here in passing that something similar to this is found in Theology in which the more we get to know God the greater becomes our recourse to the *via negationis* which is in a sense getting us farther and farther away from Him. Now if the limit towards which experimental science tends could be reached man's knowledge of the universe would be completely objective, but at the same time the universe would be completely a projection of the subject. Man's speculative knowledge of nature would be one with his practical knowledge. Nature and art would be identified. In other words, man would be God. Surely there is profound wisdom in Dante's remark: "Si che vostr' arte a Dio quasi e nipote."

> Perhaps to move
> His laughter at their quaint opinions wide
> Hereafter, when they come to model heaven
> And calculate the stars: how they will wield
> The mighty frame: how build, unbuild, contrive
> To save appearances.
>
> - - - Paradise Lost.

CHAPTER FIVE

EXPERIMENTAL SCIENCE AND DIALECTICS

1. The Problem

In the first book of the *Topics* Aristotle tells us that in seeking to discover the nature of an art it is advisable to begin by consulting those who are expert in that art. No one who attempts to follow this advice with respect to the nature of experimental science can fail to be struck by a remarkable unanimity in the opinions of those who in recent years have achieved the greatest renown as scientists. Experimental science is consistently described by them as a discourse in which from freely chosen suppositions certain conclusions are inferred. And in this hypothetical character attributed to experimental science two particular points are generally stressed: 1) it is, at least to some extent, *a priori* knowledge; 2) it never goes beyond probable knowledge.

In the foregoing pages some passages have already been cited which show that this represents the opinion which the most eminent modern scientists have of their own art. Innumerable texts of similar character could easily be adduced from the writings of such experts as DeBroglie, Le Roy, Poincaré, Eddington, Planck, etc. etc. Perhaps the following lines of Sir Jeans will serve as a typical example:

> We have seen that efforts to discover the true nature of reality are necessarily doomed to failure, so that if we are to progress further it must be by taking some other objective and utilising some new philosophical principle of which we have not so far made use. Two such suggest themselves. The first is the principle of what Leibniz described as probable reasoning; we give up the quest for certain knowledge, and concentrate on that one of the various alternatives before us which seems to be most probably true. But how are we to decide which of the alternatives is most likely to be true? This question has been much discussed of late, particularly by H. Jeffreys. For our purpose it is sufficient to rely on what may be described as the simplicity postulate; this asserts that of the two alternatives the simpler is likely to be nearer to the truth . . .
> In real science also a hypothesis can never be proved true. If it is negatived by future observations we shall know it is wrong, but if future observations confirm it we shall never be able to say it is right, since it will always be at the mercy of still further observations. A science which confines itself to correlating the phenomena can never learn anything about the reality underlying the phenomena, while a science which goes further than this, and introduces hypotheses about reality, can never acquire certain knowledge of a positive kind about reality; in whatever way we proceed, this is forever denied us. (1)

> We cannot claim to have discerned more than a very faint glimmer of light at the best; perhaps it was wholly illusory, for certainly we had to strain our eyes very hard to see anything at all. So that our main contention can hardly be that the science of to-day has a pronouncement to make, perhaps it ought rather to be that science should leave off making pronouncements: the river of knowledge has too often turned back on itself.
> Many would hold that, from the broad philosophical standpoint, the outstanding achievement of twentieth-century physics is not the theory of relativity with its welding together of space and time, or the theory of quanta with its present apparent negation of the laws of causation, or the dissection of the atom with the resultant discovery that things are not what they seem; it is the general recognition that we are not yet in contact with ultimate reality. (2)

This attitude, which Bertrand Russell characterizes as "humble and stammering", (3) is a far cry from the proud dogmatism of the classical physicists whose fundamental attitude towards experimental science had been summed up in Descartes' dictum that those who wish to find the true road in science must not occupy themselves with any object about which they cannot have certitude equal to that found in the demonstrations of arithmetic and geometry. (4) If this new attitude is correct, then Jeans is surely right in suggesting that it represents a discovery of far greater import than the amazing discoveries of modern science itself. For the former means a growth in wisdom, whereas the latter means merely a growth in science. But the full extent of this new attitude must be clearly recognized. The point is not that scientists have come to realize that modern experimental science knows nothing that is universal and necessary with absolute certitude, but rather that the nature of experimental science is such that it can never arrive at certain knowledge. In other words, the expression which Emil du Bois-Reymond made so famous must be applied to the very essence of experimental science: "Ignorabimus."

This new attitude raises a crucial problem for those who wish to establish the relevance of ancient epistemological schemes with modern science. In fact, the majority of contemporary writers both Scholastic and non-Scholastic seem to hold that this new attitude is incompatible with the epistemology of the ancient Peripatetics. The Scholastics see in this incompatibility a proof that the new attitude is false. The non-Scholastics see in it a proof that the old conception was only a provisional stage in the evolution towards modern thought. Both of these positions have consequences of great import. We believe that in the last analysis the first is a denial of experimental science and the second a denial of philosophy.

Sir Arthur Eddington has crystallized the issue

in the following terms:

> In view of the closer contact which now exists between science and philosophy, I would like to raise one question which effects our cooperation. A feature of science is its progressive approach to truth. Is there anything corresponding to this in philosophy? Does philosophy recognize and give appropriate status to that which is not pure truth but is on the way to truth. . . It is essential that philosophers should recognize that in dealing with the scientific conception of the universe they are dealing with a slowly evolving scheme. I do not mean simply that they should use it with caution because of its lack of finality; my point is that a vehicle of progress is not furnished on the same lines as a mansion of residence. The scientific aim is necessarily somewhat different from the philosophic aim, and I am not willing to concede that it is a less worthy aim.(5)

Eddington's query: "Does philosophy recognize and give appropriate status to that which is not pure truth but is on the way to truth?" may be taken in two ways. In the first place, it may mean: does philosophy grant within its own realm a place to a vehicle of progress which is not furnished on the same lines as a mansion of residence? In the second place, it may mean: does the philosophy of science recognize the progressive approach to truth which for Eddington constitutes the very essence of experimental science, and does it admit its value and its meaning? Genuine Thomistic philosophy unhesitatingly gives an affirmative answer to both of these questions. And as we have already suggested, the explanction of this answer must be sought for in the field of dialectics.

In so far as the first question is concerned it must be pointed out that Aristotle and St. Thomas in the most explicit fashion "recognise and give appropriate status to that which is not pure truth but is on the way to truth." And they do so not merely by granting this "vehicle of progress" an insignificant place within the realm of philosophy, but by admitting that it must make up the major portion of every philosophical treatise even of that which constitutes the very soul of all philosophy - - metaphysics. At the end of the first lesson of his Commentary on the Third Book of the Metaphysics Aquinas writes: "Dialecticam disputationem posuit quasi partes principiales huius scientiae."(6)

But it is evidently in the second sense that Eddington wishes his query to be understood. And here we come upon something quite different from the case just considered. Dialectics as a vehicle of progress must constitute the major portion of every philosophical treatise because the arrival at philosophical truth usually entails a long journey for the human mind. Nevertheless in philosophy there is an arrival, there is a mansion of residence furnished on different lines from the vehicle of progress, and the long journey is caused only by the

limitations of the human intellect. But in experimental science there is no arrival, there is no mansion of residence; one is committed to remain forever in the vehicle of progress. And the reason for the endless journey is not merely the limitations of the human mind, but the very nature of the object studied.

We must try to see why this is so. And our first concern will be to examine the nature of this vehicle of progress.

2. The Nature of Dialectics.

In his Commentary on the Posterior Analytics,(7) St. Thomas brings out the difference between metaphysics, logic and dialectics:

> Sciendum tamen est quod alia ratione dialectica est de communibus et logica et philosophia prima. Philosophia prima enim est de communibus, quia eius consideratio est circa ipsas res communes, scilicet circa ens et partes et passiones entis. Et quia circa omnia quae in rebus sunt habet negotiari ratio, logica autem est de operationibus rationis; logica etiam erit de his, quae communia sunt omnibus, idest de intentionibus rationis, quae ad omnes res se habent. Non autem ita, quod logica sit de ipsis rebus communibus, sicut de subiectis. Considerat enim logica, sicut subiecta, syllogismum, enunciationem, praedicatum, aut aliquid

> huiusmodi. Pars autem logicae, quae demonstrativa est, etsi circa communes intentiones versetur docendo, tamen usus demonstrativae scientiae non est in procedendo ex his communibus intentionibus ad aliquid ostendendum de rebus, quae sunt subiecta aliarum scientiarum. Sed hoc dialectica facit, quia ex communibus intentionibus procedit arguendo dialecticus ad ea quae sunt aliarum scientiarum, sive sint propria sive communia, maxime tamen ad communia. Sicut argumentatur quod odium est in concupiscibili, in qua est amor, ex hoc quod contraria sunt circa idem. Est ergo dialectica de communibus non solum quia pertractat intentiones communes rationis, quod est commune toti logicae, sed etiam quia circa communia rerum argumentatur. Quaecumque autem scientia argumentatur circa communia rerum oportet quod argumentetur circa principia communia, quia veritas principiorum communium est manifesta ex cognitione terminorum communium, ut entis et non entis, totius et partis, et similium.

The term "dialectics" has come to possess a number of meanings, but its most fundamental meaning and the one to which all others can be reduced is indicated in this text: dialectics consists in the application of an <u>ens rationis</u> to <u>ens reale</u>. That is to say, it is a process by which the intellect, starting from the <u>modus intelligendi</u> moves towards the <u>modus rei</u>. In other words, it is an attempt of the intellect to draw from mental constructs conclusions which regard reality.

This point is brought out with even greater clarity by St. Thomas when in his <u>Commentary on the Fourth</u>

<u>Book of the Metaphysics</u> he distinguishes between the dialectician and the philosopher:

> Differunt autem ab invicem. Philosophus quidem a dialectico secundum potestatem. Nam majoris virtutis est consideratio philosophi quam consideratio dialectici. Philosophus enim de praedictis communibus procedit demonstrative. Et ideo eius est habere scientiam de praedictis, et est cognoscitivus eorum per certitudinem. Nam certa cognitio sive scientia est effectus demonstrationis. Dialecticus autem circa omnia praedicta procedit ex probabilibus; unde non facit scientiam, sed quamdam opinionem. Et hoc ideo est, quia ens est duplex: ens scilicet rationis et ens naturae. Ens autem rationis dicitur proprie de illis intentionibus, quas ratio adinvenit in rebus consideratis; sicut intentio generis, speciei et similium, quae quidem non inveniuntur in rerum natura, sed considerationem rationis consequuntur. Et huiusmodi, scilicet ens rationis, est proprie subiectum logicae. Huiusmodi autem intentiones intelligibiles, entibus naturae aequiparantur, eo quod omnia entia naturae sub consideratione rationis cadunt. Et ideo subjectum logicae ad omnia se extendit, de quibus ens naturae praedicatur. Unde concludit quod subjectum logicae aequiparatur subiecto philosophiae, quod est ens naturae. Philosophus igitur ex principiis ipsius procedit ad probandum ea quae sunt consideranda circa huiusmodi communia accidentia entis. Dialecticus autem procedit ad ea consideranda ex intentionibus rationis, quae sunt extranea a natura rerum. Et ideo dicitur, quod dialectica est tentativa, quia tentare proprium est ex principiis extraneis procedere.(8)

It is clear, then, that dialectics involves a process which begins with a construct and hence <u>ab extrinsecis</u>. That is why there is a movement in dialectics - - <u>dialectica est tentativa</u>; the mind attempts to pass from the extrinsic to the intrinsic, from logical construction to reality. But as is evident from the two texts of St. Thomas just cited, there are more than one kind of construct from which the mind may attempt to reach reality. A close reading of those texts and of other passages in which Aristotle, Saint Thomas, and their medieval commentators discuss the nature of dialectics reveals that they recognized three distinct types of dialectical reasoning. The first type consists in reasoning from principles which are composed out of purely logical terms, that is to say, terms which signify second intentions. A good example of this is found in the seventh book of the *Metaphysics* in which the metaphysician employs a definition of substance which is not metaphysical but purely logical: substance is that of which everything is predicated and which is predicated of nothing.(9) The second type of dialectical reasoning is had when the principles employed are not proper to the science in which the reasoning takes place, but are common to several sciences. In this case the terms out of which the principles are constructed are not formed by the mind, but the principles themselves are, in the sense that their commonness is something that depends upon the intellect. It is only for the logician that angel and man are in the same genus, for

when things do not share in a natural genus, they can have only a logical genus in common. An example of this type of dialectical reasoning is suggested by Saint Thomas in the passage from the *Posterior Analytics* cited above: from the common principle that contraries are in the same category, one concludes that hatred pertains to the concupiscible appetite because it is the contrary of love.(10) The third type of dialectics consists in reasoning from principles which are only probable but which are accepted as if they were certain. It might not be immediately apparent why principles of this kind can be considered logical, and how reasoning based on them can realize the property of dialectics insisted upon by Saint Thomas, namely that it be *ex intentionibus*, *ex extraneis*. The answer is this: syllogistic form necessarily requires universality, and when there is mere universality *ut nunc*, that is to say a universality that is not seen in things, but is supplied tentatively by the mind, there is obviously a formation by the mind.(11)

Whenever conclusions are drawn from any of these three types of principles they are purely dialectical. For conclusions must be considered formally in the light of the principles by which they are illuminated. This is true even when only one of the premises is dialectical (in a way somewhat analogous to the case of reasoning which is formally theological even when only one of the premises is a datum of faith and the other is metaphysical). And in all reasoning of this kind the habitus employed is always the habitus of logic. That is why, if, as we shall try to show, experimental science is formally dialectics, it will be necessary to conclude that the habitus employed in it is the habitus of logic and not that of physical science. Nevertheless, it must be pointed out that while the use of dialectics in a certain matter pertains to a habitus other than the science of this matter, it is obviously necessary to have some exercise in the matter concerned in order to be able to use the dialectics. It is also worth while here calling attention to the fact that, speaking formally, the abstraction used in all types of dialectics is that of logic (i.e. a negative abstraction which falls reductive in the third degree of formal abstraction), even though the subject and predicate of the propositions may pertain to physics.

Now since all of these three types of dialectical reasoning are a functioning of a habitus that is extrinsic to the scientific habitus proper to the matter concerned,

they must from this point of view be distinguished from scientific reasoning. Yet from another point of view the first two types may be identified with scientific reasoning. For the essential property of scientific reasoning is that it is a strict demonstration, and it is evident that only the third type is lacking in demonstration.

Another way of bringing out this point is by saying that while all dialectics consists in an attempt to get at reality from a logical construct that is extrinsic to it, this construct may be extrinsic in two distinct ways. It may first of all be extrinsic from the point of view of truth, and then the reasoning is merely probable and does not give strict scientific certitude. Secondly, it may be extrinsic from the point of view of what is specifically proper to the reality concerned, and then the reasoning may give strict scientific certitude. Since a failure to grasp this important distinction may easily give rise to confusion about the way in which dialectics is employed in the study of nature, it is important to try to make it as explicit as possible. And we can best achieve this by considering the question in terms of definitions.

Definitions may be considered in two ways: either merely as definitions, or as principles of reasoning. Taken by themselves, definitions are not propositions; they do not involve predication. Hence they cannot be either true or false, but only good or bad. Now definitions may be either intrinsic or extrinsic. They are intrinsic (or proper) when they define things in terms of what constitutes them intrinsically; they are extrinsic (or dialectical) when they define things in terms of something extrinsic to them. An apt example of this distinction is found in the two definitions of substance. The proper definition of substance is: that whose nature it is to exist in itself and not in another as in a subject. The dialectical definition is in terms of something extrinsic to substance, namely predication: substance is that of which everything is predicated and which is predicated of nothing. In this distinction we have the explanation of the contrast which Aristotle draws between the physician and the dialectician at the beginning of the De Anima:

> Differenter autem definiet physicus et dialecticus unumquodque ipsorum; ut iram quid est. Hic quidem enim appetitum recontristationis, aut aliquid huiusmodi; ille autem fervorem sanguinis aut calidi circa cor. Horum autem alius quidem assignat materiam, alius vero speciem et rationem. Ratio quidem enim haec species rei. Necesse est autem hans esse in materia huiusmodi. (12)

We have seen that since sensible matter pertains essentially to mobile beings, all physical definitions must be in terms of it. That is why any definition of the things of nature which does not include sensible matter, which attempts to define them in terms of the form alone, cannot be intrinsic and proper, since it does not touch cosmic reality in what constitutes its very being. It can be nothing but extrinsic and dialectical, for the forms of natural things can exist independently of sensible matter only in the mind; the very quod quid est of these forms demands matter.

Definitions however may not only be considered in themselves, but also in relation to the thing defined. In this sense they are virtual propositions and can become principles of syllogisms, as St. Thomas points out in the Posteriora Analytica: "Principium autem syllogismi dici potest non solum propositio, sed etiam definitio. Vel potest dici quod licet definitio in se not sit propositio in actu, est tamen in virtute propositio quia cognita definitione, apparet definitionem de subjecto vere praedicari." (13) Considered in this way, definitions may be either scientific or dialectical. They are scientific if the connection with the thing defined is necessary, in other words if they are virtual propositions that are true. They are dialectical if the connection is not necessary, in other words if they are virtual propositions that are merely probable.(14) It is clear, then, that definitions can be truly scientific and at the same time dialectical in the first sense of the term. It is likewise clear that they can be truly physical and natural, and at the same time dialectical in the second sense. Hence it is extremely important to keep distinct these two ways in which the term "dialectics" is employed by Aristotle and St. Thomas in relation to natural doctrine.

