per accidens and those that are sensible *per se*. Objects are said to be sensible *per accidens* when although they themselves are incapable of being sensed, they are connected with something that is the actual object of sensation. Thus,for example, substance cannot be actually sensed; nevertheless in so far as it is the substratum of the accidents that are sensed, it is said to be sensible *per accidens*. Objects that are sensible *per se* are those which are actually sensed in themselves. They are divided into two types: proper sensibles and common sensibles. It is this latter distinction that interests us particularly.

The proper sensibles are those which constitute the specific object of each individual external sense, and are consequently the exclusive property of only one sense, as, for example, color for the eye, sound for the ear, etc. The common sensibles are those which are the common property of more than one sense. There are five principal common sensibles: figure, motion, rest, number and magnitude; and to these are added three others: time, which is connected with motion and rest; position which is connected with external figure; and place, which is connected with magnitude.

These common sensibles comprise all of the predicaments except two. Action and passion are included under motion and rest; quantity comes in under number and magnitude; quality under figure; *habitus* is taken in by figure; *situs* has already been enumerated as one of the common sensibles; and *ubi* and *quando* are directly reducible to place and time. The only two predicaments not included are substance, which, as we saw is only a sensible *per accidens*, and relation, which cannot be sensed because it involves something that is proper to the intellect, an ordering of one thing to another. Hence, in so far as experimental science is based upon the common sensibles it will be incapable of attaining the substances of things or true predicamental relations. And yet quantity provides a substitute for both substance and predicamental relation. Because of the unique position which it occupies as the first accident and consequently the one closest to substance there is a quasi substantiality about it which, as we saw in the last Chapter, explains why it alone of all the accidents is capable of being the object of a special science. Because "in solo quantitatis genere, aliqua significantur ut subjecta, alia ut passiones" quantity can constitute a world apart. And in this world mathematical order substitutes for real predicamental relation.

Now perhaps the most important aspect of these common sensibles as far as we are concerned is that they are all reducible to quantity.[26] Number and magnitude are species of quantity; figure is a quality that is proper to quantity, since it consists in the termination of magnitude; motion (rest) and time are modes of quantity, "ex eo quod dividuntur secundum quantitatem ad divisionem alicuius quantitatis";[27] and position and place, by being connected with figure and magnitude are reducible to quantity. The fundamental reason for this reductibility to quantity is that quantity by being the first accident is the matrix of all the others and hence contributes to them a quantitative mode. This common matrix on the part of the object is the foundation of the common sensibility on the part of the senses. The very homogeneity in which all of the common sensibles are rooted makes them common to several senses and prevents them from being proper to any one sense.

In connection with the proper sensibles a distinction must be made the importance of which will be apparent later. Among the external senses there is a

hierarchy in which sight occupies the highest place and touch the lowest. Of all the external senses sight is the most perfect because it is the most immaterial and the most objective. It is the sense which enables us to know the greatest number and the greatest variety of objects. Of all the senses it is the most detached from its object.[28] Touch, on the other hand is the most material and the most subjective of all the sense faculties. It is the least detached; it has the weakest capacity for apprehending things in their distinctions. And yet it has a quality which makes it excel all the other external faculties. Professor DeKoninck has analyzed with great accuracy and clarity this characteristic quality:

> C'est pourtant le toucher qui nous enracine le plus directement et le plus sûrement dans les choses. Il est pour ainsi dire un prolongement en nous des choses telles qu'elles sont dans leur concrétion propre. Il coincide le plus avec elles, dans l'espace et dans le temps; il revêt davantage leur condition. Pour cette raison, il est aussi, par excellence le sens de l'expérience et de l'intelligence. Au point de vue certitude, c'est le toucher qui l'emporte. Un signe en est que nous demandons de toucher les choses comme critère ultime. L'ouïe, et davantage encore la vue, à cause de leur proximité de l'imagination, peuvent être sujets d'illusion. Le toucher, au contraire est davantage soumis au choc des choses dans leur concrétion épaisse. Il est, d'après l'expression des anciens 'grossior' et 'crassior', mais cette grossièreté lui donne des avantages au point de vue de la sobre certitude. En tant qu'elle implique 'subir' la connaissance