And now, having made these necessary distinctions between the various meanings of dialectics, we must try to see in what sense experimental science can be called dialectical. From all that was said in the last Chapter it should be evident that the most fundamental way in which experimental science is dialectical consists in this that in it the mind attempts to get at the truth about nature by means of hypothetical and hence probable reasoning. Consequently in this Chapter we shall concentrate upon the meaning of dialectics in which it is opposed to what is strictly scientific, that is to say, to what involves true demonstration, and leave the consideration of other ways in which physics is dialectical to future contexts. Taken in this sense, dialectics is defined by Aristotle at the opening

of the first book of the Topics as: "methodus per quam possimus argumentari de omni proposito problemate ex probabilibus et ipsi disputationem sustinentes nihil dicamus repugnans." The central notion that must be analyzed in this definition is obviously that of probability.

There are two kinds of probability: real and dialectical. The former belongs to objective reality independently of knowledge, and it arises from the indeterminism of nature. The existence of chance in nature means that there are some future events which are not completely predetermined in their causes. These events are not necessary, and hence are at best only probable. Only conjectural knowledge can be had of them.(15) Even the most perfect created intelligence is unable to foresee them with certitude. Of course a created intelligence can judge with certitude of the present probability of the future, and in this sense real probability can be the foundation of a true proposition. But the truth of the future event does not follow from the truth of the present probability. Dialectical probability is not founded as real probability is upon an indetermination inherent in things, but upon an indetermination proper to the intellect which must move from potency to act. And it is with this type of probability that we are concerned in the dialectics of experimental science.

Aristotle defines dialectical probability in the following terms: "Probabilia autem sunt, quae videntur omnibus vel plerisque vel sapientibus, atque his vel omnibus vel plerisque vel maxime notis et claris."(16) The important word in this definition is "videntur". Probability must be defined in terms of appearances. As Aristotle points out in the fourth book of the *Topics*,(17) the probable is not a species of being. It must not be defined in terms of being, but in terms of that which has the likeness of being -- that which appears to be. Just as being gives rise to truth in the mind, so the likeness of being gives rise to the likeness of truth. That is why in the *Rhetoric* Aristotle defines probability as that which is similar to the truth.(18) Probability means verisimilitude. In other words, just as truth is the adequation of the mind with what is, so probability is the adequation of the mind with what appears to be. And this explains why, as Aristotle suggests in the *Rhetoric*,(19) the same natural impetus which moves the mind to seek after truth and take delight in it, likewise moves the mind to seek after its likeness and take delight in it, even though this delight is not completely satisfying. In his commentary on the *Topics* Sylvester Maurus

writes:

> Respondet Aristoteles Dialecticam distingui a Philosophia per hoc, quod licet dialecticus versetur circa res omnes et circa omnia problemata, sicut philosophus scientificus, adhuc differunt in modo considerandi. Philosophus enim non est contentus apparentia, sed examinat omnia secundum veritatem, ac quaerit, propria principia et proprias causas rerum; dialecticus e converso contentus est quadam apparentia veri et procedit ex communibus et probabilibus, quae causant solam opinionem. (20)

A first reading of Aristotle's definition cited above may make one wonder why in it he gives so much attention to the various kinds of knowers. But from what has just been said it should be clear that probability must necessarily be defined in terms of the knower and not in terms of the thing known. In other words, it is essentially related to appearances and hence to the apprehension of the knower and not to objective reality.(21)

The judgment which is the subject of the qualification "probable" is known as *opinion*. Just as a truly scientific judgment is necessarily true, so an opinionative judgment is necessarily probable. Opinion is opposed to certitude as indetermination to determination. And the indetermination that is proper to opinion is in the mind and not in things.(22) In other words, the object of opinion considered formally as such exists only in the apprehension.(23) By the indetermination found in opinion the mind is opposed to reality as logical being is opposed to real being. In other words the mind interposes itself so to speak between itself and reality. And the attempt to arrive at reality from this state of indetermination will be a dialectical process.

There was profound wisdom in the recognition by the ancient Greeks of the fact that at least much of the study of nature was merely doxa and not episteme in the strict sense of the word. For a study which can never rise above the appearances presented by experience except by having recourse to hypotheses which are never more than probable and whose sole purpose is to "save the phenomena", can never rise above the state of opinion, can never become a science in the strict sense of the word. In this connection St. Thomas writes:

> ... ita et in processu rationis, qui non est cum omnimoda certitudine, gradum aliquis inveniitur, secundum quod magis et minus ad perfectam certitudinem acceditur. Per huiusmodi enim processum, quandoque quidem etsi non fiat scientia, fit tamen fides vel opinio propter probabilitatem propositionum, ex quibus proceditur: quia ratio totaliter declinat in unam partem contradictionis, licet cum formidine alterius, et ad hoc ordinatur Topica, sive Dialectica. Nam syllogismus dialecticus ex probabilibus est, de quo agit Aristoteles in libro Topicorum.(24)

cum omnimoda / inveniitur, secundum quod / magis et minus ad

But before turning to consider the way in which the dialectics of probable reasoning is employed in experimental science, we must try to determine a bit more accurately its precise nature. It should be fairly evident from what has already been said that it pertains to what the schoolmen termed logica utens, as opposed to logica docens which merely gives the rules for the application of scientific principles that are already given and which does not enter into the very construction of these principles. But the term logica utens is employed in a variety of ways, and John of St. Thomas has brought out with great clarity the sense in which it must be understood here:

> Tertius usus Logicae est ipsi specialissimus, quatenus praebet usum in aliis scientiis seu materiis probabiliter disputandi sine hoc, quod procedatur demonstrative et resolutive usque ad prima principia. Et tunc proprie dicitur Logica utens, ut distinguitur a demonstrante et docente, eo quod demonstrans non praecise utitur discursu sistendo in eo sed pervenit resolvendo usque ad prima principia, quae discursu non probantur, sed sunt terminus discursus. Utens autem discursu, sed non demonstrans, ita utitur et sistit in discurso, quod non pervenit ad terminum discursus, qui est resolutio usque ad prima principia, et hoc pertinet ad processum disputativum seu tentativum, quando inquirendo, non autem resolvendo proceditur. Et ita vocatur probabilis processus, quia non cum certitudine ultimae resolutionis usque ad principia fit. Hic est actus Logicae utentis, et sic explicat illum D. Thomas opusc. 70, q.6. art.1 dicens

non pervenit / ad terminum discursus

> Logicam utentem esse, quae utitur discursu, sed non termino discursus, qui terminatur in principia per se nota, ubi cessat usus rationis discurrentis . . .
> Logica utens tertio modo accepta solum versatur circa partem topicam et sophisticam, id est processu non resolutivo, sed probabili seu probativo et disputativo. Et si talis usus fiat in aliis scientiis ex principiis talium scientiarum disputando ex illis et non resolvendo, talis usus pertinet ad Logicam solum directive; si autem procedat ex principiis ipsius Logicae talis disputatio non resolutiva, non solum directive, sed elicitive erit a Logica, quasi actus secundarius et imperfectus . . .
> Expressius autem hoc tradit D. Thomas opusc. 70 cit. q. 6, art. 1., ubi docet, 'quod aliquando dicitur processus rationalis ex termino, in quo sistitur procedendo. Ultimus autem terminus, ad quem rationis inquisitio perducere debet, est intellectus principiorum, in quae resolvendo iudicamus; quod quidem quando fit, dicitur demonstratio. Quando autem inquisitio rationis usque ad ultimum terminum non perducit, sed sistitur in ipsa inquisitione, quando scilicet quaerenti adhuc manet via ad utrumlibet, sic rationalis processus distinguitur contra demonstrativum. Et hoc modo procedi potest rationabiliter in qualibet scientia, ut ex probabilibus paretur via ad necessarias conclusiones. Et hic est alius modus, quod Logica utitur in aliis scientiis, non ut est docens, sed ut utens.' Sic D.Thomas . . . .
> Et si hoc faciat praebendo principia propria tali discursui et disputationi, elicitive totum illum discursum producet Logica, quia non solo praebet modum disputandi, sed etiam materiam seu principia. (85)

In order to understand that passage correctly it is necessary to recall the distinction made above between the two ways in which the extrinsic character of dialectics can be understood. When John of St. Thomas suggests that

the use of dialectics which he terms <u>directivus</u> does not provide the principles for the process of reasoning, but merely the <u>modus disputandi</u> he obviously has in mind the meaning of extrinsic in which it signifies something exterior to the matter that is specifically proper to the science involved, as in the case of the definition of anger in terms of form alone, or of substance in terms of predication. For if extrinsic were understood in the other sense, then even the dialectics of probable reasoning must be said to provide the principles.

In any case, it is in the use of logic which John of Saint Thomas calls <u>directivus</u> that we are now particularly interested. Later we shall have occasion to see that mathematical physics also involves a use of logic that is similar to what he terms <u>elicitivus</u>, in so far as an attempt is made to explain natural phenomena in terms of logical constructs.

It is clear that a study which remains within the dialectical discourse just described without ever being able to emerge from it can never be a science in the strict sense of the word. It is not a science in its own right, since it never achieves strict demonstration. Nor can it be considered a logical science, since the logic involved is not logica docens but utens. The following passage from St. Thomas' Commentary on the Metaphysics is relevant here;

> Licet autem dicatur, quod philosophia est scientia, non autem dialectica et sophistica, non tamen per hoc removetur quin dialectica et sophistica sint scientiae. Dialectica enim potest considerari secundum quod est docens, et secundum quod est utens. Secundum quidem quod est docens, habet considerationem de istis intentionibus, instituens modum quo per eas procedi possit ad conclusiones in singulis scientiis probabiliter ostendendas; et hoc demonstrative facit, et secundum hoc est scientia. Utens vero est secundum quod modo adiuncto utitur ad concludendum aliquid probabiliter in singulis; et sic recedit a modo scientiae. Et similiter dicendum est de sophistica; quia prout est docens tradit per necessarias et demonstrativas rationes modum arguendi apparenter. Secundum vero quod est utens deficit a processus verae argumentationis. Sed in parte logicae quae dicitur demonstrativa, solum doctrina pertinet ad logicam, usus vero ad philosophiam et ad alias particulares scientias quae sunt de rebus naturae. Et hoc ideo, quia usus demonstrativae consistit in utendo principiis rerum, de quibus fit demonstratio, quae ad scientias reales pertinet, non utendo intentionibus logicis. Et sic apparet, quod quaedam partes logicae habent ipsam scientiam et doctrinam et usum, sicut dialectica tentativa et sophistica; quaedam autem doctrinam et non usum, sicut demonstrativa. (26)

From all that has been said thus far it follows that the meaning which the term "knowledge" has for us when applied to experimental science coincides exactly with the sense in which it is understood by Sir Arthur Eddington;

> Some writers restrict the term 'knowledge' to things of which we are quite certain; others recognise knowledge of varying degrees of uncertainty. This is one of the common ambiguities of speech as to which no one is entitled to dictate, and an author can only state which usage he has himself chosen to follow. If 'to know' means 'to be quite certain of', the term is of little use to those who wish to be undogmatic. I therefore prefer the broader meaning; and my own usage will recognize uncertain knowledge. (27)

Enough has been said to show that if we wish to discover the principles which reveal the true nature of experimental science it is to the Topics especially that we must turn. And it is extremely significant that this part of logic has been almost completely neglected by modern scholastics. In fact, the teaching of logic has been almost exclusively limited to the Prior and Posterior Analytics. And we believe that there is a connection between the scholastics' neglect of dialectics and their neglect of movement towards concretion in the study of nature. This disregard for the importance of dialectics goes back as far as John of St. Thomas himself;

> In secunda vero parte agemus de his quae pertinent ad materiam logicalem seu ad posterioristicam resolutionem, maxime in demonstratione, ad quam praecipue ordinatur. (28)
> Quae enim pertinent ad partem topicam, quae agit de probabilibus, et quae pertinent ad libros Elenchorum qui agunt de parte sophistica, omittuntur in praesenti, quia non agunt de certa et perfecta

> resolutione iudicii, et ideo solum libri Priorum et Posteriorum vocantur analytici ab Aristotle.(29)

At the time that these lines were written the modern development of experimental science was already underway. Without realizing it, men like Galileo had already discovered in dialectics a potent intellectual instrument for the advancement of the study of nature in the direction of concretion. It remains for us to see just how this dialectical instrument is employed by experimental science.

2. Dialectics and Experimental Science.

As we have already explained, the propositions that are proper to experimental science are devoid of intrinsic and objective universality. But because the intellect cannot remain imprisoned in singularity, the scientist is lead to confer universality upon them <u>ab extrinseco</u>. In order to get at the reason for the regularity appearing in nature, the scientist is lead to act <u>as if</u> these propositions were universal. In so doing he is applying the principle laid down by Aristotle in the <u>Topics</u> : "quaecumque in omnibus aut in plurimis apparent, sumenda sunt quasi principia et probabiles theses."(30) In this way

he uses the principle <u>dici de omni</u> in the sense in which it is employed in the <u>Priora</u> where it is not restricted to science in the strict sense of the term, but is common to both science and dialectics:

> Ad quod sciendum est quod <u>dici de omni</u>, prout hic sumitur, addit supra <u>dici de omni</u>, prout sumitur in libro Priorum. Nam in libro Priorum accipitur <u>dici de omni</u> communiter, prout utitur eo et <u>dialecticus</u> et demonstrator. Et ideo non plus ponitur in definitione eius, quam quod praedicatum insit cuilibet eorum quae continentur sub subiecto. Hoc autem contingit vel <u>ut nunc</u>, et sic utitur quandoque <u>dici de omni</u> dialecticus; vel simpliciter et secundum omne tempus, et sic solum utitur demonstrator.(31)

We have already pointed out that these propositions which are <u>posed</u> by the scientist instead of being <u>imposed</u> upon him are purely functional. Their position must lead to something beyond themselves. They are instruments - - principles of research. In other words, they are dialectical. The mind uses them in order to get at reality.

But as we explained in the last Chapter, these universalized propositions do not satisfy the mind, for they do not "save the phenomena". That is to say, they merely state the connection between subject and predicate without giving the reason for it. Consequently, the mind is lead to reach out for the <u>propter quid</u> by constructing hypotheses which will give a provisional explanation of the experimental propositions. In other words, purely experimental propositions contain an implicit problem, and in order to solve this problem we transform propositions into questions which anticipate experience. In connection with this use of hypothesis it is worth while pointing out, lest confusion arise, that the term "hypothesis" (suppositio) usually meant for Aristotle and St. Thomas something quite different from the sense in which it is now understood. It did not mean something that was lacking in certainty, and that as a consequence could not be demonstrated. On the contrary, it meant something that was absolutely certain, but that was accepted without demonstration either because of its self-evidence or because of its demonstration in another science, or at least because of its acceptance by the adversary or the disciple with whom he who used it had to deal. (32) It is clear, however, from the passages cited in the last Chapter from the De Coelo etc. with regard to the planetary systems that the ancients also recognized the use of hypothesis in the modern sense of the term. Taken in this sense it means, as we have already suggested, a proposition or a group of propositions posed by the mind in order to save sensible phenomena by offering a provisional explanation of the reason behind experimental propositions.

An hypothesis never goes beyond probability; it is, as someone has said "an educated guess" - - an anticipated solution of a problem. It is essentially the product of the creative imagination and of scientific construction. From hypotheses of this kind posited as premises, the mind seeks to deduce conclusions which square with sensible experience and thus explain it. It is clear that these hypotheses are purely dialectical; they are constructions by which the mind attempts to arrive at the nature of reality.

The scientist accepts what is similar to the truth as if it were the truth and uses it as a principle of research. In doing so he is following the natural appetite of the mind which as we saw above must seize upon what is similar to the truth when it cannot have the truth. The student of nature must multiply without end his conjectures and must fix attention more upon their operative, functional, instrumental value than upon anything else.

> Les théories n'ont pas pour but de nous révéler la véritable nature des choses; . . . leur but unique est de coordonner les lois physiques que l'expérience nous fait connaître... Peu nous importe que l'éther existe réellement: c'est l'affaire des métaphysiciens; l'essentiel pour nous, c'est que tout se passe comme s'il existait, et que cette hypothèse est commode pour l'explication des phenomenes." (33)

Ever remaining within the realm of the conjecturable, the experimental scientist must carry on a methodical interrogation of nature which never has any final issue. The art which guides this methodical interrogation is dialectics.