expérimentale est essentiellement imparfaite, mais elle l'emporte chez nous en tant qu'elle est pour nous origine de toute connaissance, et principe de toute certitude; 'veritas principiorum quantumcumque per se nota, in nobis semper est reducibilis ad sensus ex quibus originatur, et eorum universalitas ex inductione facta per sensus dependet.' (Jean de S.Thomas, Curs. Theol., T. I. p.392b) C'est sous ce rapport qu'il répond le plus pleinement à la première exigence de l'intelligence. Il a par là une affinité à l'intelligence, qui se traduit même dans l'organe. "Homo secundum tactum, multum differt in certitudine cognitionis ab aliis animalibus. Unde quia homo habet optimum tactum sequitur quod sit prudentissimum omnium aliorum animalium. Et in genere hominum ex sensu tactus accipimus, quod aliqui ingeniosi sunt, vel non ingeniosi et non secundum aliquem alium sensum. Qui enim habent duram carnem, et per consequens habent malum tactum, sunt inepti secundum mentem; qui vero sunt molles carne, et per consequens boni tactus, sunt bene apti mente. (In II de Anima, lect.19 nos.482 - 485) (29)

It is clear, then, that though from different points of view we may say that both sight and touch are at once the most objective and the most subjective sense faculties, the objectivity of touch has a very special significance for experimental science. In spite of its lack of distinction, it provides us with the greatest certitude, and in this it is like something that is found in the intellectual order: the most confused knowledge has the greatest certitude for us.

Now in so far as the sense of touch is the sense

of homogeneity, the sense which comes closest to the quantitative aspects of material objects, the sense that comes closest to pure corporeity and pure exteriority, it is the sense that is the most closely allied to mathematical physics. Modern science wants to reduce its sense experience with the universe to the minimum that is found in the sense of touch, and that means not merely to the generic sense of touch which includes perception of temperature, etc. but to pure taction, that is to say to pure contact of point to point.

This brings us to the consideration of a final distinction that has a bearing upon our problem. - - the distinction between external and internal experience. External experience consists in the experience of the external senses of which we have been speaking. Internal experience consists in the experience had of one's own proper reality through the operations of the internal senses and the mind. Now all too often it seems to be taken for granted that the study of nature depends only upon external experience. This is far from being the case, especially when it is a question of the study of living nature. As a matter of fact it is true to say that in a certain sense the study of psychology is based principally upon internal experience. We come to know what life is originally and primarily through our own proper experience of living. St. Thomas brings out this point in his _Commentary on the De Anima_ of Aristotle: "Hoc enim quilibet experitur in seipso, quod scilicet habeat animam, et quod anima vivificet." (30) This internal experience is so important that if one were to abstract completely from his own personal experience of living, he could not speak of life existing in anything. And it is important to insist upon the fact that this internal experience is not the flimsy and untrustworthy thing that many modern scientists attempt to make of it. On the contrary it enjoys the greatest certitude. In the text just cited, St. Thomas bases the eminent certitude which psychology possesses precisely upon the fact that life is known through internal experience. In comparison with the certitude which we have of our own life, our knowledge of the existence of life in other things, which depends upon external sensation, has only a greater or less degree of probability. It is precisely because psychology is based upon the experience we have of our own soul that the basic Aristotelian treatise on living nature is called _De Anima_. In it the soul is considered _in quadam abstractione_ - - not in the sense that it is studied in complete abstraction from the sensible matter with which it is united, for then

it could not form a part of natural doctrine, but in the sense that it is considered to some degree in and by itself. And this dependence upon internal experience introduces a new factor into the ordering of the natural treatises about which we spoke in Chapter IV. Since the basic methodological principle is to begin with what is best known to us, the study of living nature must start with the soul as it is experienced by us, _in quadam abstractione_, and then pass on to things that are more intimately bound to matter. That is why _De Sensu et Sensato_ comes after the _De Anima_. In the introduction to his _Commentary on De Sensu et Sensato_ St. Thomas explains this ordering. (31) Vegetative life which is not attainable by direct internal experience is the most hidden form of life: "vita in plantis est occulta." (32)

But it would be a mistake to believe that internal experience enters only into the treatises on living nature. It is also used in the _Physics_. For example, in book three when Aristotle is looking for an illustration of motion, he has recourse to the example of a man building a house. One might be tempted to wonder why he deliberately chose the example of the becoming of an _artefactum_ and not of a natural generation. But the illustration like all of the illustrations of Aristotle, is not without its profound significance. For in the example of the building of a house we have a case of motion in which both external and internal experience enter. As a matter of fact, the striving of an agent for an end, which is so essential to the true concept of motion, is most clearly apprehended by us in our own internal experience. When this internal experience is completely set aside, it is all too easy to lose sight of the fact that motion involves the coming into being of a new actuality which is the end of an agent, and to look upon it as a pure degradation. As a matter of fact many modern scientists have come to look upon motion merely in terms of the second law of thermodynamics which states that the world is continually in a state of degradation, that is to say, continually losing actuality, and consequently destined ultimately to arrive at a state of thermodynamic equilibrium in which all of cosmic reality will be in a state of utter chaotic diffusion and formless homogeneity. In connection with this question of entropy which constitutes time's arrow for the scientists, it is interesting to note that in his commentary on Aristotle's treatise on time in the fourth book of the Physics St. Thomas teaches that if we abstract from the agent of motion and from its intention, time is a degrading factor: "mutatio est ad peiora ex natura sua." (33)

mutation and time must be joined with the idea of an agent acting for a certain end in order to have the generation of a new actuality.

All this may appear to be an irrelevant digression, but as a matter of fact it is very a propos. For it serves to bring out the fact that the starting point of mathematical physics is diametrically opposed to that of philosophy of nature. Mathematical physics seeks to take its start from a minimum of experience. It excludes internal experience, and it reduces external experience to its very lowest form: pure corporeal contact. And out of this minimum of experience it seeks to construct the whole universe. Philosophy of nature on the other hand, has as its point of departure a maximum of experience. It employs not only the whole range of external experience, but also internal experience. And in connection with its dependence upon internal experience, it must be pointed out that this method of investigating problems is neither anthropomorphism nor subjectivism. On the contrary it enjoys a high degree of objectivity. For one's own internal states and experiences are as objective as anything in the universe.

This contrast between the points of departure of mathematical physics and the philosophy of nature brings into relief a striking paradox. While from the point of view we have had in mind in this discussion philosophy of nature depends upon a maximum of experience and mathematical physics upon a minimum of experience, from the point of view from which we considered the problem of experience in Chapter IV the situation is completely reversed: a minimum serves as a starting point for philosophy, while a maximum is required for mathematical physics and all branches of experimental science. We may say, then, that because of a significant effort on the part of the intellect to shake itself loose from its dependence upon the senses, mathematical physics tends towards a minimum of experience. This tendency is seen first in the vast use of hypothesis by which the mind seeks to anticipate reality. It is carried forward by a reduction of sense experience to its lowest form: pure taction. But it is a tendency that can seek its end only by binding the intellect down to a maximum of experience.

But in order to become aware of all that is involved in this question it is not sufficient to consider the difference between the starting points of mathematical physics and philosophy of nature; we must also consider the terminal points at which they aim. Precisely because

philosophy of nature begins with a maximum of experience it has as its ultimate goal and as its most important object the noblest being existing in nature, the being which in some sense transcends nature, and yet is a part of it. The being which possesses the highest degree of heterogeneous interiority in the universe is the spiritual soul of man. (34) On the other hand, precisely because mathematical physics begins with a minimum of experience, its ultimate goal must be to reduce the whole cosmos to pure homogeneous exteriority, to a state of pure otherness without any formal distinctions. As we shall have occasion to point out a little later, if mathematical physics could actually arrive at the goal towards which it is constantly striving, it would succeed in reducing the cosmos to a state of pure emptiness.

It should be obvious that this question is closely connected with the divergent forms of measurement employed in the philosophical sciences and in the experimental sciences, to which we alluded in Chapter I and which we shall consider in greater detail in Chapter IX. The method of mathematical physics has its many advantages and its rich returns, but when, as has often happened, the knowledge that it provides is proposed as the only valid knowledge of nature, then we are asked to accept an epistemological monstrosity, an exaltation of the superficial, a radical form of nihilism.

3. Science and Sensibility.

We are now in a position to consider the problem of science and sensibility. From what was said above it is clear that it is especially in relation to the proper sensibles that the ever widening gap between science and the sensible world has occurred. We must now try to see what has created this gap. Perhaps enough has already been said to show that it is not an artificial and arbitrary creation, nor a fortuitous occurrence, but something that has come inevitably from the very nature of experimental science and the nature of sensibility.