The mind is therefore free in the construction of these hypotheses. We have already quoted several passages from Einstein which show that the premises of experimental science are free inventions, creations. This freedom is not absolute, to be sure, for the dialectics of experimental science must always be kept in tow, so to speak, by constant recourse to experience. Nevertheless there is liberty and creativity in this dialectics. The scientist is free to choose between contrary or contradictory hypotheses the one which seems to serve his purpose best at the moment. He is, for example, free to choose between the opposing corpuscular and wave theories of light the hypothesis which gives him the greater help in achieving his task. All this recalls what St. Thomas has to say about the dialectician:

> Secundo, ibi: Dialectica etc., ponit differentiam inter dialecticam propositionem et demonstrativam, dicens quod cum propositio accipiat alteram partem enunciationis, dialectica indifferenter accipit quamcumque earum. Habet enim viam ad utranque partem contradictionis, eo quod ex probabilibus procedit. Unde etiam et in proponendo accipit utramlibet partem contradictionis et quaerendo

> proponit. Demonstrativa autem propositio accipit alteram partem determinate, quia nunquam habet demonstrator viam, nisi ad verum demonstrandum. Unde etiam semper proponendo accipit veram partem contradictionis. Propter hoc etiam non interrogat, sed sumit, qui demonstrat quasi notum. (34)

Because these hypotheses are never more than probable, experimental science must ever call into question not merely its conclusions but its very principles. And this characteristic of dialectics, as St. Thomas points out in his <u>Commentary on the Posterior Analytics</u>:

> Sciendum tamen est quod interrogatio aliter est in scientiis demonstrativis et aliter est in dialectica. In dialectica enim non solum interrogatur de conclusione, sed etiam de praemissis: de quibus demonstrator non interrogat, sed ea sumit quasi per se nota, vel per talia principia probata: sed interrogat tantum de conclusione. Sed cum eam demonstraverit, utitur ea, ut propositione, ad aliam conclusionem demonstrandam. (35)

This brings out the difference in the way dialectics is employed by philosophy and by experimental science. In philosophy it is used merely as an instrument to search out principles which, when found, impose themselves upon the mind by their certitude. In experimental science, dialectics is employed not merely in the search for principles but in the very choice and positing of the principles. (36) This ties up with what we saw in chapter IV about the difference between the Thomistic and the Kantian meaning of <u>a priori</u>.

In all this we have the reason why experimental science is essentially variable and transitory — a vehicle of progress and not a mansion of residence. And in this connection De Broglie writes:

> Il ne faut pas s'étonner si souvent la découverte d'un ordre nouveau de phénomènes vient renverser comme un château de cartes nos plus belles théories, car la richesse de la nature dépasse toujours nos imaginations. Les savants sont bien hardis de vouloir reconstruire par la pensée des portions de l'univers; la grande merveille, c'est qu'ils y ont parfois réussi. (37)

As Dotterer has remarked, "the first principles of the sciences must be regarded as postulates; and there is a sense in which all science is founded on faith". (38) It was because Claude Bernard recognized the dialectical character of experimental science that he made doubt the great experimental principle : "The great experimental principle, therefore, is doubt, the philosophical doubt which leaves the mind its freedom and initiative, and from which come the most valuable qualities in an investigator in physiology and in medicine." (39)

Experimental science advances by a gradual rationalization of irrational elements; but this demands a continual reorganization of its rational system. Both the method employed and the corpus of doctrine achieved must ever

remain open to revision. The only way that experimental science has to develop is by a continual process of substitution. It can grow only by passing through crises and revolutions. (40)

It is clear, then, that all the propositions which make up the structure of experimental science are reducible to what St. Albert the Great calls "interrogatio consensus in probabile".

> Sed dialectica propositio est interrogatio consensus in probabile, nec consensus requireretur si probari non deberet: manifeste autem falsum probari non potest, et manifeste verum non indiget probari, sed ad alterius alicuius assumitur probationem.
> In diffiniendo ergo propositionem dialecticam secundum potissimum suum statum dicimus, quod propositio dialectica est interrogatio probabilis, ita quod probabilis sit genitivi casus, hoc est, interrogatio de probabili, quod est materia propositionis dialecticae. In probabili enim (quia ponitur in judicio eius cui proponitur, utrum sic videatur vel non) oportet quaerere respondentis judicium et consensum, antequam procedere possit opponens. Sic ergo dialectica propositio interrogatio est probabilis. Et hac ratione etiam Boetius in diffinitione syllogismi dicit, quod est oratio in qua quibusdam positis et concessis, respiciens ad propositionem syllogismi dialectici. Cuius causa est, quod probabile de se non habet sufficientem causam consequentiae vel inferentiae, et causam inferentiae sufficientem accepit a concessione respondentis. Haec igitur est tota diffinitio propositionis dialecticae.
> (41)

Sir James Jeans has brought out the dialectical character of the scientist's interrogation of nature:

> Such an experiment, like every other, amounts in effect to asking a question of nature. This question can never be -- 'Is hypotheses A true?' but 'Is hypothesis A tenable?' Nature may answer our question by showing us a phenomenon which is inconsistent with our hypothesis or by showing us a phenomenon which is not inconsistent with our hypothesis. She can never show us a phenomenon which proves it; one phenomenon is enough to disprove a hypothesis but million million do not suffice to prove it. For this reason, the scientist can never claim to know anything for certain, except direct facts of observation. Beyond this, he can only proceed by building up hypotheses, each of which covers more phenomena than its predecessor, but each of which may have to give place to another hypothesis in due course. Strictly speaking, the time for replacing a hypothesis by a claim to certainty never arrives.(42)

As von Uexkull has pointed out,(43) the art of interrogation of nature, the art of research is characteristic of the experimental scientist. We feel that enough has been said to show that this art is substantially the same as the "logica interrogativa", "tentativa", "inquisitiva", "inventiva" of the ancients, i.e. the dialectics of the Topics.(44) And it is extremely significant to note the similarity between the following passage of von Uexkull and the lines quoted earlier in this Chapter from St. Thomas' Commentary on the Fourth book of the Metaphysics: "dialecticus autem procedit ad ea consideranda ex intentionibus rationis, quae sunt extranea a natura rerum. Et ideo

dicitur, quod dialectica est tentativa, quia tentare proprium est ex principiis extraneis procedere."

> In the present book I have endeavoured to frame the theoretical considerations concerning biology, in such a way that there can no longer be any doubt that, in their very nature, biological doctrines always remain unsolved problems.
> In nature everything is certain; in science everything is problematical. Science can fulfill its purpose only if it be built up like a scaffolding against the wall of a house. Its purpose is to insure the workman a firm support everywhere, so that he may get to any point without losing a general survey of the whole. Accordingly, it is of the first importance that the structure of the scaffolding be built in such a way as to afford this comprehensive view; and it must never be forgotten that the scaffolding does not itself pertain to Nature, but is always something extraneous. (45)

The comparison of science with a scaffolding, which had already been employed by Goethe, is, as we suggested in the last chapter, very exact. It brings out the fact that experimental science is essentially a logical construction which the mind uses in an attempt to get at reality. As we shall point out in chapter XI, it is not a formal sign of nature, but purely an instrumental sign. Just as a scaffolding can be made to approach closer and closer to the form of the house and thus be brought to take on gradually a greater likeness to the house, so experimental science can approach ever closer and closer to nature and in so doing take on a greater likeness to nature. But just as a scaffolding can never become the house and must ever remain an extrinsic construction, so science must ever remain an extrinsic construction of the mind. In fact, as we suggested in chapter IV the closer it gets to nature the more extrinsic it becomes, because of the fact that the subjective construction constantly increases. As we shall point out in chapter XI, there is a great deal of similarity between the dialectical approach of science to nature and the dialectical movement of a regularly inscribed polygon with constantly increasing sides towards a circle. Just as the multiplication of the sides of the polygon makes it more like a circle, and at the same time more of a polygon and hence more unlike a circle (which has only one "side") so the movement of science towards nature makes it at once more objective and more subjective.

A number of objections may suggest themselves in regard to this identification of experimental science with dialectics. In the first place, one may be tempted to ask: if experimental science is dialectics, in what sense can it be considered as a part of natural doctrine?

The answer is: experimental science is natural doctrine principally because of the limit towards which its dialectical movement is orientated, i.e. nature. In other words, it is natural doctrine not so much because of what it has achieved at any given stage of its development as because of what it is at all times attempting to achieve. To get back to the example used above — the circle is the limit of the polygon only in so far as the latter is in a state of movement through the successive multiplication of its sides. If this movement should stop at any one given polygon, no matter how far advanced it may be in the series, the circle can no longer be considered as the limit. Similarly, natural doctrine, in so far as it is built upon hypothesis, must ever remain in a state of movement towards its limit which is nature, that is to say, the absolute world condition. No given stage of the development of experimental science can be considered natural doctrine in an absolute sense. To so consider it would be to identify a subjective construct with objective nature — which would be comparable to identifying a polygon with a circle. Nevertheless, just as a given polygon that is far advanced in the series which tends towards the circle is already in some way a revelation of the nature of the circle, so any given stage of the construction of experimental science is in some measure a revelation of objective nature. And just as a polygon of a million sides is closer to the circle than a polygon of ten sides, so modern physics knows nature better than the physics of the four elements. We shall return to examine these notions in fuller detail in chapter XI. For the moment, it is interesting to compare what has just been said with the following passage from von Uexkull:

> A man may have assimilated the conclusions of natural science in the form of doctrine, and may know how to employ them in speculation, according to the rules of logic; but he still knows nothing whatsoever concerning Nature — or at any rate, infinitely less than does any peasant or gardener who is in daily intercourse with her. (46)

This statement, which at first sight appears to be an extreme exaggeration, can be accepted if viewed in the light of our foregoing remarks. In so far as experimental science is a subjective hypothetical construction, the scientist may be said to know nothing about nature in its purely objective condition. Nevertheless, because this subjective construction is in some measure a reflection of nature, von

Uexkull is correct in immediately qualifying his initial absolute statement. And there is a sense in which it is true to say that experimental scientists know infinitely less about nature than gardeners and peasants, who are, though in an extremely obscure way, in contact with objective nature. The actual vegetables with which gardeners deal are certainly not constructed according to the hypotheses of biology. This would suppose that biology had achieved a knowledge of the true essence of living things. "Scientific vegetables" are not edible.

A second objection to our identification of experimental science with dialectics might be that in innumerable places Aristotle and St. Thomas condemn the Platonists and the Pythagoreans for proceeding "logice sive dialectice in naturalibus." (47) An attentive examination of these texts, however, will immediately reveal that they do not condemn the use of dialectics as such in the study of reality. As a matter of fact, both of them have frequent recourse to it. What they do condemn is the abuse of dialectics, which consists in granting priority to principles over experience, when, as a matter of fact, the former should ever remain in complete dependence upon the confirmation of the latter. Instead of rejecting principles in order to save appearances, the Platonists made it a practice of rejecting sensible appearances in order to save their preconceived principles. This is evident from the passage from the third book of the De Coelo quoted in the last chapter. In other words, the condemnations of Aristotle and St. Thomas are levelled against the logical error of confusing a formal consequence with an argument, which would make dialectics self-sufficient and independent in the study of nature.

A final objection which might be brought to bear against the identification of experimental science with the Aristotelian dialectics of the Topics is that the very definition which the Stagirite gives of the latter (48) seems to indicate that it is essentially a method of discussion with adversaries and that consequently it presupposes a dialogue. It is true that dialectics essentially involves a kind of dialogue, since, as we have seen, its principles are always "interrogationes probabiles." It may also be granted that in writing the Topics Aristotle had principally in mind the use of dialectics which involves a plurality of persons. But the dialogue of dialec-

tics does not necessarily suppose such a plurality. In dialectical reasoning one person can start with what seems probable to him and seek his own assent to it. Moreover, even without a plurality of persons there is always an adversary, namely the other part of the contradiction.

In this dialectical character of experimental science we find the basic reason why physics inevitably issues into mathematical physics. Not finding scientific certitude within its own realm, it attempts to acquire for itself a substitute certitude by reaching up to mathematics. From this point of view, Bertrand Russell is correct in saying that "physics is mathematical, not because we know so much about the physical world, but because we know so little." (49) What we have been saying in this chapter also brings to light the reason why mathematics in the modern sense of the term is a natural prolongation of the dialectics of experimental science. Dialectics bestows upon physics the hypothetico-deductive method which is so characteristic of modern mathematics. And in this connection it is extremely interesting to compare what we have said about the nature of dialectical reasoning from freely chosen hypotheses with Bertrand Russell's famous definition of mathematics:

> Pure mathematics consists entirely of assertions to the effect that, if such and such a proposition is true of __anything__ then such and such another proposition is true of that thing. . . Thus mathematics may be defined as the subject in which we never know what we are talking about, nor whether what we are saying is true. (80)

This brings us to the task of analyzing the proper nature of mathematics.

CHAPTER SIX

THE NATURE OF MATHEMATICAL ABSTRACTION

1. Mathematical Abstraction.

History has played with the term "mathematics" in a way similar to that in which it has played with the term "science". We have seen that the latter term now has a meaning quite distinct from, and to a certain extent opposed to, the meaning it had for the ancients: it no longer signifies certain knowledge of things in their causes, but a purely dialectical type of knowledge that is lacking in certitude. In somewhat the same way, the meaning of the term "mathematics" has undergone a profound change. For the ancients it signified a strictly unified science specified by a definite formal object, namely quantity. But in recent years mathematics has been divorced from its essential relation to quantity and given a range that extends indefinitely beyond its confines.

> In former days, it was supposed (and philosophers are still apt to suppose) that quantity was the fundamental notion of mathematics. But nowadays, quantity is banished altogether, except from one little corner of Geometry, while order more and more reigns supreme. The investigation of different kind of series and their relations is now a very large part of mathematics, and it has been found that this investigation can be conducted

> without any reference to quantity, and, for the most part, without any reference to number. All types of series are capable of formal definition, and their properties can be deduced from the principles of symbolic logic by means of the Algebra of Relatives. (1)

Mathematics is no longer a strictly unified science; it no longer has a definite formal object. And the result is that most of what is now considered mathematics is not mathematics in the original sense of the term; it is dialectics. In this chapter we shall try to analyze the nature of mathematics in the strict and formal sense of the term, in the sense in which it was understood by the ancient Thomists. (2)

One of the objections brought against the relevance of Peripateticism for the question of science is that it necessarily minimizes the importance of mathematics because of the fact that it considers quantity merely as one out of ten predicaments. (3) As a matter of fact, however, Peripatetics have always accorded to quantity a unique position among all the categories. For of all the nine accidents it is the one closest to substance. And it is the only one of the accidents that can be the subject of a special science. For all science deals with a subject manifesting itself through certain definable properties, and quantity is the only accident in

which there is found both subject and properties. This explains why quantity and the quantitative can constitute, in relation to knowledge, a closed universe apart from everything else:

> Sciendum autem est quod quantitas inter alia accidentia propinquior est substantiae. Unde quidam quantitates esse substantias putant, scilicet lineam et numerum et superficiem et corpus. Nam sola quantitas habet divisionem in partes proprias post substantiam. Albedo enim non potest dividi, et per consequens nec intelligitur individuari nisi per subjectum. Et inde est quod in solo quantitatis genere aliqua significantur ut subjecta, alia ut passiones. (4)

But in order to get at the nature of this special science it is necessary to point out that it is not quite accurate to call mathematics the science of quantity. For the other two speculative sciences, metaphysics and philosophy of nature, also deal with quantity in some way. Metaphysics deals with it in so far as it is a principle of being -- one of the nine accidents. Philosophy of nature deals with it in so far as in nature there is mobility in the genus of quantity, which is characteristic of those mobile beings which have life. Consequently, in order to get at the intrinsic nature of mathematics, it will be necessary to consider the particular way in which it deals with the notion of quantity, it will be necessary to analyze as accurately as possible the special nature of mathematical abstraction.

A number of things were said about the nature of this abstraction in Chapter II. Before pushing ahead in our analysis let us recapitulate briefly the points already laid down.

Mathematical abstraction is the second degree of formal abstraction. It stands midway between physical and metaphysical abstraction, and shares to some extent in both. Yet from another point of view, it is not midway between the first and third degrees of abstraction, in the sense of being in direct line with them. Rather it is out of line, off to one side, so to speak. And in this connection it is interesting to note that while the term "metaphysics" is an historical accident, it is an extremely happy accident in the sense that it characterizes quite accurately the nature of the science it has been chosen to designate. From this point of view it is highly significant that mathematics, though coming directly after physics in the degrees of abstraction, is not called metaphysics. Nor is metaphysics called metamathematics, though it comes immediately after

mathematics. And yet when physics begins to seek a substitute cause and reason to explain its facts, it is not to metaphysics that it naturally turns, but to mathematics. This is a paradox upon which we must endeavor to throw some light.

Mathematical abstraction prescinds from all sensible matter, though not from intelligible matter. By sensible matter we understand matter with sensible qualities, and hence apprehensible by the senses. It is important to distinguish between mathematical quantity and the common sensibles. As we shall see there is a close connection between the two, but they are not identified, precisely because the common sensibles are sensible. A mathematical line, a number, etc. are by definition not sensible. (5) By intelligible matter we mean the substance considered as the subject of quantity, which is the order of the parts of the substance. This abstraction gives to mathematics an object which depends upon sensible matter for its being, but not for its "being known", that is, it is conceived by the mind and defined independently of all sensible matter, but in order for it to exist outside the mind, it must be realized in sensible matter.

As we pointed out in Chapter II, this profound dichotomy between subjective and objective existence is something peculiar to mathematical abstraction. It is found neither in physical abstraction, in which the object is dependent upon sensible matter both for its existence in the mind and its existence outside the mind, nor in metaphysical abstraction, in which the object is independent of sensible matter both for its existence in the mind and its existence outside the mind. We suggested that this dichotomy found in mathematical abstraction is extremely significant, and the time has now come to explore that significance.