The first cause of the withdrawal of science from the sensible world is obviously the subjectivity of sense cognition. Natural science is orientated completely towards the absolute world condition, and its whole inner finality urges it to draw ever closer to this goal. The inherent subjectivity of the ministrations of the senses

is a direct obstacle to this tendency. For the deliverances of the senses present an anthropomorphic world, a world that has been refashioned, to some extent at least, according to the structure of man's sense organs. They consequently present a relative world, a world of appearances. If science is to be true to its inner urge to strive for the absolute world condition, it must find a way to disanthropomorphize these deliverances; it must, as we have suggested, strive to transform the "uti apparent" of Kant to "sicuti sunt". And it does this by means of a double substitution: one on the part of the subject and one on the part of the object. On the part of the subject, it puts in the place of organic instruments of perception inorganic artificial instruments of measurement especially designed for the purpose in accordance with scientific theories. On the part of the object there is a corresponding substitution of quantitative for qualitative determinations. The scientific world that is built up by means of these artificial inorganic instruments of measurement will inevitably draw farther and farther from the sensible world that is built up by the organic instruments of perception. (38)

It is to be noted that the subjectivity of the senses is an individual subjectivity. The corresponding sense of ten different subjects will not necessarily represent the same object in the same way. Ten different men, for example, may get ten different perceptions of the temperature of the same body of water. Now this is contrary to one of the ideals of science, which has come to be known in recent years as intersubjectivibility. And science has found that by the double substitution mentioned above almost perfect intersubjectivibility can be achieved. Norman Campbell has shown that the only exact judgments with regard to perceptions that are universally accepted are those that are based on quantitative determinations, and particularly those which have to do with the three categories of space, time and number.[36]

Another important reason for the withdrawal of science from the world of sense is that from the point of view from which experimental science approaches the cosmos, the proper sensibles are irrationals. And that for two reasons. In the first place, these proper sensibles cannot be defined. It is impossible to define heat; it is impossible to define a color or a sound. They are utterly incapable of analysis. They possess no inherent communicability. It is impossible to explain to a man born blind what red and blue are. And the reason for this is that the

proper sensibles are the primary and immediate data of sense cognition. Hence there are no prior notions in terms of which they may be defined; there are no more fundamental elements into which they may be analyzed.

Now it is different for the mind to rest satisfied with this state of affairs. It has an instinctive desire to define, to express to itself the quod quid est of things. That is why there have always been attempts to liberate the proper sensibles from the incommunicability that is native to them. The medieval Scholastics made attempts of this kind. For example, they defined white as disgregativum visus. But it is evident that such attempts can never yield strict definitions.

Similarly, the proper sensibles are indemonstrable. There are no prior principles in the sensible order from which they may be deduced. At the same time, they themselves are not principles of demonstration. Nothing can be deduced from them. However, it is only through them that the common sensibles can be perceived. That is why they may in a way be compared to what is known in the intellectual order as the supreme dignitates, which are necessary for every demonstration, but which are not in themselves the principles of any demonstration. Indefinable, incapable of analysis, indemonstrable, incapable of being a source of demonstration, the proper sensibles are merely given. Is it any wonder that science instinctively draws away from them?

The second source of their irrationality is very closely allied with the first; by the very fact that they are proper sensibles, they are irreducibly heterogeneous; they are isolated one from the other; they are not unified by a logical pattern. As we shall attempt to explain presently, not all types of heterogeneity are essentially and completely irrational. Nevertheless, in the measure in which heterogeneity is incapable of being reduced to some kind of unification it always presents an element of irrationality to the mind. Meyerson has laid considerable emphasis upon the isolation of the proper sensibles:

> Il suffit en effet, de réfléchir à la nature de la qualité pour se rendre compte à quel point elle se prête difficilement aux tentatives consistant à relier, mentalement, le divers à l'identique, qui constituent l'essentiel de toute explication du réel. Car toute qualité nous apparaît comme quelque chose de complet en soi; non seulement le fait de son existence ne postule rien en dehors d'elle-même, mais elle est quelque chose d'intensif et ne paraît donc point susceptible de se combiner, de s'adjoindre à autre chose. (37)

Material qualities lend themselves admirably to

descriptive knowledge, but they seem refractory to explanatory knowledge. They appear to be closer to sentiency, whereas quantity seems closer to rationality. Once again from this point of view, the proper sensibles are merely given, and this givenness is in direct opposition to the necessity that science seeks. Not being able to find this necessity in the realm of the proper sensibles, it will look for it elsewhere.