We know of no better point of departure for this exploration than a consideration of a text of Saint Thomas which at first sight might appear somewhat confusing, but which actually contains the key to the nature of mathematical abstraction. As we noted in Chapter II, in the third article of the fifth question of the De Trinitate, Aquinas seems to restrict the expression "formal abstraction" to the type of abstraction found in the mathematical sciences. He points out in fact that there are two kinds of abstraction: the abstraction of a form from matter, and the abstraction of a universal from a particular. The former he considers to be proper to mathematics, while the latter is common to all

the sciences. (6) We have already explained in a general way how this passage must be interpreted. But at this juncture it is necessary to analyze the mind of Aquinas with greater exactness, for by so doing we shall be able to lay bare the proper nature of mathematical abstraction.

In simple apprehension the intellect is able to separate certain things which in reality are not separated. It is in this way that the mind gets at the things which form the objects of the mathematical sciences. Objects such as line and number can be separated by the mind from the sensible matter with which in reality they are necessarily united. Now precisely because this union in reality is necessary the separation effected by the mind in simple apprehension cannot be transposed to the second operation of the mind, the judgment. For the essence of the judgment is the copula, and this expresses existence, reality. That is why from the conception of a line separated from sensible matter we cannot pass on to the judgment: " the line exists without sensible matter." What about the judgment: "the line exists with sensible matter?" Such a judgment can be made, of course, but then we are no longer speaking about the separated line, the abstract line. There is, therefore, a kind of indifference in this abstraction.

On the one hand, it does not say that the line is with sensible matter. But on the other hand, it does not say that it exists separated from it.

This brings out the characteristic feature of mathematical abstraction, and explains what is meant by saying that quantity depends upon sensible matter in order to be, but not in order to be conceived. For on the one hand, in the case of the sensible qualities which enter intrinsically into the study of nature, there is no possibility of separation "secundum intellectum" since sensibility pertains to their very concept. Material substance, which is the object of the science of nature, even though as substance it is the first subject of all the determinations connected with it, cannot be conceived as material substance without mobility, and mobility necessarily involves quantity with sensible matter. On the other hand, while the objects with which metaphysics deals are separated "secundum intellectum", they are also separated "secundum esse", and that is why in metaphysics we can transpose the separation found in simple apprehension to the operation of judgment. "Considerare substantiam sine quantitate, magis pertinet ad genus separationis quam abstractionis . . Et haec competit scientiae divinae, sive metaphysicae."(7)

All this helps us to see why St. Thomas is justified in calling the abstraction found in mathematics formal abstraction in a very special sense. In it alone there is a form lifted out of matter to which it is necessarily united in reality. And this enables us to grasp the difference between the formal abstraction characteristic of mathematics, and the "universalizing" abstraction found in the other sciences. For it follows from what has just been said that mathematical entities in one sense can and in another sense cannot be realized in nature. They may be said to be realized in nature in the sense that there are triangles, lines, etc. actually existing in the world of reality. But mathematical entities as such, that is, in their state of abstraction from sensible matter, cannot exist in reality. This point is important, for not only does it reveal the special nature of mathematical abstraction, but it also enables us to understand the true nature of mathematical physics. For as we have already pointed out, the application of mathematics to physics consists in the application of mathematical entities as such, that is, in their abstract state. It is not merely a question of finding in nature quantitative determinations as they exist in union with sensible matter.

But perhaps it is not sufficiently clear yet just how mathematical abstraction differs from the abstraction found in the other sciences. For all the sciences deal with abstractions, and abstract things as such, that is, in their state of abstraction cannot be realized in nature, even though they may be realized by the removal of this state. In what way, then, do mathematical entities differ from the abstract things with which the other sciences deal? There is a vast difference between mathematics and the other sciences. For, although all sciences deal with abstract things, only mathematics deals with abstract things as abstract. That is to say, the abstractions found in all the other sciences may be predicated directly of things existing in reality. Mathematical entities, on the other hand, can be predicated directly of nothing existing in reality, precisely because they are defined in a way in which they cannot exist, that is, as separated from sensible matter. In other words, the only difference between the abstract entities found in the other sciences and reality is that of universality and particularity. But in mathematics there is much more than this. Not only do universal mathematical entities not exist in reality, but even particular mathematical entities

do not exist. This point has been summed up with great exactness by Cajetan:

> Cum ergo in littera dicitur quod mathematica non subsistunt, non est interpretandum quod universalia mathematica universaliter sumpta non subsistunt (hoc enim esset ridiculum pro ratione afferre); sed quod mathematica ut sic particulariter sumpta, non subsistunt; seu, quod idem est, quod mathematica ut sic, non habent aliquod individuum existens in rerum natura. Et propterea neque sunt in universali, neque in particulari: ac per hoc bona esse non possunt. Quod de aliis rebus universaliter sumptis dici non potest. Et sic patet nullitas consequentiae ad oppositum factae: et quare singulariter dicatur de mathematicis quod non habent esse.(6)

This, then, is the essential difference between mathematical abstraction and the other types of scientific abstraction: In physical abstraction there is a kind of separation from matter through simple apprehension. But the only kind of matter from which separation is made is individual matter. All the matter pertaining to the essence of the thing abstracted is retained. And this explains two things. First it explains why the separation cannot be transposed to the operation of judgment, for only individuals exist, and things which have matter in their essence must have individual matter to exist. Secondly, it explains why we can, nevertheless, make a judgment which predicates the abstract essence of actually existing things, for the predicate of a predication is a universal nature, and through physical abstraction nothing has been removed from the nature except individuation.

In metaphysical abstraction there is a separation from all matter, and this separation can be transposed to the operation of judgment, since there are beings existing without any matter. For the same reason, we can predicate metaphysical entities in their very state of separation or abstraction of actually existing things. As Cajetan points out: "Metaphysicalia secundum propriam abstractionem sumpta subsistunt: quoniam habent in rerum natura individua abstrahentia ab omnia materia sensibili et intelligibili, ut patet de intelligentiis."(9) Metaphysical abstraction differs from physical abstraction in that in the latter the separation cannot be transposed to the operation of judgment, and though the abstract entities can be predicated of reality, they cannot be predicated in their very state of separation.

In mathematics there is something different from either of these two types of abstraction. Like physics and unlike metaphysics, mathematics deals with things which depend upon sensible matter for their existence outside the mind (in the sense explained above). Like meta-

physics and unlike physics, it deals with things which are independent of sensible matter for their conception and definition. Like the case of physics and that of metaphysics there is separation from matter. Like the case of physics and unlike that of metaphysics this separation cannot be transposed to the judgment. Unlike both the case of physics (because the separation now has to do with matter which pertains to the very essence of things abstracted in so far as those things are real) and that of metaphysics the things abstracted cannot be predicated of reality.

But even this does not bring out with complete clarity the distinctive character of mathematical abstraction. Following leads given us by Cajetan(10) and John of St. Thomas(11) we can push the question a little further:

> Advertendum est ex Cajetano quod quantitas potest dupliciter abstrahi. Uno modo secundum abstractionem generis vel speciei ab individuis, remanente tota natura et quidditate quantitatis, sicut omnes aliae naturae quando in universali concipiuntur: et haec abstractio fit ab intellectu universalizante naturam; et hoc modo quantitas in abstracto consideratur a metaphysico et sic non omittit rationem perfectionis neque boni. Alio modo fit abstractio quantitatis denudando illam a sensibilitate, et fit per imaginationem: sicut imaginamur distantiam quantitatis in vacuo, lineas aut superficies in eo imaginantes; et talis abstractio non est universalis a particulari, sed solum quantitatis interminatae, seu imaginatae, a sensibili...(12)

We have already had the occasion to point out that it does not pertain to mathematics to consider the nature of quantity in itself, nor its ontological properties, nor even the nature and ontological properties of its two species: continuous and discrete. All this belongs to metaphysics. For quantity is a principle of being, one of the ten predicaments, and therefore comes under the object of metaphysics whose object is the being that is distributed through the ten categories. It is evident, then, that the mind is able to lay hold of quantity by another kind of abstraction than that found in mathematics. And it is clear from the passage just cited from John of St. Thomas that this abstraction is the kind we have been opposing to mathematical abstraction since the beginning of this discussion, that is, the universalizing abstraction, which considers quantity as a universal genus of being, apart from the real individuals in which it is realized. This abstraction lays hold of quantity in so far as it is a certain essence, a certain reality that exists ontologically. It considers quantity precisely in so far as it exists in reality as a principle of being, and not in so far as it is set off in a state in which it cannot exist in reality. It is to be noted that the metaphysical consideration of

quantity in some way abstracts from sensible matter (otherwise it would be a physical and not a metaphysical consideration). But it does not, like the mathematical consideration, explicitly separate it from sensibility, "denudando illam a sensibilitate," and explicitly set it off in a world apart from the real world. Rather, while not taking account of its sensible determinations, it considers it as it exists in reality along with the other accidents which constitute the structure of physical being. Mathematical abstraction, on the other hand, considers quantity not in so far as it is a principle of being, or a category of reality, or a certain form or essence, but from the point of view of the relations of order and measure that result when it is separated from all sensibility and set apart by itself.

It must be kept in mind that physical abstraction also lays hold of quantity in some way. For since quantity is the first accident, it is the matrix of all the sensible qualities, which consequently cannot be conceived of except in relation to it. All the mobility in the cosmos is inextricably bound up with quantitative determinations, and from this point of view quantity enters into the object of the study of nature. These quantitative determinations, incidentally, form the basis of the mathematization of nature. But they are only the basis, for in mathematical physics they are considered from the point of view of the mathematician and not that of the physicist. Quantity is also studied by the philosopher of nature in a very particular way, in so far as in living mobile beings there is found a special kind of mobility pertaining to the genus of quantity.

It is obvious that this consideration of quantity is quite different from that of the mathematician.

> Mathematicae ex vi suae abstractionis et conceptus, excludunt a quantitate statum sensibilem, nec considerant quantitatem secundum illam realitatem qua potest cadere sub sensu, sed secundum extensionem imaginabilem praecise; quia, ut diximus, ad demonstrationes mathematicas sufficiunt lineae et figurae in imaginatione formatae, quantum ad id quod extensionis, proportionis vel continuitatis considerari potest; non vero quantum ad id quod sensibilitatis est in tali quantitate, seu in quantum ens naturale est. (13)

There would seem, then, to be three distinct ways in which quantity may be laid hold of scientifically by the mind. First it may be considered explicitly in relation to sensible determinations, and in this way it is the object of the science of physics. Secondly, it may be considered as an ontological accident in so far as it exists in reality along with the sensible accidents - - abstracting

from them in some way, i.e. not explicitly as determined by them, and yet not explicitly as separated from them. In this way it is the object of the science of metaphysics. Finally, it may be considered as separated from all sensibility, set off in a state in which it cannot have actual reality, and contemplated precisely in terms of this abstract state. In this way it is the object of the sciences of mathematics.

All this makes it clear that mathematics not only deals with abstract things like the other sciences, but it deals with them precisely in so far as they are abstract. In this sense, Whitehead is justified in saying that "mathematics is the science of the most complete abstractions to which the human mind can attain." (14) The particular nature of the abstraction found in the mathematical sciences has not been generally recognized. Professor Lenzen, for example, writes: "The relational structure is a complex universal which may be exemplified in various instances, and hence the problem of the reality of mathematical objects is that of the reality of universals." (15) We hope that enough has been said to show that the problem of reality which results from the special kind of formal abstraction found in the mathematical sciences is something quite different from the problem connected with the "universalizing" abstraction found in the other sciences.

This consideration of the abstract character of mathematics brings us to an interesting paradox. In a sense it is true to say that by the very fact that it is the most abstract of all the sciences, it is also the most concrete. What we mean by that is that in a sense the mathematical universal is the same as the mathematical particular. For mathematical particulars abstract from sensible matter just as the universal does. "Materia sensibilis non includitur in intellectu mathematicorum neque in universali, neque in particulari." (16) Nothing extrinsic is added to a mathematical particular to individuate it. A particular circle _a_ or _b_ may be considered the universal circle.

This truth has considerable importance for our problem of mathematical physics as may be gathered from the following passage of Ernst Cassirer. While not subscribing to everything contained in this passage we believe that it brings out effectively the point we are trying to make;

> In his criticism of the logic of the Wolffian school, Lambert pointed out that it was the exclusive merit of mathematical 'general concepts' not to cancel the determinations of the special case, but in all strictness fully to retain them. When a mathematician makes his formula more

> general, this means not only that he is _to retain_ all the more special cases, but also to _be able to deduce_ them from the universal formula. The possibility of deduction is not found in the case of the scholastic concepts, since these, according to the traditional formula, are formed by neglecting the particular, and hence the reproduction of the particular moments of the concept seems excluded. Thus abstraction is very easy for the 'philosopher', but on the other hand, the determination of the particular from the universal so much the more difficult; for in the process of abstraction he leaves behind all the particularities in such a way that he cannot recover them, much less reckon the transformations of which they are capable. This simple remark contains, in fact, the germ of a distinction of great consequence. The ideal of a _scientific_ concept here appears in opposition to the _schematic general presentation_ which is expressed by a mere _word_. The genuine concept does not disregard the peculiarities and particularities which it holds under it, but seeks to show the _necessity_ of the occurrence and connection of just these particularities. What it gives is a universal rule for the connection of the particulars themselves. Thus we can proceed from a general mathematical formula, — for example, from the formula of a curve of the second order, — to the special geometrical forms of the circle, the ellipse etc., by considering a certain parameter which occurs in them and permitting it to vary through a continuous series of magnitudes. Here the more universal concept shows itself also the more rich in content; whoever has it can deduce from it all the mathematical relations which concern the special problems, while, on the other hand, he takes these problems not as isolated but as in continuous connection with each other, thus in their deeper systematic connections. The individual case is not excluded from consideration, but is fixed and retained as a perfectly determinate step in a general process of change. It is evident anew that the characteristic feature of the concept is not the 'universality' of a presentation, but the universal validity of a principle of serial order. We do not isolate any abstract part whatever from the

manifold before us, but we create from its members a definite relation by thinking of them as bound together by an inclusive law. And the further we proceed in this and the more firmly this connection according to laws is established, so much the clearer does the unambiguous determination of the particular stand forth. Thus, for example, the intuition of our Euclidian three-dimensional space only gains in clear comprehension when, in modern geometry, we ascend to the 'higher' forms of space; for in this way the total axiomatic structure of our space is first revealed in full distinctness. (17)

The mathematical universe is indeed a strange universe. Its abstract character gives it a high degree of intelligibility. And yet this intelligibility is extremely inadequate, for from the abstract mathematical entities we cannot arrive at actually existing things. The separation from matter gives it a perfection which the physical universe does not have. And yet, unlike the case of the separated substances, this removal of matter does not contribute to the perfection of natures. In fact, the separation from matter prevents mathematical entities from being natures. And yet, it is in the light of these entities that we shall try to understand the natures existing in the cosmos.

In order to add further precision to our notion of mathematical abstraction, it seems worth while, before leaving this question, to compare the way in which mathematical en-

tities are abstracted from the world of sensible matter and the way in which dialectical entities, such as the one discussed in the last chapter: the form of anger considered independently of the sensible matter to which it pertains, are abstracted. In both cases we have the abstraction of a form from the matter to which it belongs. But there is a vast difference in the way this abstraction takes place. In the case of the dialectical definition of anger, we have the form of a natural thing which is essentially inseparable from matter both for its being and for its "being known". Hence when it is set off by itself, it is in a purely logical state; it is a mere construction of the mind. Mathematical entities, on the other hand, are by their very nature separable from sensible matter *secundum intellectum*, even though they are not separable *secundum esse*. Consequently, when they are considered as seperated, they are in their natural state; they are not dialectical. Anger as a pure form is an *ens logicum*. A mathematical entity as a pure form is an *ens naturae*.

This brings us to the important question of the relation between mathematics and existence.

2. Mathematics and Existence

The question of the relation between mathematics and existence has been an acute philosophical problem ever since the time of the ancient Greeks. The analysis of the nature of mathematical abstraction has already thrown some light upon it. But the question demands closer attention. In fact, what we have seen thus far in a sense only serves to throw the problem into sharper focus. For if mathematical entities cannot exist as such in reality, must we not conclude that mathematics deals with <u>entia rationis</u> — logical beings? John of St. Thomas has gone to great pains in the <u>Cursus Theologicus</u>[18] to settle this question. Let us consider briefly his solution.

By a logical being we understand: "ens habens esse objective in ratione, cui nullum esse correspondet in re." Consequently, if mathematical entities were logical beings it would be absolutely contradictory for them to exist in reality. Now, from what we have seen about the nature of mathematical abstraction it should be evident that we cannot say in absolute fashion that the real existence of mathematical entities always involves a contradiction. For there is a sense in which it is true to say that some mathematical entities may exist in reality, not indeed in their state of separation from sensible

matter. We may say some mathematical entities, because there are obviously a good many mathematical entities, which are evidently mere logical beings, and whose real existence would necessarily involve a contradiction. An example such as the square root of minus one comes readily to mind. In fact, the whole point of John of St. Thomas' analysis is to show that mathematics, by the very nature of the abstraction it employs, remains indifferent to whether the entities it deals with are real or logical beings.