Another reason for the withdrawal of science from the sensible world arises from the extremely restricted nature of the senses. The crudity of our sense organs allow us to perceive only an infinitesimally small part of the cosmic occurrences. By the substitution of inorganic instruments of measurement for the organic instruments of perception the scope of science is increased immeasurably.

> In general, then, we may say that we experience the outer world through small samples of it coming into contact with our sense-organs... Yet not all samples of the outer world affect our sense organs. Our ear-drums are affected by ten octaves, at most, out of the endless range of sounds which occur in nature; by far the greater number of air-vibrations make no effect on them. Our eyes are even more selective; speaking in terms of the undulatory theory of light, these are sensitive to only about one octave out of the almost infinite number which occur in nature . . .
> Science has of course provided us with methods of extending our senses both in respect

of quality and quantity. We can only see one octave of light, but it is easy to imagine light-vibrations some thirty octaves deeper than any our eyes can see. While philosophy is reflecting how different the world would appear to beings with eyes which could see these vibrations, science sets to work to devise such eyes - - they are our ordinary wireless sets. We also have means for studying vibrations far above any our eyes can see. Actually a range of vibrations extending over about 65 octaves can be detected and has been explored - - 65 times the range of the unaided eye. And even this limit is not one of the resources of science, but of what nature provides for us to see. In the same way, the spectroscope makes good the deficiency of our eyes for analyzing a beam of light into its constituent colours, and further enables us to measure the wave-length of each colour of light to a high degree of accuracy.
Science has extended the range and amplified the powers of our other senses in similar ways, in quality as well as in quantity. We cannot touch the sun to feel how hot it is, but our thermocouples estimate its temperature for us with great accuracy. We cannot taste or smell the sun, but our spectroscopes do both for us - or at any rate give us a better acquaintance with the substance of the sun than any amount of smelling or tasting could do. We are entirely wanting in an electric sense, but our galvanometers and electroscopes make good the deficiency.(38)

As Hermann Weyl has pointed out, this crudity of the senses leads us to identify things which are physically distinct and thus runs counter to one of the most basic principles of science:

For the question forces itself upon us: why is

physics not content with this domain of perceived colors which has only two dimensions, what urges it to put oscillations of the ether or something similar in their place? After all, from our visual perceptions we know nothing about the oscillations of the ether; what we are given are precisely only these colors, the way we encounter them in our perception. Answer: To light rays which cause the same impression to the eye are in general distinct in all their remaining physical and chemical effects. If, for example, one illuminates one and the same colored surface with two lights which visually appear as the same white, the illuminated surface usually looks quite different in both cases. Red and green-blue together give white light, equally light brown together with violet. But the first light produces a dark hue on the photographic plate, the second a very light one. If one sends two lights which visually appear as the same white through one and the same prism, the intensity distribution in the spectrum arising behind the prism is different in both cases. Therefore physics cannot declare two lights which are perceptually alike to be really alike, or else it would be involved in a conflict with its dominating principle: equal causes under equal circumstances produce equal effects. Perceptual equality therefore appears to physics only as a somewhat accidental equality of the reactions which physically distinct agencies produce in the retina. The accidental equality of the reaction rests upon the particular nature of this receptive apparatus.(39)

In connection with this point it is not superfluous to add that the deliverances of the sense are extremely fluctuating and unstable. As Meyerson has remarked: "le retour de sensations véritablement identiques est excessivement rare." (40) That is why science must look for a source of permanence which is so essential to its nature.

Moreover, the qualitative determinations of nature permit of only general and loose propositions. In order to achieve accuracy, and in order to make its propositions capable of unambiguous confirmation or refutation, science must have recourse to quantitative determinations. For example, the statement: "fire causes water to boil," is not true unless a number of precise determinations be added with regard to temperature, pressure, respective masses of the water and fire, surface of radiation of the fire, etc. A certain arrangement of these conditions could actually keep water from boiling.

It seems necessary to add one final observation before we leave this question. The whole material universe is a mixture of qualitative and quantitative determinations. As we go up the scale of perfection in cosmic reality, the qualitative determinations assume an ascending importance, for they manifest the increasing triumph of form over matter. That is why they are so important in the biological sciences. But in inorganic matter it is the quantitative aspect that is in the ascendency. And that can perhaps be adduced as a further reason why physics as it progresses becomes more and more immersed in the quantitative.