And he illustrates this point by having recourse to the example of predicamental relation. The essence of a relation consists in the ordering of one thing <u>to another</u>. But a relation may be of two kinds: it may be either real, that is, existing in reality, or it may be only logical, that is, created by the mind. A real relation is one of the nine accidental categories, and like all of the other accidental categories it has a real existence <u>in</u> the subject which it relates to something else. A logical relation does not have a real existence <u>in</u> the subject related, since it is the mind which creates the ordering. Now since the proper essence of relation which distinguishes it from all the other categories consists in the ordering of one thing <u>to</u> another, or in

Scholastic terminology, in the *ratio ad*, it is indifferent to either real existence (the *ratio in*) or purely logical existence. The *ratio ad* is common to both of these types of existence. In somewhat the same way mathematics is indifferent to whether the entities it deals with have real or only logical existence. In this way it differs from all the other sciences, and is a kind of medium between the science of nature and metaphysics on the one hand, and logic on the other. For both the science of nature and metaphysics deal necessarily with real beings. Logic deals necessarily with logical beings. Mathematics deals with either or both. It is true that *entia rationis* enter into both the science of nature and metaphysics, but their existence in these studies is purely functional, that is, the whole *raison d'être* of the construction of these *entia rationis* is to enable the philosopher of nature or the metaphysician to get to know reality; they do not constitute the object of these sciences, and are not considered for their own sake. In mathematics, however, the *entia rationis* are considered for their own sake. In this respect, mathematics is similar to logic. It differs from it, however, in that the *entia rationis* it considers are based on real beings which also constitute its object. In this sense Meyerson is justified in saying: "...chez le mathématicien,

réel et idée semblant en quelque sorte se confondre, on ne distingue pas immédiatement s'il traite de l'un ou de l'autre ...C'est là, encore un coup, la conséquence directe de l'accord de l'intellect et du concret dans la mathématique, et c'est ce qui fait de cet élément la vraie et unique 'substance intermédiaire,' dans le sens de Platon."(19)

As has already been suggested, this indifference on the part of mathematics to real or logical existence is something that arises out of the very nature of mathematical abstraction. As John of St. Thomas explains, it is precisely because mathematics considers quantity stripped of the definite determination and formation that it has in its state of union with sensibility that mathematical entities can be simple concepts capable of being realized in sensible matter, or concepts that have been elaborated by the mind into a state which cannot be realized in nature.

> Mathematica ex vi suae abstractionis et conceptus, excludens a quantitate statum sensibilem, non considerant quantitatem secundum illam realitatem qui potest cadere sub sensu, sed secundum extensionem imaginabilem praecise; quia, ut diximus, ad demonstrationes mathematicas sufficiunt lineae et figurae in imaginatione formatae, quantum ad id quod est extensionis, proportionis vel continuitatis considerari potest: nonvero quantum ad id quod sensibilitatis est in tali quantitate, seu in quantum ens naturale est. Et sic apud Aver-

> rorem et alios antiquos considerabatur quantitas interminata et terminata; et illa interminata dicitur quae praecise extensionem considerat secundum quod praecise sequitur ad materiam, quantum ad id quod de extensione potentiali et formabili dicit; terminata vero quantitas est illa quae sub certa terminatione et formatione concipitur, et sic redditur sensibilis; . . . ita mathematica considerat quantitatem quantum ad id praecise quod habet de extensione interminata, et secundum id quod habet a materia; non secundum terminationem et modum quem habet a forma, ratione cuius redditur sensibilis. Quare quantitas mathematica habet conceptum positivum quantitatis interminatae, eo modo quo quantitas potest inveniri, sive imaginarie, sive sensibiliter in ratione entis veri. Unde permissive se habet ad rationem entis realis et veri; neque positive includendo et considerando adaequate, neque positive excludendo per repugnantiam, realitatem ipsius quantitatis. Et in hoc differt a quantitate pure imaginaria, quae est ens rationis; haec enim repugnanter se habet ad quantitatem realem, quia ens rationis est. At vero quantitas mathematica non repugnanter se habet, sed indifferenter; quia aeque bene potest facere suas demonstrationes in eis realibus, vel imaginariis; sicut si relatio consideretur secundum rationem *ad* praecise, nondum consideratur ut ens rationis; nec tamen ut determinate ens reale: sed indifferenter ad illud; quia non consideratur adaequata ratio eius ex omni parte quae requiritur ad realitatem, ad quam etiam requiritur ratio *in*: sed ex ea parte qua indifferens est ad realitatem, et solum explicat rationem *ad*. Sic quantitas consideratur a mathematico inadaequate, et sub ea ratione praecise extensionis interminatae: quae indifferenter se habet ad imaginariam et realem, et sic non excludit rationem entis, sed permittit; neque repugnanter se habet ad illud, sed indifferenter. Unde nec ens rationis est determinate, nec ens reale determinate: sed indifferenter et permissive se habet ad utrumque. Quod non solum contingit in ratione entis in communi, quae abstrahit ab ente reali, et rationis: sed etiam in relatione, quae abstrahit a reali, et rationis, secundum inadaequatum conceptum *ad*: et in quantitate

> quae abstrahit ab imaginaria et sensibili, sub inadaequate conceptu extensionis interminatae. (80)

All this helps us to understand more accurately the meaning of the phrase to which we have already given some consideration: "mathematica dependent a materia secundum esse". The primary meaning is that while it doesn't pertain to the essence of a mathematical entity to be capable of realization, whenever it is capable, the realization always takes place in matter. But there is another important meaning which can also be attached to this phrase: in every mathematical entity, capable of realization or not, there is always an essential relation to matter. If prime matter were impossible, mathematics would also be impossible. Since prime matter is the principle of homogeneity, and since homogeneity is the fundamental postulate of all mathematics, there is obviously no possibility of mathematical science without an intrinsic reference to prime matter. But the important point is that while always intrinsically dependent upon matter, mathematical entities are not always necessarily capable of realization in matter, for the capability of realization does not enter into their intrinsic formality.

It is equally false to say that mathematical entities have this capability, or that they do not have it. In themselves they are indifferent.

But this may seem to involve us in a contradiction, or at least in a sophism. For in discussing the nature of mathematical abstraction we stated that mathematical entities as such are not capable of realization in nature, and now we seem to admit the possibility of their realization. The contradiction here is only apparent; both statements are correct, provided they be rightly understood. And it is precisely because the mathematical world is so strange that it gives rise to apparent contradictions of this kind. In the first place, it is obvious that abstract things are not capable of realization in their abstract state. In this sense not even the concepts arrived at by mere univerisalizing abstraction which lifts them out of individuation have such capability. But as we saw above, mathematical entities are incapable of realization in a deeper sense than this. For not only does mathematical abstraction lift them out of the accidental determinations of individuation, but it separates them from an element that pertains to their very essence if that essence is to be real. Mathematical entities are not

capable of realization, therefore, in the sense that they cannot exist in their state of separation from sensible matter. On the other hand there is a sense in which they are capable of realization, for there are actually existing lines and circles and a plurality of quantified things. These may be considered the realization of mathematical lines, circles and number. It is true that the realization is not perfect. Mathematical entities cease to be truly mathematical once they are realized. The realization robs them of the ideal purity and perfection they possess in their state of abstraction. The straight lines in nature are not perfectly straight, nor are natural circles perfectly circular. It would be a mistake to identify the mathematical zero with the philosophical concept of nothingness, or to confuse mathematical number with a plurality of natural beings. And all this results from the nature of mathematical abstraction which does not seize upon the ontological essence of the things it abstracts. On the other hand, the relation between mathematical lines and circles and the lines and circles existing in nature is not the same as that existing between logical beings and their foundation in reality. We cannot say that logical beings are realized in their objective foundation, as we can say that mathematical lines and circles are realized in the lines and circles of nature.

All this makes it clear that mathematical being is a medium between possible being, arrived at by universalizing abstraction, and logical being. Possible being prescinds only from the actual exercise of existence; it retains an intrinsic order to real existence. Mathematical being, by the very fact that it is indifferent to either real or logical existence, prescinds not only from the actual exercise of existence, but also from any intrinsic order to existence; on the other hand, it does not absolutely exclude the possibility of actual existence. Logical being not only prescinds from real existence; it positively excludes it.

The mathematical world is indeed a strange world. In it mind and nature, the real order and the ideal order are in some sense fused. On the one hand, mathematical being is not a pure creation of the mind; on the other hand it is not a pure discovery of the mind. For since mathematical abstraction never lays hold of quantity in its ontological essence, a mathematical entity is never a property of reality.

On the one hand, mathematical entities prescind not only from actual existence but from an intrinsic order to real existence. On the other hand, mathematical being has a necessary relation with the real, and the character of this relation is unique, for it never retains the ontological essence of the thing with which it is connected. Even the mathematical entities which are capable of realization in nature have an ideal character about them which they lose by this realization. Even those which are not capable of realization in nature are in one way or another elaborations of something that is capable of realization. At the basis of the whole mathematical structure is something found in reality: quantity taken by itself with its proper forms and specifications and relational structures. But right from the start the mind lays hold of this quantity in such a way as to establish its own priority and its own autonomy. For, as has been said repeatedly, it does not grasp its ontological nature; to do that would mean a complete submission of mind to ontological reality. Rather, it transforms quantity into a condition that is especially congenial to its own nature: it establishes it in an abstract state and deals with it precisely as abstract. By so doing the mind acquires for itself a freedom that is almost unlimited. Though dealing with things originally connected with sense matter, it no longer has to be concerned with having its processes terminate in the external senses. There remains an intrinsic connection with the intuitive imagination, but as the mind exploits its freedom and pursues its process of intellectual elaboration this connection can be stretched to extreme limits of tenuity. And as the intellect takes fuller advantage of its liberty, it will tend more and more to impose its own nature upon the mathematical world. There will be an inevitable growth in spiritualization. The concreteness and potentiality of the continuum will tend to be absorbed by the greater abstractness and actuality of number. There will even be a reaching out beyond the confines of quantity itself to transcendental multitude and pure logical relations. And all this is perfectly legitimate, provided the intellect remains critically conscious of what it is doing. And in this intellectual movement, the mind is not bound down to dealing with real entities; it has at its disposal the vast possibilities of logical being. But in the last analysis it remains true that all logical mathematical beings are founded upon real mathematical beings, and that these real beings have by a process of mathematical abstraction been lifted out of actual experience with the real world. Thus the whole mathematical structure is rooted in

real quantity — the same quantity which the philosopher grasps ontologically.

All this is extremely important for the problem of mathematical physics. As has already been suggested, mathematical physics does not mean the discovery of the mathematical world in the physical world. Nor does it imply the direct realization of the mathematical world in the physical world. Rather, it is a question of application. And by application we mean an intellectual interpretation of the cosmos which always remains in some sense extrinsic to the cosmos. This is true even when physics employs mathematical entities which are real beings and which are consequently capable of realization in the sense defined above. For, as we have already pointed out, when these entities are employed in physics they retain their mathematical character. In other word, they are applied to the physical world in their abstract state. It is the mathematically perfect straight line that the physicist has in mind when he tells us that light is propagated in a straight line.

If the use of mathematical entities which are real beings is always an extrinsic application, that is <u>a fortiori</u> true of the use of those entities which are merely logical beings. And it is extremely significant to understand that by the very fact that it is a question of an extrinsic application, it is possible for logical mathematical beings to be more fruitful in the interpretation of the cosmos than real mathematical beings. As we have already pointed out, mathematical physics is essentially a doctrine of als ob. That is why a logical being may be able to "explain" better than a real being. And this point has a direct bearing upon the highly disputed question about whether the cosmos is Euclidian on non-Euclidian. We do not wish to attempt a solution of this question here. But there are a few things that must be pointed out as to the meaning this problem must have. First of all, to say that our cosmos is Euclidian cannot mean that Euclidian geometry as such, that is, in its ideal geometrical state is realized in nature. Nor does it necessarily imply that Euclidian geometry is capable of "explaining" the cosmos with greater accuracy and fruitfulness than any other geometry. It can only mean that the mathematical entities which make up the structure of the Euclidian system are real beings and are capable of realization in nature in the sense explained. Moreover, this question cannot be solved by an appeal to the relative explanatory powers of the different geometries.

For it is possible for a Euclidian universe to be more rational for us when interpreted in terms of Riemannian geometry than when interpreted in terms of Euclidian geometry. That is why most of the arguments adduced by those who try to prove that the physical universe is non-Euclidian are inefficacious. The question is further complicated by the highly ambiguous meaning of "physical universe." But we do not wish to enter into the problem at this point.

In connection with this problem and with the general question of the relation between mathematics and existence, the oft-quoted remark of Sir James Jeans comes to mind: the cosmos was created by a pure mathematician. As we know, Jeans was lead to this conclusion because of the remarkable way in which modern physicists have been able to fit the most abstruse constructions of higher mathematics upon the material universe. But from what has just been said it is clear that this successful and fruitful application does not constitute a sufficient premise for such a conclusion. Moreover, it is worth while pointing out that there is a profound opposition between the concepts of a pure mathematician and a creator of a material universe. The pure mathematician is indeed a creator, but a creator in the abstract speculative order. And the world he constructs is, as we have seen, not only cut off from concrete existence, but even from any intrinsic order to concrete existence. He deals with the abstract as abstract, and the whole movement of his science is in the opposite direction from any embodiment in the matter and motion which go to make up the substance of the material universe. In another work Jeans states: "Kronecker is quoted as saying that in arithmetic God made the integers and man made the rest; in the same spirit we may perhaps say that in physics God made the mathematics and man made the rest." (21) Our analysis of the nature of mathematical abstraction has led us to a somewhat different conclusion, and while it would not be completely true, it would be much closer to the truth to say: in physics, man made the mathematics and God made the rest.

And now perhaps enough has been said to make it clear that mathematics and logic cannot be identified. The confusion between the two generally derives from a confused notion of the nature of logic. Nor are those who maintain this identity with such zeal always anxious to explain what they mean by logic. The science of logic is essentially a reflective science in the sense that its object is what

is known in scholastic terminology as "second intentions." That is to say, it considers what the mind knows in the other sciences, precisely as known by the mind. Mathematics is not a reflective but a direct science. It does not deal essentially with second intentions. It has as its object a proper realm of knowable "natures". That is why it cannot be identified with logic.

This discussion of the relation between mathematics and existence would not be complete unless at least passing mention were made of the question of whether mathematical beings have the property of goodness. The ancient Thomists paid considerable attention to this question. In fact it is principally in connection with it that they discussed the problem of the relation of mathematics to existence. And briefly their solution was this: precisely because mathematical being prescinds not only from existence, but even from any intrinsic order to existence, it necessarily lacks the property of goodness. For the good is whatever can be the object of an appetite, and appetite has a necessary connection with the existential order. Or, to present the question in a slightly different fashion: because the mathematical world prescinds from all order to existence, it is an immobile world of pure essences - - essences which in no sense are natures. Consequently, in this world there is no becoming, no seeking for ends, no finality. And without finality there is no goodness. For the good is formally defined as: perfectivum alterius per modum finis.

> In immobilibus non contingit aliquid esse per se bonum. Unde in mathematicis nihil per hanc causam probatur, neque est aliqua demonstratio.(22)

> Mathematica non subsistunt separata secundum esse; quia si subsisterent, esset in eis bonum, scilicet ipsum esse ipsorum; sunt autem mathematica separata secundum rationem tantum, prout abstrahunt a motu et a materia; et sic abstrahunt a ratione finis, qui habet rationem moventis. (23)

This doctrine must be taken in the strictly formal sense in which it was understood by the ancient Thomists. It refers only to mathematical being considered intrinsically. For it is evident that extrinsically finality may enter into mathematics, and with it goodness. Mathematical being can be an end and a means to an end, and thus in both ways involve finality. In the first place, it can be an end in the speculative order in so far as there is truth in mathematics and truth is the good of the mind. But as John of St. Thomas points out,(24) this does not make mathematical beings intrinsically good, just as the knowledge of evil things may be a good for the mind without making the evil things good. Mathematics may be good as a means in relation

to the practical order, as is evident from the large part that mathematics plays in technology. It may also be good as a means in the purely speculative order. In this sense mathematics is a good for the physicist in so far as it becomes for him an instrument to open up the meaning of the universe. In fact, it is the goodness of a mathematical theory which primarily determines its acceptance or rejection by the physicist. For, as we shall see in Chapter XI, there is a sense in which it is true to say that theories are neither true nor false; they are only good or bad. From this point of view a scientist is essentially a pragmatist.

And this brings us to the question of whether or not there is truth in mathematics. Since the world of mathematics is a world of essences which constitute an object knowable by the mind, it is evident that there is truth in mathematics. But since this world of essences is separated off by itself without even an intrinsic order to existence, it is likewise evident that this truth is of a very special sort.(25) For the definition of truth as the conformity of the mind with existing reality cannot be characteristic of a world which is cut off from existing reality, and in which logical beings are accepted on equal terms with real beings. The truth characteristic of such a world cannot consist essentially in a relation, one of whose terms is found in existence, but in a relation, both of whose terms are found within the realm of essence, or in other words, in intrinsic coherence. And that explains why mathematics is the most deductive of all the sciences. Free of any necessity of conforming to an objective order, it can follow out rigorously its own inner logic. It does not, like philosophy, have to keep in constant touch with experience. It affords the one chance that the mind has to triumph completely over mere givenness. It is worthwhile noting here that the coherence notion of truth is proper to the science of mathematics. Every other science, including logic, employs the conformity notion. From this point of view, mathematics is even more detached from the real than logic, although from another point of view, as we saw above, it is in closer relation to it. It is also worth while pointing out that the word "real" is often substituted for the word "true". For a mathematician whatever is mathematically true may be considered real. And this adds to the ambiguity of the question whether real space is Euclidian or non-Euclidian. The special meaning which truth has in mathematics is of

great importance for our problem. For a physicist by the very fact that he is a student of nature, must adhere in so far as he is able to the conformity notion of truth. What happens when these two notions of truth are brought together in mathematical physics we shall see later when we come to discuss the relation between the physico- mathematical world and the absolute world condition.