And now, having considered the relation that exists between science and sensibility, we must try to see the way in which the mind triumphs over the limitations of the senses.

4. Science and Homogeneity.

In order to understand the part that homogeneity plays in science it is necessary to begin by making an important distinction between two types of heterogeneity. There is first of all a kind of heterogeneity which is found on the part of the object of knowledge and which we shall call "natural". This is the heterogeneity that exists between man and brute, between the numbers two and three, between the different angelic species, between the logically distinct rationes formales of the divine essence. This type of heterogeneity obviously springs from a difference of form (in the broad sense in which it signifies a ratio formalis). It is consequently a heterogeneity that is essentially rational. It has its source in intelligibility. And the more perfect an intellect is, the more perfectly does it grasp things in their proper and irreducible heterogeneity.

There is another type of heterogeneity that may be termed "noetic" because it is found not on the part of the object of knowledge but on the part of the intelligence itself. It consists in the multiplicity of media or concepts, or intelligible species which the intellect needs to employ in order to know reality. The more imperfect an intellect is, the greater is this multiplicity. This heterogeneity therefore is essentially irrational. It is a reflection of the original potentiality of the intellect. It is clear that perfect knowledge will consist in knowing natural heterogeneity in all the fullness and richness of its proper specific distinctness by means of absolute noetic homogeneity.

It is only in divine knowledge that this perfection of knowledge is found. In a unique intelligible species which is His essence God sees all the individual natures in existence, exhaustively and in their ultimate specific concretion. Here there is no possibility of any conflict between heterogeneity and homogeneity. In fact, it is only because God sees things in the one species which is Himself that he is able to grasp them in their absolute heterogeneity. But as we descend the scale of

beings, noetic heterogeneity gradually increases. The higher separated substances can know a large number of individual natures in their specific distinctness through a small number of intelligible species. In the lower separated substances a great multiplicity of media are required. And the limit of this process is found in the human intelligence which because it partakes of the diffusion of matter with which it is united, can know things in their distinctness only through a multiplicity of intelligible species equal to the multiplicity of ontological species.

In the intellect of man there is a profound conflict between homogeneity and heterogeneity. On the one hand, he is incapable of sharing in noetic homogeneity. He can, indeed, attempt to triumph over this limitation by having recourse to the dynamic method of limits, and this method is not without its fruitfulness. But it always remains only an attempt, since dialectical limits cannot be attained. On the other hand, natural heterogeneity, though something basically rational, will always present to him an irrational aspect in the measure in which it remains in its pure isolated givenness, in the measure in which it cannot be reduced to some kind of unification, to some type of homogeneity. It must be remembered that even though the source of natural heterogeneity is fundamentally something rational, in so far as it is found in the material universe it also involves an irrational element in the sense that a plurality of really distinct forms is possible only because they are imperfect and limited.

The problem of the human intellect then, is to see the heterogeneity of nature in terms of some type of homogeneity. Here we are touching upon a conflict in the intellectual order of which there is something strangely analogous in the sensible order. We refer to the distinction pointed out above between the faculties of sight and touch. As we saw above the first is a faculty of heterogeneity in that, better than any other sense, it is capable of grasping things in the richness of their specific distinctness. The second is a faculty of homogeneity in that it has the least capacity for grasping distinctions and in that it seems to come into closest contact with the quantitative determinations of nature. It is also the most important sense faculty from the point of view of certitude, and this carries out the analogy still further, since, as we have seen, it is only by remaining in the homogeneity of generality that the mind is able to arrive at true scientific certitude in relation to the cosmos. The ideal towards which man will ever strive

will be a union of this distinctness and this certitude. In the sensible order this is possible, since sight and touch can be brought into a combined operation on the same object: "unless I see in his hands the print of the nails, and put my finger into his side, I will not believe." But in the intellectual order separate faculties of distinctness and certitude, or heterogeneity and homogeneity do not exist. Hence the mind will have to discover some other means of striving towards its ideal. Let us see how it goes about it.

There are two important ways by which man tries to triumph over the heterogeneity of reality through homogeneity. The first is by retreating into generality and consequently into logical potentiality. It is in this way that philosophy of nature studies the cosmos. By reducing the specific heterogeneity of the universe to the logical homogeneity of generalities, it is able to procure for itself a number of important advantages. It is able to get at the fundamental, common structure of the physical world, and to know it with certitude. It is able to view the cosmos in terms of unity and in terms of what is most knowable for it. But the price it has to pay for these advantages is great. For all the concrete richness of the universe remains untouched. At the limit of this process of logical homogenization the universe would be reduced to the emptiest, most vague and most potential concept - - that of being, abstracted by mere total abstraction.

It is in order to get at the richness of nature that the mind starts its march towards concretion. But by advancing in this direction it soon gets involved in an intellectual crisis. For its gain from the point of view of heterogeneity means a loss from the point of view of homogeneity, and hence an increase in irrationality. And this increasing irrationality forces the mind to seek for some kind of homogeneity through which to triumph over it. But it will have to be a homogeneity that is quite different from the one from which it is emerging, i.e. one that will not lead it back into generalities, but will carry it forward into concretion. It will have to be a homogeneity that is not logical but ontological. It will have to be something which will afford at the same time both a unity to provide for what is lost by drawing away from generalities, and a distinctness to enable the mind to press forward towards concreteness. It will have to be something that

will make it possible for the mind to see nature in terms of what is most knowable for it (and thus make up for what is lost by drawing away from generalities) and at the same time in terms of what is most knowable in se (and thus make up for the deficiencies of purely generic knowledge. And the mind finds a basis for what it is seeking for in a general substructure of cosmic reality, in a common matrix in which the heterogeneous determinations of the physical world are rooted. This, we believe, is the most fundamental significance of the mathematization of the universe.(41)

New science gets at this homogeneous matrix by displacing its object from the realm of the proper sensibles to that of the common sensibles. And these common sensibles serve its purpose excellently by the very fact that, while they are not quantity in themselves, they are all reducible to quantity. Since they are sensibles, and hence not quantity specifically, the science which studies them is able to remain within the realm of physics. On the other hand, since they are all reducible to quantity, the mind is able to find the homogeneity it is seeking for, and physics b ecomes mathematical physics. Since quantity is the primary accident and the one closest to substance, all the specific determinations of cosmic reality are rooted to it, and hence they all assume a quantitative mode. Because of the principle "quidquid recipitur ad modum recipientis recipitur," quantity necessarily modifies the qualities that are received into it.(42)

In order to understand the nature of these quantitative modes it must be noted that in the structure of physical reality, the qualitative and the quantitative determinations are not related to each other after the manner of two contiguous layers. Rather, there is an intimate, dynamic union between them. And this explains why the qualitative determinations can be "translated" into quantitative equivalents, why the colors and sounds and heat of the universe can become functions of the space, time, mass and other derivative relationships that exist between the various parts of nature. By getting at these quantitative modes, science is able to construct a physics that can be informed and rationalized by mathematics.

But at this point it must be noted that it is possible to study qualitative perfections in a quantitative way without having recourse to a physical quantitative mode. Intelligence, for example, is studied in experi-

mental psychology in terms of quantitative measurements based on an association between certain psychological reactions and a scale of numbers. Mathematical physics is primarily concerned not with an extrinsic and artificial correlation of this kind, but with an intrinsic correlation which springs from the very structure of physical reality. This intrinsic correlation is not a discovery of modern science; it was clearly recognized by the ancients, and was the basis of their mathematical physics.(43)

But in order to understand this point accurately it is necessary to introduce a distinction here, which will not only help us to clarify the present issue, but will also be useful for us in the next Chapter when we come to discuss the relation between science and measurement. We have in mind the distinction between predicamental and transcendental quantity. St. Thomas explains this distinction with great preciseness in the following passage:

> Duplex est quantitas. Una scilicet, quae dicitur quantitas molis, vel quantitas dimensiva, quae in solis rebus corporalibus est. Unde in divinis personis locum non habet. Sed alia est quantitas virtutis, quae attenditur secundum perfectionem alicuius naturae, vel formae. Quae quidem quantitas designatur, secundum quod dicitur aliquid magis, vel minus calidum, inquantum est perfectius vel minus perfectum in tali caliditate. Huiusmodi autem quantitas virtualis attenditur primo quidem in radice, idest in ipsa perfectione formae, vel