Though without goodness, mathematical beings possess beauty as well as truth. For as St. Thomas points out: "pulchrum proprie pertinet ad rationem causae formalis." (26) And thus Aristotle writes:

> The chief forms of beauty are order and symmetry and definiteness, which the mathematical sciences demonstrate in a special degree. And since these (e.g. order and definiteness) are obviously causes of many things, evidently these sciences must treat this sort of causative principle also. (i.e. the beautiful) as in some sense a cause. (27)

These remarks are not gratuitous, for the beauty of mathematics sometimes prevents the scientist from recognizing the essentially functional role that mathematics plays in physics. When that happens, the end of mathematical physics is made a means, and the means an end, and the scientist becomes, as Professor Babin has remarked, "un artiste égaré ou frustré."

This consideration of the nature of mathematical abstraction and of the detachment from existence that is consequent upon it helps us to understand the kind of causality that is found in the mathematical world. A world which is the result of formal abstraction in the strictest sense of the term, that is, an abstraction which detaches pure forms from the material embodiment in which they belong and sets them off by themselves, can be endowed with formal causality alone. In other words, in abstracting from matter, the mathematical world excludes material causality. Furthermore, the abstraction from matter involves abstraction from mobility, since mobility follows upon matter. Hence the mathematical world prescinds from both efficient and final causality, which are, as it were, the two causal terms of mobility. Or, to put the matter in a slightly different way, in detaching itself from existence the mathematical world detaches itself from coming into existence, or becoming, and only formal causality can exist where there is no becoming, since the other three causes have an analytical relation with coming into existence.

This point is of supreme importance for a correct understanding of the nature of mathematical physics. For the scientist, by the very fact that he is a physicist,

must endeavor to know the cosmos in terms of all four causes. But by the fact that he is a mathematical physicist, and that he must interpret the cosmos in the light of mathematics which is the formal element in his study, whereas the physical is only material, he can see things only in terms of formal causality. What happens when these two tendencies meet we shall consider in some detail in Chapter IX.

The paradox of studying a universe in which efficient, final and material causes are essential in the light of a science which positively excludes all but formal causality is in the last analysis reducible to the paradox of introducing into a science whose object is essentially mobile being the principles of a science which absolutely excludes all mobility. We do not intend to consider this problem here, but perhaps it would be well at this point to eliminate a possible source of confusion. For it might be argued that there is mobility in the mathematical world, since the infinitesimal, vectorial and tensor calculus, for example, deal with the idea of variable quantities and the function concept. Thus we can speak of an infinitesimal as a quantity which approaches zero as its limit. Moreover, the inherent constructibility of mathematical entities seems to involve motion, for we can speak of a surface being generated by a moving line.

There is indeed motion of a sort in the mathematical world. But it is merely dialectical and not real. It is a purely imaginary and intramental thing, and does not involve becoming in the true sense of the word. Mathematical entities do not come into being; and they are neither the principle nor the terminus of becoming. We may have recourse to an imaginary movement in order to generate the figures, but that is due to the imperfection of our knowledge. The figures themselves do not originate that way.

Moreover, the exclusion of real motion from the mathematical world does not eliminate the possibility of an application of mathematics to real motion. For, as we have already pointed out, quantity is the primary accident and the matrix of all the others. And that is why all of the determinations of mobile being are endowed with a quantitative mode. This quantitative mode may be laid hold of, and treated mathematically. But we shall come back to this point later.

3. Mathematics and the Intuitive Imagination.

It is clear from the foregoing that, unlike physics, mathematics does not receive its subject from the external senses. It is true that mathematical entities are derived originally from sense experience. For example, we form our notion of a circle only after having experienced a concrete perceptible circular object such as a ball. But this sense experience has only a pre-scientific function. It is required by mathematics only as a presupposition, not as an intrinsic element in the science itself, as it is required by physics. Once derived from sense experience, mathematical notions by virtue of mathematical abstraction, become independent of sense experience. They are stripped of the experiential context in which they were discovered and invested with a new, idealized, non-sensible character. That is why mathematical judgments do not have to terminate in sense experience.

Recently a number of authors have called into question this detachment of mathematics from sense experience. For example, Professor Hogben whose popular book, Mathematics for the Million, is written from the point of view of dialectical materialism even to the extent of being overt propaganda, says: "The statement AB = CD does not mean 'the line AB is exactly equal to the line CD,' because no one knows how to make exactly equal lines with any actual compass or rule. Its correct translation is '<u>measure</u> AB <u>to get</u> the length of CD as accurately as you need it.'"(28) And as a refutation of the proposition that a straight line is the shortest distance between two points he cites the example of an experiment made on a shrimp whose directional movements are controlled by a certain organ connected with the nervous system. If this organ is filled with steel fillings, the shrimp swimming in a magnetic field will move in curves since the lines of force in the magnetic field are curved. Consequently for the shrimp a straight line is not the shortest distance between two points.(29) We do not consider it necessary to give an explicit refutation of this view of the nature of geometry. So much has already been said about the essential abstraction of mathematics from sensibility that it would be superfluous to labor the point any further. Nor does recourse to the etymology of the word geometry which signifies the science of surveying afford any rational basis for the advocates of "physical" geometry. In recent years the so-called "concrete" methods

of teaching geometry have become increasingly popular. Whatever we may think of these methods as a pedagogical device to gradually prepare the mind for the effort of mathematical abstraction it is evident that one does not really enter into the realm of geometry until this abstraction has been achieved.

Einstein's views on the nature of geometry are relevant here. In his book <u>Geometry and Experience</u> he divides geometry into two distinct branches. The first consists in purely formal knowledge based on axioms that are free creations of the human mind and made up of schematic concepts that are empty of all content. The second is called practical geometry; it is a natural science, and is in fact the most ancient of all the branches of physics. Taken as it stands, this opinion of Einstein is really a denial of the true nature of geometry. For his first branch of geometry seems to be nothing but dialectics, and if his second branch is identified with physical science, there is no place left for a specifically distinct and proper science of geometry.

Once again we do not feel it necessary to enter into a refutation of these views. They have been introduced here to bring into focus the point to be discussed in this section, namely that while on the one hand mathematics is independent of sense experience and hence not to be identified with physical science, on the other hand it is not independent of all reference to sense, as dialectics may be.

Though detached from the external senses, mathematics has an essential connection with the internal sense of imagination. It is in the intuitive imagination that all the judgments of mathematics must terminate, either directly and immediately, or at least reductively. And this brings home to us once again the intermediary character of mathematics. Unlike physics and like metaphysics it is independent of external sense experience. But unlike metaphysics and like physics it still retains a terminal connection with sense life. Mathematics is at once both more free and less free than metaphysics. It is more free in that unlike metaphysics it not only does not have to terminate in sense experience, but its judgments do not have to correspond with anything that is given in objective reality. It is less free in that it has to terminate in the intuitive imagination. It is because of having abandoned this intrinsic connection with imaginative intuition that

modern mathematicians have arrived at the notion of mathematics as a science that is empty of any objective content, as a science that is in the last analysis identified with logic. It is evident that the true view of the nature of mathematics holds a middle course between the "concrete" notion of mathematics which seeks to establish an intrinsic connection between it and external sense experience, and the purely axiomatic notion which severs all connection with the internal sense. Both of these extreme views will evidently have repercussions upon our problem. By holding the first position one would be lead to believe that mathematical physics consists in discovering the mathematical world in the physical world. By holding the second one would be forced to conclude that mathematics provides the empty forms to which physics gives objective content, or that mathematics reveals the essential rules of the game which the scientist plays with the physical universe.

Mathematics and the imagination hold a parallel relation to external sense experience. Like mathematics, the imagination is dependent upon the external senses only as a presupposition. Once it has received its material from them, it can to some extent detach this material from the perceptual context from which it was drawn, that is to say from the external physical conditions which embodied it originally; like mathematics, it can construct and reconstruct this material into new forms and patterns; it can create new entities only remotely connected with the material to which they owe their origin. And the reason why mathematics must retain some connection with the imagination is that though freed from the determinations of sensible qualities, it is not freed from all materiality and hence it must in some way remain bound up with a cognitive power related to materiality. Though prior to the whole sensible order by reason of its being the primary accident, quantity is nevertheless known to us only through sensible determinations, and hence even after it has been detached from sensible qualities there is still something of sense clinging to it. It is the imagination which, though a sense faculty and thus essentially distinct from the intellect, is nevertheless in the existential order bound up so inextricably with the workings of the intellect, which makes it possible for mathematics to retain its orientation towards sense, even though it is so far advanced in the order of intelligibility. The object of mathematics is never purely intelligible.

But this connection of mathematics with the

imaginative intuition must be rightly understood. In the first place, the intuitive schemes which the imagination presents are not in themselves the object of mathematics; they are only the sensible illustration of that object. Morever, not all branches of mathematics are equally dependent upon these intuitive schemes. As has already been pointed out, arithmetic, because of its more abstract character, is more remotedly connected with the imagination than geometry. For any kind of phantasm will serve to represent number, provided there is plurality; but only a very definite kind of phantasm will serve to represent a circle of a triangle. And as mathematics takes fuller advantage of its inherent liberty, and as it follows its natural tendency towards higher abstraction and spiritualization, the connection with the imagination becomes increasingly attenuated. It would be ridiculous to maintain that all mathematical entities must be capable of direct and perfect reconstruction in the imaginative intuition, and that in this sense all of the judgments of mathematics must terminate immediately in the imagination. Such an assertion would limit mathematics to an infinitesimal fraction of its actual range.

But it is impossible to have an adequate notion of the orientation of mathematics towards the imagination without seeing the essential relation which the imagination has with intelligible matter, which enters intrinsically into mathematical abstraction. We have explained that mathematics, while prescinding from sensible matter, clings to intelligible matter. "Non possunt (mathematica) considerari sine intellectu substantiae quantitati subjectae; quod esset eas abstrahi a materia intelligibili communi."(30) By intelligible matter is understood the material substance as determined by quantity in so far as quantity is the order of its parts. Why it is called intelligible matter is explained by St. Thomas: "substantia enim remotis accidentibus non remanet nisi intellectu comprehensibilis, eo quod sensibiles potentiae non pertingunt usque ad substantiae comprehensionem. Et de his abstractis est mathematica."(31) Though this matter is rightly called intelligible, it has an intrinsic connection with the imagination, precisely because it is matter. For mathematical forms are not purely intelligible as metaphysical forms are. They are like natural forms in that they are in matter. "Sicut naturalia habent formam in materia, ita et mathematica."(32) And just as the presence of sensible matter in the object of the study of nature makes it necessary for

sense experience to enter into the understanding of this object, so the presence of intelligible matter in the object of mathematics makes it necessary for the imagination to play a part in mathematical intellection.

> In his quae sunt per 'abstractionem, idest in mathematicis quorum ratio abstrahit a materia sensibili, rectum se habet sicut simum. Haec enim mathematica habent materiam, sicut et naturalia. Rectum enim mathematicum est, simum autem naturale. Ratio enim recti est cum continuo, sicut ratio simi cum naso. Continuum autem est materia intelligibilis, sicut simum materia sensibilis. Unde manifestum est, quod aliud est in mathematicis res et quod quid erat esse, ut rectum et recte esse; unde oportet quod alio cognoscat quod quid erat esse horum, et alio ipsa. . .
> Unde sicut per naturalia ostenditur, quod intellectus, qui cognoscit quidditates naturalium, sit alius a sensu qui cognoscit ipsa naturalia singularia, ita ex mathematicis ostenditur quod intellectus qui cognoscit quod quid est ipsorum, sit aliud ab imaginative virtute, quae apprehendit ipsa mathematica. (33)

It is clear from this last quotation that intelligible matter plays the part of the material element in mathematical definitions.(34)

The principal role played by the imagination in mathematics in connection with intelligible matter has already been pointed out in Chapter II. From what has been said about the nature of intelligible matter it is evident that it provides the homogeneous exteriority that is at the basis of the whole mathematical structure. Now homogeneous exteriority means a multiplication of the same form - such a multiplication is impossible without individuation. And this individuation must take place in the imaginative intuition. For since mathematical entities are stripped of sensible qualities, the individuation cannot be effected by qualitative determinations grasped by the senses. On the other hand, the intellect of itself has to do with pure form separated from matter, and hence if it alone functioned in mathematics we could have no notion of homogeneous multiplicity. For things that are outside each other because of the form are formally different, hence heterogeneous. Speaking of Plato's doctrine of the intermediary position of mathematics, Aristotle says: "Further, besides sensible things and Forms he says there are the objects of mathematics, which occupy an intermediate position, differing from sensible things in being eternal and unchangeable, from Forms in that there are many alike, while Form itself is in each case unique."[35]

There remains just one last point of which passing mention must be made before we bring this discussion to a close. In his Commentary on the Posterior Analytics

St. Thomas explains that intelligible matter is *ipsa continuitas*.[36] Taken in its strictest sense, then, it is essential only to geometry. Nevertheless, even arithmetic must terminate in the imagination in some way, in so far as number is caused by a division of the continuum.

4. Mathematics and the Human Mind.

There are a number of reasons why physics inevitably reaches out to mathematics for illumination, and some of them have already been touched upon. But at this point we wish to call particular attention to one of the most significant causes of this natural gravitation: the profound congeniality existing between mathematical science and the human mind. Since the time of the Renaissance when mathematics commenced the phenomenal development which has brought it to its present high point of perfection, and when physics began to be increasingly quantified, the fact of this connaturality has been clearly recognized. Kepler is quoted as saying that our minds are so constructed that they can know nothing perfectly except quantities. "Just as the eye was made to see colours, and the ear to hear sounds, so the human mind was made to understand, not whatever you please, but quantity." (37) And Descartes' insistence on the close relation between the mind and mathematics is too well known to need being mentioned. But while the fact of this congeniality has become obvious, the reason for it has not been so clearly recognized. It is significant that while in comparison with modern developments mathematical science and the quantification of physics were only in an incipient state at the time of Aristotle and St. Thomas, both of these philosophers not only grasped the fact of the intimate relationship between the intellect and mathematics, but also gave a clear and adequate explanation for it. (38)

As Aristotle points out, (39) difficulties which stand in the way of the mind's perfect union with a scientific object may come either from the mind or from the object. In the case of metaphysics, the difficulties come from the weakness of the human mind. For metaphysical objects because of their complete separation from all matter are of all scientific objects the most knowable in themselves. But in relation to the human mind they are the least knowable. For their high degree of immateriality keeps them from being within easy reach of an intellect which is essentially united with matter

and which must derive all its knowledge from the material world through the medium of organic faculties. In relation to metaphysical objects, as Aristotle goes on to explain, the human mind is like the eye of the owl for which the light of day is too bright to see well, and which can see with greater clarity in the obscurity of night. And this explains why for Aristotle and St. Thomas metaphysical wisdom was something too divine to be possessed by man except in a very inadequate and precarious fashion, something rather loaned to man than actually given to him outright.

In the case of physics, on the other hand, the difficulties come from the object. For cosmic things, immersed as they are in matter and in the flux of mobility, are essentially obscure. It is true that by remaining in generalities the mind may triumph over this obscurity to some extent. But as it pursues its inevitable progress towards concretion, the light and certainty deriving from generality gradually fades. Now modern experimental physics is a stage in the study of the cosmos that is far advanced towards concretion. That is why its object is doubly obscure. It is obscure first of all because it is cosmic reality of matter and motion; it is obscure, secondly, because it attempts to get at this cosmic reality in its concretion. In experimental physics the human intellect is caught in a kind of anguish. From a certain point of view, it is in a realm that is most proper to it. For since it is human, its proper object is the essence of material things; and since it is an intellect it is impelled to know them not just in a general way but in their proper specific concretion. And yet by following this instinct of its nature it inevitably becomes immersed in deeper and deeper obscurity.

Now mathematical science occupies a privileged position between these two extremes. On the one hand, since it abstracts from matter and motion, its object is more intelligible *in se* than that of the science of nature. On the other hand, since it is not completely immaterial, since it always retains an essential connection with the imagination from which the human intellect derives all its concepts, it is more intelligible for us than that of metaphysics. "Sed mathematica sunt abstracta a materia, et tamen non sunt excedentia intellectum nostrum: et ideo in eis est requirenda certissima ratio." [40]

Another reason for the connaturality of mathematics

with the human mind is given by Aristotle and Saint Thomas in the sixth book of the *Ethics*. [41] The intellect finds the science which deals with sensible things difficult because it demands a great deal of experience; it finds the study of metaphysics difficult because it transcends the imagination and is free of all reference to sense. In between these two extremes stands mathematics, "quae nec experientia indigent, nec imaginationem transcendunt." One of the signs of this connaturality is the comparatively frequent occurrence of child prodigies in mathematical science — a phenomenon that is not found in the other speculative sciences. [42]

This profound attraction which mathematics has for the intellect can constitute a danger. For it is easy for the mind to try in one way or another to reduce all knowledge to mathematical knowledge, and to reject whatever does not prove amenable to this reduction. Descartes, we know, fell a prey to this tendency. As St. Thomas remarks, "quidam non recipiunt quod eis dicitur, nisi dicatur eis per modum mathematicum." [43] It is true, as Aquinas goes on to explain, that a similar monistic tendency is sometimes found with regard to other types of knowledge. But the danger is more acute in connection with mathematics because of the connatural attraction of which we have been speaking. And that is why Aristotle and St. Thomas insist that the study of nature must not be reduced to a kind of mathematics:

> Ostendit quod ille modus, qui est simpliciter optimus, non debet in omnibus quaeri; dicens quod 'acribologia' idest diligens et certa ratio, sicut est in mathematicis, non debet requiri in omnibus rebus, de quibus sunt scientiae; sed debet solum requiri in his, quae non habent materiam. Ea enim quae habent materiam, subjecta sunt motui et variationi: et ideo non potest in eis omnibus omnimoda certitudo haberi. Quaeritur enim in eis non quid semper sit, et ex necessitate; sed quid sit ut in pluribus."(44)

From all that has been said thus far it is clear that this passage does not intend to exclude the possibility of an application of mathematics to the study of nature. It is merely trying to point out that this application is not an identification.

But we have not yet fully explained the connatural attraction which mathematics exercises over the intellect. There is an innate tendency in the human mind to see one thing in another. This is the root of all scientific endeavor, whose purpose is to see things in their causes. And the source of this tendency we know: every intellect is a reflection

of the divine intellect which sees all things in their proper specification and in their ultimate concretion in the light of the one divine essence. And not only does every intellect seek to grasp one thing in another, it also seeks to construct otherness out of sameness. It strives to become like the divine intellect by constituting itself prior to things, by making itself the creator of its own object. Because the human intellect is human it will always in some measure be subjected to givenness; but because it is an itellect it will strive to triumph over this givenness by making itself the source of the things it knows, thus dominating its object completely. Now the unlimited constructibility of the mathematical world provides the fullest freedom for this tendency of the mind. In mathematics the intellect is able to construct its own object. From a point it is able to construct a line, from a line a plane, from a plane a solid, etc. And it is only after the construction of the subject that the properties of the subject become manifest. Thus the mind constructs the source of these properties. It does not as in the other sciences merely discover the properties and allow them to lead it to a knowledge of a given subject. In all the other sciences the subject is givenness there is obscurity.

Mathematical abstraction has this unique privilege that the most knowable *in se* is the most knowable for us. In the other two types of formal abstraction, the most knowable for us is the least knowable *in se*. Unlike the fundamental principles of the other speculative sciences, the principles of mathematics are at the same time universal *in praedicando* and universal *in causando*. And that is why the whole mathematical world is deducible from a few fundamental principles and postulates. And this explains why in some way mathematics is like wisdom, as Cournot has remarked: *sophias germana mathesis.* For it is the property of wisdom to reveal all things in the light of an original source, and the perfect deductibility of mathematics enables the mind to see the whole mathematical world as flowing out of the original postulates. And since, as we explained above, mathematical particulars are abstract, and in some sense identified with universals, this process of mathematical wisdom is able to reach even particulars. In a way, mathematics satisfies the mind's instinct for wisdom even better than metaphysics, for since in metaphysical abstraction the best known for us is the least known *in se*, the whole

metaphysical world can not be drawn out of the original principles. That is why after the mind has pursued its course from the original generalities up through the angelic universe to the divine being it must, in order to satisfy its quest for wisdom, complete its study by having recourse to a dialectical process by which the multiplicity of things are derived from the divine source.

In our introductory chapter we pointed out that Plato conceived the mathematical world as occupying a kind of intermediary position, and we suggested that this was an extremely profound and fruitful insight. There are, in fact, many ways in which mathematical being is truly a medium. Some of them have been touched upon and others could easily be adduced.(45) But here we wish to call attention to one particular aspect of this intermediary character of mathematics, for it will serve to throw light upon the point we are trying to develop.

Mathematical being is a medium between purely material and purely immaterial being, and it participates in the nature of both. In the first place, although it is distinct from material being because of the nature of mathematical abstraction which frees it from sensible matter, it remains inseparable from it in the sense of always being linked to it by an intrinsic and essential bond. As a matter of fact, if the material world were impossible, the mathematical world would likewise be impossible. For it is only in a world of composed essences, in which formal oppositions are incomplete because of the common matrix of prime matter that the mathematical world can originate. It is this common matrix that provides the source of the homogeneity, and consequently of the univocal relations which are essential to mathematics. The mathematical world is a world of formality, but it is a strange formality, a kind of material formality, since it is immersed in homogeneity. It is something quite different from the heterogeneous formality of the world of separated substances. Because of the homogeneity and the common matrix found in the mathematical world there is a lack of the perfect unity and the pure distinctions found in the separated substances. But at the same time the homogeneity provides a substitute for this lack of unity by being the source of the relations out of which the mathematical world is constructed. On the other hand, the mathematical world is a world of formality even though this formality is not pure. And that is why

it transcends the world of contingency and obscurity, and becomes a world of rationality and necessity. This brings it close to the spiritual world and transpositions from one to the other become possible. It was indeed a profound intuition on the part of Plato to give to mathematics an intermediary position between the "Same" and the "Other".(46) By its very nature mathematics appears to us as an principle of reconciliation between reason and material nature.(47) And all this enables us to understand more clearly why the mathematization of the cosmos can lead, and often has led, to both materialism and idealism. It is only by understanding the true nature of mathematical abstraction and the intermediary character of the science that results from it that these two extremes can be avoided.

Now it is this intermediary character of mathematics that makes it the ideal instrument for physics. Because it is without matter secundum intelligi it participates in the immobile world of necessity and rationality; because it is with matter secundum esse it is applicable to cosmic reality. Hence it is the perfect instrument by which physics may be lifted out of its natural obscurity and contingency into the realm of perfect science, and even into a state that is in some respects similar to wisdom. And while being a medium between the material and spiritual, it is at the same time a medium between the objective and the subjective, as we saw in our discussion of the relation it bears to existence. This adds immeasurably to its effectiveness as a scientific instrument. For it leaves the mind free to work out its own rational schemes, and yet it provides the possibility of these schemes being applied to cosmic reality. The following remark of Meyerson is extremely relevant here;

> C'est que le mathématique, se détachant du reste du réel a l'air de pouvoir progresser sans faire appel à son comportement; c'est ce qui semble en faire la vraie 'matière intermédiaire' entre la pensée et le réel, et ce qui explique aussi l'attrait que le panmathématisme, en dépit du fruste irrémédiable de l'image de l'univers qu'il construit, exerce et exercera sans doute éternellement sur l'esprit humain.(48)

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CHAPTER SEVEN

SCIENCE, SENSIBILITY, AND HOMOGENEITY

1. The Problem.

This chapter marks a turning point in our study. In the last three chapters we have been concerned with a delineation of the salient characteristics of the two sciences whose union constitutes the intermediary science of mathematical physics. Whatever else this delineation has accomplished, it has certainly brought into clear relief the profound antithesis which lies between these two sciences: on the one hand, a science which sees everything in terms of mobility and sensible matter, a science of contingency and obscurity; on the other hand, a science which prescinds essentially from mobility and sensible matter, a science of necessity and rationality. A more radical antithesis could hardly be imagined than the one which exists between these two studies. And yet out of this antithesis must come a synthesis if mathematical physics is to exist. It is to the nature of this synthesis that we must now turn our attention. We shall devote three chapters to an analysis of how this synthesis is effected.

In the remaining chapters of our study we shall consider the results of this synthesis.

The general problem which immediately confronts us, then, is this: how does the mathematical world lay hold of the world of sensible phenomena and transform it into its own image and likeness? Anyone at all acquainted with science knows that the answer to this problem lies in the one word: measurement. But before we can come to an analysis of the process of measurement, a preliminary question imposes itself: what is there in nature itself which makes it amenable to this transformation through measurement into a system of mathematical symbolism? Measurement is the instrument of the mathematization of the cosmos. But there must be in the cosmos itself a basis for this mathematization.

Duhem has posed the question which confronts us here in the following terms:

> "Pour qu'une théorie physique se puisse présenter sous la forme d'une enchaînement de calculs algébriques, il faut que toutes les notions dont elle fait usage puissent être figurées par des nombres; nous sommes ainsi amenés à nous poser cette question: A quelle condition un attribut physique peut-il être signifié par un symbole numérique?"(1)

And to this question he gives the following general answer:

> Cette question posée, la première réponse qui se présente à l'esprit est la suivante: Pour qu'un attribut que nous rencontrons dans les corps puisse s'exprimer par un symbole numérique, il faut et il suffit, selon le langage d'Aristote, que cet attribut appartienne à la catégorie de la quantité et non pas à la catégorie de la qualité; il faut et il suffit, pour parler un langage plus volontiers accepté par le géomètre moderne, que cet attribut soit une grandeur. (2)

This general answer is fairly obvious, and was already implicit in what we saw in the last chapter about the nature of mathematics and the link which binds it to reality. But it is only a general answer, and it stands in need of a good deal of explication. And perhaps we can orientate ourselves towards a more definite solution by presenting the issue in the following terms: Since mathematical physics consists in the union of a sensible world with a world which prescinds from sensibility, the suture which knits the two together must be along the lines of something which is at once connected with sensibility and independent of it, something which while not sensible in the fullest sense of the word, is nevertheless sensible in a secondary sense. Presented in this way, the problem immediately calls to mind the Thomistic doctrine of proper sensibles and common sensibles, of which the latter are all reducible to quantity, even though in themselves they are not quantity, by the very fact that they are sensible. We believe that it is in this doctrine that the fundamental solution of our problem is to be found.

And we know of no better way of bringing the question into proper focus than by having recourse to the well-known adventure of Sir Arthur Eddington's elephant:

> Let us then examine the kind of knowledge which is handled by exact science. If we search the examination papers in physics and natural philosophy for the more intelligible questions we may come across one beginning something like this: 'An elephant slides down a grassy hill-side...' The experienced candidate knows that he need not pay much attention to this; it is only put in to give an impression of realism. He reads on: 'The mass of the elephant is two tons.' Now we are getting down to business; the elephant fades out of the problem and a mass of two tons takes its place....Let us pass on. 'The slope of the hill is 60°.' Now the hill-side fades out of the problem and an angle of 60° takes its place.... Similarly for the other data of the problem. The softly yielding turf on which the elephant slid is replaced by a coefficient of friction, which though perhaps not directly a pointer reading is of kindred nature...
> We have for example, an impression of bulkiness. To this there is presumably some direct counterpart in the external world, but that counter-

> part must be of a nature beyond our apprehension, and science can make nothing of it. Bulkiness enters into exact science by yet another substitution; we replace it by a series of readings of a pair of calipers. Similarly the greyish-black appearance in our mental impression is replaced in exact science by the readings of a photometer for various wave-lengths of light. And so on until all the characteristics of the elephant are exhausted and it has become reduced to a schedule of measures. (5)

This remarkable passage brings out with great exactness the fact that it is through the instrumentality of various types of measurement that the cosmos is mathematicized. But it also suggests what the basis of this mathematization is. For it is evident from the concrete example here given that when the mathematician seeks to lay hold of the material universe all the attributes of this universe which are known in Thomistic terminology as proper sensibles and in modern terminology as secondary qualities slip through his fingers. And no matter how many efforts he makes to recapture them, they continue to elude his grasp. With their passing, the very natures of the things he is dealing with vanish. The characteristic qualities of the hill-side, the greeness of the grass, the softness of the turf, etc. fade out of the picture of the physicist — and the hill-side fades with them. And the same is true of the elephant itself.

Yet it is clear that the exact scientist lays hold of something in the material universe, otherwise his science could in no sense be called physics. It is likewise clear that he lays hold of something which though in a sense independent of sensibility is at the same time essentially connected with it. He does not grasp the greyish-black colour of the elephant in its proper nature, yet the wave-lengths of light which register on his photometer are essentially connected with this greyish-black colour. And evidently the thing which he lays hold of can be approached through the avenues of more than one sense. For, a blind scientist can have a perfect knowledge of optics,(4) a deaf scientist can be expertly proficient in acoustics, and if it were possible to live and have sentiency without the faculty of touch there would be nothing to preclude the possibility of the science of thermodynamics. This common character of the object with which exact science directly deals manifests its nature: it reveals the fact that it is intimately bound up with homogeneity. And all of these considerations lead us

to this conclusion: mathematical physics prescinds from proper sensibles; its object falls within the domain of the common sensibles.

The views of modern scientists and philosophers of science confirm this conclusion, even though these views are not expressed in Thomistic terminology. Max Planck, for example, has this to say:

> Now all physical experience is based upon our sense perceptions, and accordingly the first and obvious system of classification was in accordance with our senses. Physics was divided into mechanics, acoustics, optics, and heat. These were treated as distinct subjects. In course of time, however, it was seen that there was a close connection between these various subjects, and that it was much easier to establish exact physical laws if the senses are ignored and attention is concentrated on the events outside the senses — if, for example, the sound waves emanating from a sounding body are dealt with apart from the ear, and the rays of light emanating from a glowing body apart from the eye. This leads to a different classification of physics, certain parts of which are re-arranged, while the organs of sense recede into the background. According to this principle the heat rays emanating from a hot stove ceased to be the province of heat and were assigned to optics, where they were dealt with as though entirely similar to light waves. Admittedly such a re-arrangement, neglecting as it does the perceptions of the senses, contains an element of bias and arbitrariness. (5)

But this concentration upon primary qualities to the exclusion of secondary qualities is by no means peculiar to modern science. A definite movement in that direction is discernible almost from the beginning of the systematic study of the cosmos. It is true, as Planck points out, that in the first stages of its development natural science identified the sensible and the physical. This was inevitable, since, as we have seen, pure natural science is a study of reality in terms of sensible matter. Physics took its origin when man began to observe and analyze perceptible properties and to express the results in descriptions. This enabled him to introduce order into his cognitions by means of classification. Regular recurrences in his sensory experiences (e.g. hot bodies become cold; a swinging object comes to rest etc.) made it possible for him to arrive at general laws based on qualitative uniformities. But the persistent attempt to perfect this rudimentary knowledge, to analyse these classifications and uniformities with greater exactness, and to render them more rational inevitably led to a dissolution of the relation of identity between the sensible and the physical, and a gradual abandonment of sensorial categories in the explanation of the physical world. In some cases this abandonment became not only methodological,

but philosophical. Already in Democritus and Lucretius we have an explicit denial of the ontological existence of what were later to be known as proper sensibles or secondary qualities. It is only by opinion or convention that they can be said to exist. At the time of the Renaissance this doctrine of the ancient atomists was revived by such men as Vives, Sanchez, and Campanella, and this revival, together with the astounding success of the new mathematical method in physics, had a profound influence on the epistemological views of subsequent scientists. As we saw in Chapter I, Kepler, while admitting the objectivity of the qualitative determinations of nature, maintained that they were somehow less real and fundamental than the quantitative determinations. Galileo went further than Kepler and made the secondary qualities subjective. For him the quantitative determinations of nature were absolute, objective, and immutable, and the object of true knowledge, whereas the qualitative determinations were relative, subjective, fluctuating and the source of mere opinion and illusion. Descartes' expulsion of qualitative determinations from both the physical and the geometrical world, and Newton's subsequent discovery of measurable cor- (6) relates of colour in terms of differently refrangible rays provided both a theoretical and experimental foundation for this position. And it remained for Hobbes (7) and Locke (8) to lend the weight of their authority to make it the generally accepted philosophical and scientific view. In mechanism the divorce between the sensible and the physical was accepted as a fundamental dogma. And wherever mechanism was accepted as a philosophy, the denial of the ontological existence of the secondary qualities usually resulted.

Contemporary science has continued to maintain the divorce between the sensible and the physical. Max Planck sees the evolution of Physics as a progressive withdrawal from the world of sense:

> But at the same moment the structure of this physical world consistently moved farther and farther away from the world of sense and lost its former anthropomorphic character. Still further, physical sensations have been progressively eliminated, as for example in physical optics, in which the human eye no longer plays any part at all. Thus the physical world has become progressively more and more abstract; purely formal mathematical operations play a growing part while qualitative differences tend to be explained more and more by means of quantitative differences. . .
> As the view of the physical world is perfected, it simultaneously recedes from the world of sense: and this process is tantamount to an approach to the world of reality. (9)

The gap between the world of sense and the world of physics has become so wide that authors dispute whether "qualitative physics" might not be considered a contradiction in terms, or whether such qualitative propositions as "copper conducts electricity;" "the melting point of ice is lowered by pressure," can be called physical laws. (10)

Recent physics has introduced a new and significant aspect into this progressive recession from the world of sense. In classical physics, although the gap between the world of science and the world of external sensibility had already grown wide, there still remained a direct and immediate relation between the scientific world and the imagination. The scientific constructions of classical physics were susceptible of direct representation through concrete images. That is why mechanism was essentially a physics of models. Lord Kelvin's well-known remark that he had to be able to make a model of a thing before he could understand it is typical of classical physics. But in recent years science seems to have made a direct break not only with external sensibility, but even with the imagination. This break was first effected by the introduction of the theory of Relativity and the theory of Quanta. And more recent developments have served to widen the gap immeasurably. The theories of Schrodinger and Dirac, for example, seem to be completely incapable of imaginative representation.

It is important to recognize the fact that this progressive withdrawal from the world of sense has sprung from a finality intrinsic to experimental science itself. It was not brought about by arbitrary, extrinsic influences. In particular, it did not grow out of any idealistic bias. When Galileo made the secondary qualities subjective, he understood subjective in the sense of intra-organic and not in the sense of psychic. They were for him the product of an interraction between an external object and a sense organ. Even Descartes, who might perhaps be suspected of a bias towards idealism, admitted the objective existence of a reality which caused the secondary qualities. (11) It is true that idealistic philosophers have seized upon this particular development of science as grist for their mill. But science cannot be held responsible for the interpretations and generalizations of philosophers.

And yet the directions in which science develops have great significance for philosophy. The particular development we have just sketched presents several important problems which we must try to solve if we are to understand the true nature of mathematical physics.

This should be evident from all that was said in Chapter II about the essential relation between physics and sensible matter. In some way physics seems to depend upon the senses for its very subject, (12) and yet as it develops it draws farther and farther away from the deliverances of the senses. What then is the precise relation between physical science and sensibility? Why has progress in science produced an ever widening gap between the sensible and the physical? In withdrawing from the world of sense, what is it that science is actually laying hold of in the cosmos? What is the nature and validity of the knowledge that results from this prescinding from the determinations of the cosmos that are presented by the senses? Is Planck correct in stating that this withdrawal from the world of sense is tantamount to an approach to the world of reality? Has the progressive desensibilization of physical science demonstrated that the objective world is devoid of qualities or that qualities may in some way be reduced to quantities? What is it that the intellect is attempting to achieve fundamentally in pursuing this progressive desensibilization? Does this development in any way favor idealism? These are some of the questions that demand our attention.

At the beginning of this chapter we suggested that the key to our general problem might be found in the Thomistic doctrine of proper and common sensibles. But the recent developments in physics to which we alluded above might seem to challenge this statement. For some authors see in this break with the imagination a demonstration of the illusory character of the common sensibles, just as they see in the previous withdrawal from external sensibility a demonstration of the illusory character of the proper sensibles:

> Or on constaté sans peine que le discernement entre le sensible et le physique, si bien commencé jadis, n'avait pas été poussé aussi loin qu'il aurait pu, et que sans doute il aurait dû l'être. De quel droit affirme-t-on la valeur immédiatement physique des qualités premières et des autres données mathématiques perçues? La force, et l'inertie, sont des notions issues directement de l'expérience sensible. Et l'image, car c'est bien d'une représentation imaginative qu'il s'agit, l'image d'un corps à trois dimensions, dans l'espace euclidien, d'un corps qui se déplace sans se déformer et qui demeure impénétrable, dépend indubitablement des conditions

> particulières de l'expérience sensorielle de l'homme. Notions anthropomorphiques donc, et qui ne sont pas moins liées à la structure particulière de notre sensibilité que ne l'était la couleur orangée ou le parfum de la violette. Il s'agit d'ailleurs de ce que les anciens appelaient des sensibles communs, qui ne sont jamais perçus qu'en liaison avec les sensibles propres; si donc ces derniers sont transposés du fait de la sensation, il est normal que les sensibles communs subissent le même sort."(13)

Perhaps the best way of coming to grips with these problems is by considering the relation between science and sensibility. But in order to understand this relation it will be necessary to recall a few fundamental notions about the nature of sense cognition.

2. The Nature of Sense Cognition.

Sensation is in many respects an anomalous thing. It represents the first confused awakening of matter to conscious life. It is at once an act of knowledge (which is defined in terms of immateriality) and an act of a material body. While on the one hand transcending pure corporeality, it remains immersed in it. By the fact that it is knowledge it involves a kind of immaterial trans-subjective union between subject and object. But because it is also an act of a material body, this union is bound up with a material subjective union produced by a physical movement.

Now all knowledge is by its very nature objective, for to know is to become another thing in its very otherness. But not all knowledge is equally objective, for there is a direct proportion between the objectivity of knowledge and its perfection. Only divine knowledge is completely objective, for it alone is perfect. This does not mean that knowledge which is imperfect is subjective precisely in so far as it is knowledge. It merely means that its objectivity is conditioned by a certain measure of subjectivity.

Since sensation is the lowest form of knowledge, it is necessarily the most subjective. It is immersed in matter, and matter is by its very nature a subject and the furthest removed from the state of object. It is to be borne in mind that an object is an object not in so far as it acts physically upon a knower, but in so far as it specifies an act of knowing. As we have just suggested, sensation is dependent upon matter not only from the point of view of its object as the intellect is, but even in its own intrinsic nature. For the senses are not purely psychic powers; they are psychosomatic. Sensation is an _actus coniuncti_, and matter enters into it not merely as a necessary condition, but as a co-cause. That is why it cannot

possess the otherness necessary for pure objectivity for; "intus existens prohibet extraneum." In the measure in which cognitive powers must conform to their object in its entitative state, they cannot conform to it in its objectivity.

Professor DeKoninck has brought out with great exactness the profoundly subjective character of sense cognition:

> Alors que l'intelligence est une faculté séparée qui atteint les choses sans leurs conditions matérielles individuantes, le sens reste, à tous les niveaux, lié à ces conditions de la matière. Et cela est le plus manifeste dans les sens externes. Ceux-ci sont pour ainsi dire diffusés sur les choses dans leur concrétion matérielle, et, par conséquent dans ce qu'elles ont d'obscur en soi, sous ce rapport, ils participent aux conditions mêmes de l'objet dans ce qu'il comporte d'irréductiblement entitatif: la sensation en est liée à un organe corporel. On le voit le mieux dans le toucher. L'organe de la température a lui-même une température; il a lui-même dureté et mollesse; est étendu, et il est mesuré par le temps; il a sa masse à lui; il se répand sur l'objet étendu; il cède à l'objet dur, et il en épouse la figure; il s'imprime dans l'objet qui l'enveloppe; etc. Bien que les premiers philosophes se soient trompés dans leur explication de la connaissance par une similitude entitative qui serait requise de la part du connaissant, ils ont néanmoins énoncé un principe qui se vérifie du sens. Mais il s'y vérifie dans la mesure où le sens s'éloigne de la pure objectivité. La connaissance sensible est imparfaite parce qu'elle demande cette immixtion de l'organe à la chose matérielle. Le sens sera moins parfaitement l'autre dans la mesure où il demande au préalable une assimilation entitative dans laquelle le sens même est passif. Le toucher ne

peut sentir une température sans que l'organe ne prenne lui-même cette température. Cette passibilité, où nous sommes, pour ainsi dire, assimilés par une autre chose, est, comme telle, à l'extrême opposé de la connaissance: celle-ci est, en effet, une opération vitale; motus ab intrinseco. L'immixtion aux choses dans leurs conditions matérielles reste purement instrumentale. (14)

The subjectivity of sense cognition is so evident that it has become proverbial; de gustibus et de coloribus non est disputandum. The same subject may receive different sensations of the same object, as when, for example a person touches a piece of metal and a piece of wood in a cold room: though both are of the same temperature, the first will feel much colder than the second. The same subject may likewise receive the same sensation from different objects, as when one's hands have a different temperature and are brought into contact with bodies of different temperature.

Now we can best get at the exact nature of this subjectivity by having recourse to some fundamental principles laid down by St. Thomas. "Nam sentire, quod etiam videtur esse operatio in sentiente, est extra naturam intellectualem, neque totaliter est remotum a genere actionum quae sunt ad extra." (15) Sensation is at the point in the universe where immanence first emerges from the transitive activity of material natures. It does not completely emerge

from it; it remains inextricably bound up with it. For in every act of sensation a physical, material interaction takes place between the material object and the material organ. Out of this interaction comes a "product" whose nature is determined both by the character of the stimuli which impinge upon the organ (and these are dependent upon the nature of the medium) and the character of the organ which receives them. It is this "mixture" of external stimuli (already a "mixture" arising out of the interaction between the distant object and the innumerable, indefinable elements which go to make up the medium) and the complex structure of the material organ which constitute the direct object of sensation. What is immediately sensed is not an absolute, distant object exactly as it exists in itself, but something intra-organic.

One of the most fundamental principles of cognition established by Aristotle and St. Thomas is that the sensible object in act is the same as the sense in act. There is a similar principle governing intellectual cognition: the intelligible object in act is identified with the intellect in act. But there is a vast difference between the significance of these two principles. For because of the material interaction of which we have been speaking, the transition of the sensible object from the state of potency to that of act is not a pure actualization which leaves its intrinsic nature unchanged. The sensible object in act is <u>physically</u> different from the sensible object in potency. St. Thomas explains this point in the following significant passage:

> Probat (Philosophus) quod supposuerat; scilicet quod unus et idem sit actus sensibilis et sentientis, sed ratione differunt, ex his quae sunt ostensa in tertio Physicorum. Ibi enim ostensum est, quod tam motus quam actio vel passio sunt in eo quod agitur, id est in mobili et patiente. Manifestum est autem, quod auditus patitur a sono; unde necesse est, quod tam sonus secundum actum, qui dicitur sonatio, quae auditus secundum actum, qui dicitur auditio, sit in eo quod est secundum potentiam, scilicet in organo auditus. Et hoc ideo, quia actus activi et motivi fit in patiente, et non in agente et movente. Et ista est ratio, quare non est necessarium, quod omne movens moveatur. In quocumque enim est motus, illud movetur. Unde si motus et actio, quae est quidam motus esset in movente, sequeretur, quod movens moveretur. Et sicut dictum est in tertio physicorum, quod actio et passio sunt unus actus subjecto, sed differunt ratione, prout actio signatur ut ab agente, passio autem ut in patiente, ita supra dixit, quod idem est actus sensibilis et sentientis subjecto, sed non ratione. Actus igitur sonativi vel soni est sonatio, auditivi autem actus est auditio.
> Dupliciter enim dicitur auditus et sonus; scilicet secundum actum et secundum potentiam. Et quod de auditu et sono dictum est, eadem ratione se habet in aliis sensibus et sensibilibus. Sicut enim actio et passio est in patiente et non in agente, ut subjecto, sed solum ut in principio a quo, ita tam actus sensibilis quam actus sensitivi, est in sensitivo ut in subjecto. (16)

Sensation, then, is the result of a physical, material action which takes place within the material organ, and which produces there a material motion, and this involves a physical, material <u>passio</u> on the part of the organ which, paradoxically, is the source of both the objectivity and the subjectivity of sensation. It is the source of objectivity because it is the reception of an action coming from an external object; it is the source of subjectivity because it involves a physical change on the part of the instrument of sensation and a reaction which contributes to the constitution of the object immediately sensed. As St. Thomas points out, "non enim oportet quod actio agentis recipiatur in patiente secundum modum agentis, sed secundum modum patientis et recipientis." (17) On a number of occasions both Aristotle and St. Thomas state that sensation consists in a modification, an alteration of the sense organ; it is this alteration that is immediately sensed. "Sentire consistit in moveri et pati. Est enim sensus in actu quaedam alteratio: quod autem alteratur, patitur et movetur." (18)

Whitehead, then, is justified in remarking: "It is an evident fact of experience that our apprehensions of the external world depend absolutely on the occurrences within the human body .... We have to admit that the body is the organism whose states regulate our cognizance of the world."[19] By naively attributing absolute objectivity to our sense cognition we are, as Sir Arthur Eddington has remarked, "continually making the mistake of the man who on receiving a telegram, thinks that the handwriting is that of the sender."[20] And in the same context he points out that to attribute the taste we experience in eating an apple to the apple itself is something like saying that the pain we experience in a dental operation is in the dentist's drill. It is necessary then to recognize the enormous distance which separates us from the things that are the closest to us. The very physical proximity of sensible things is a sign of their distance in the order of knowledge.

It is important to note that this subjectivity of sense cognition in no way gives aid and comfort to the idealists, as some might be led to think. For, as we have already pointed out, the very source of the subjectivity is at the same time the guarantee of objectivity. That is why Aristotle, after pointing out that sensations are really nothing but "modifications of the perceiver" immediately adds: "but that the substrata which cause the sensation should not exist even apart from sensation is impossible. For sensation is surely not the sensation of

itself, but there is something beyond the sensation, which must be prior to the sensation; for that which moves is prior in nature to that which is moved."[21]

Moreover, to say that the qualities that are immediately sensed are intra-organic is not the same as saying that they are psychic. As a matter of fact, they are completely physical and independent of consciousness.[22] They are a part of the physical world, even though they do not exist in the place in which they are localized by the naive view. And the reason why they are where they are is determined by the very physical structure of the universe. Bertrand Russell brings out this point in Mysticism and Logic:

> The view that sense-data are mental is derived, no doubt, in part from their physiological subjectivity, but in part also from a failure to distinguish between sense-data and 'sensations'. By a sensation I mean the fact consisting in the subject's awareness of the sense-datum. Thus a sensation is a complex of which the subject is a constituent and which therefore is mental. The sense-datum, on the other hand, stands over against the subject as that external object of which in sensation the subject is aware. It is true that the sense-datum is in many cases in the subject's body, but the subject's body is as distinct from the subject as tables and chairs are, and is in fact merely a part of the material world. So soon, therefore, as sense-data are clearly distinguished from sensations, and as their subjectivity is recognized to be physiologi-

cal not psychical, the chief obstacles in the way of regarding them as physical are removed. (23)

We have laid considerable emphasis upon the nature of sensation both because it is of great importance for the problem we are undertaking to solve, and also because the majority of modern Scholastic philosophers have presented sensation as though it possessed the same purity of objectivity as intellectual cognition. It is extremely important to realize that sense and intellectual knowledge differ generically and not merely specifically. From the point of view of objectivity there is a vast difference between sense and intellectual knowledge. Kant brings out this difference rather accurately when he writes: "Sensitive cogitata esse rerum repraesentationes, uti apparent, intellectualia autem, sicuti sunt." (24) The senses have to do with phenomena, with things as they appear and not as they are in themselves. Their object is not an essence — something absolute as it exists <u>in se</u> in the external world, but something essentially relative to the sense organ itself. It is true that when the intellect is brought to bear up sense data there will be an instinctive attempt to assimilate them to the condition of intellectual objects, that is to lift the "uti apparent" to "sicuti sunt", and as we shall

point out presently, this is precisely what the intellect is trying to do in its mathematization of the sensible world, but the fact remains that in themselves the sense data are purely phenomenal. To lose sight of this and to project into the external world the sense data as sensed by us is tantamount to identifying the sensible in act with the sensible in potency. As we pointed out above, because of the material nature of the sense organ, there is a difference between the two, not only from the metaphysical point of view, but even from the physical and material point of view. We cannot say just how great this difference is. To do that it would be necessary for us to know actually the sensible in potency, which is a contradiction. Only the separated substances know actually the <u>sensibilia in potentia</u>, and, we may add, they know the <u>sensibilia in actu</u> in the only way in which they can be known: as sensed by material subjects, as existing within the organs of beings endowed with sense life. But even though we cannot say just how much difference there is between the sensible in act and the sensible in potency we know that there is a difference. Things do not exist exactly as they are sensed by us. And we cannot insist too much upon the fact that we never sense the sensible in potency, that is the separated object in its own absolute existence. Perhaps we can sum up this point succinctly in the following terms. On the one hand only the sensible in potency exists (i.e. outside the sense organ); on the other hand, only the sensible in act is known by us. Consequently, there is a real gap between the sensible and the physical (i.e. the extra-organic world). And the withdrawal of science from the sensible world is a clear recognition of this gap.

Paradoxical as it may seem, the attribution to sensation of the pure objectivity proper to intellectual knowledge comes closer to idealism than the clear recognition of the subjectivity that is characteristic of all sense operations. For in the last analysis this attribution consists in projecting into the external world something that is the product of the sentient subject. In other words, idealists identify the sensible in potency with the sensible in act; those who attribute pure objectivity to the senses identify the sensible in act with the sensible in potency. Ultimately, the two positions coincide. Aristotle and St. Thomas point out the consequences of this fatal identification:

> Si omne apparens est verum, nec aliquid est verum nisi ex hoc ipso quod est apparens sensui, sequetur quod nihil est nisi inquantum sensibile est in actu.

> Sed si solum sic aliquid est, scilicet inquantum est sensible, sequetur quod nihil sit si non erunt sensus. Et per consequens si non erunt animata vel animalia. Hoc autem est impossible. Nam hoc potest esse verum quod sensibilia inquantum sensibilia non sunt, idest si accipiatur prout sunt sensibilia in actu, quod non sunt sine sensibus. Sunt enim sensibilia in actu secundum quod sunt in sensu. Et secundum hoc omne sensibile in actu est quaedam passio sentientis, quae non potest esse si sentientia non sunt. Sed quod ipsa sensibilia quae faciunt hanc passionem in sensu non sint, hoc est impossibile. (25)

If the sensible in act and the sensible in potency are identified, either the objective world depends for its existence on sensation, or everything in the objective world is actually and constantly sensed, or nothing is sensed. This last consequence follows because in order for an object to be sensed there must be a physical mutation produced in the organ, and this mutation necessarily involves a transition from a potential to an actual state of sensibility. It is only by clearly distinguishing betweenthe sensible in potency and the sensible in act that we can escape idealism and angelism.

And now a few notions relative to the object of sensation must be touched upon before we can consider the relation between science and sensibility. Aristotle and St. Thomas distinguish between objects that are sensible