> naturae; et sic dicitur magnitudo specialis, sicut dicitur magnus calor propter suam intensionem, et perfectionem. Et ideo dicit August. 6 de Trinit. cap. 18. quod in his quae non mole magna sunt, hoc est maius esse, quod est melius esse. Nam melius dicitur, quod perfectius est. Secundo autem attenditur quantitas virtualis in effectibus formae. Primus autem effectus formae est esse; nam omnis res habet esse secundum suam formam. Secundus autem effectus est operatio: nam homo agens agit per suam formam. Attenditur igitur quantitas virtualis et secundum esse, et secundum operationem. Secundum esse quidem, inquantum ea quae sunt perfectioris naturae, sunt maioris durationis. operationem Secundum operationem vero, inquantum ea, quae sunt perfectioris naturae, sunt magis potentia ad agendum. (44)

The more or less of transcendental or virtual quantity is based on heterogeneity while that of predicamental or formal quantity is based on homogeneity. And it is interesting and helpful to view the latter as the dialectical limit towards which the former tends as the hierarchy of immaterial things descends towards the realm of corporeality. The difference of forms gradually diminishes and at the limit the definition of each part is the same as the definition of the whole. The diversity is no longer formal; it is purely material. In all material things both types of quantity are found together. The heterogeneity of the one is rooted in the homogeneity of the other and takes on its modes and determinations.

In Quaestiones Disputatae de Virtutibus in Communi, St. Thomas explains that besides the magnitude which qualities and forms are said to possess *per se*, there is another magnitude that is attributed to them *per accidens*. It is this quantity *per accidens* that is of special significance for mathematical physics:

> Omnibus qualitatibus et formis est communis ratio magnitudinis quae dicta est, scilicet perfectio earum in subiecto. Aliquae tamen qualitates, praeter istam magnitudinem seu quantitatem quae competit eis per se, habent aliam magnitudinem vel quantitatem quae competit eis per accidens; et hoc dupliciter. Uno modo ratione subiecti; sicut albedo dicitur quanta per accidens, quia subiectum eius est quantum; unde augmentato subiecto, augmentatur albedo per accidens. Sed secundum hoc augmentum, non dicitur aliquid magis album, sed maior albedo, sicut et dicitur aliquid maius album . . . Alio modo quantitas et augmentum attribuitur alicui qualitati per accidens, ex *parte obiecti in quod agit*; et haec dicitur quantitas virtutis; quae magis dicitur propter quantitatem obiecti vel continentiam; sicut dicitur magnae virtutis qui magnum pondus potest ferre, vel qualitercumque potest magnam rem facere, sive magnitudine dimensiva, sive magnitudine perfectionis, vel secundum quantitatem discretam; sicut dicitur cliquis magnae virtutis qui potest multa facere . . . Sed considerandum est, quod eiusdem rationis est quod aliqua qualitas in aliquid magnum possit, et quod ipsa sit magna, sicut ex supra dictis patet; unde etiam magnitudo perfectionis potest dici magniudo virtutis.(45)

It is clear from this passage that in so far as forms and qualities are found in corporeal beings they may become quantitative *per accidens* in relation to predicamental quantity. And from the last line of St. Thomas just cited it is evident that there can be a direct relation between the transcendental *per se* quantity of these forms and qualities and the predicamentally quantitative modes which make them quantitative *per accidens*. This makes it possible for science to deal with the transcendental quantity of the specific perfections of reality in terms of predicamental quantity.

By fixing its attention upon the quantitative modes of the specific determinations of the cosmos, physics obtains for itself innumerable advantages. For, in the first place, nothing seems so real to common sense as quantity. As Spaier has remarked, "c'est la quantité qui représente la réalité la plus solide . . En un mot, le réalisme habituel est avant tout un réalisme de la quantité" (46)

By adopting the quantitative method the mind enjoys an experience that is in some way similar to that of being able to reach out and touch and handle an object of sense. Whether or not along with this there is the advantage of being able to grasp things in their distinct-

ness in a way that would be similar to the perfection of the sense of sight is a question which we shall consider a little later. Moreover, nothing is capable of being so abstract and ideal as quantity. And this gives almost unlimited scope for the mind's desire for perfect rationality.

This reveals the profound significance of the homogenization of the cosmos. Because man is composed of both matter and spirit there are two fundamental tendencies in him: to draw everything from matter, and to draw everything from mind. The persistent recurrence of the extremes of materialism and idealism in the history of philosophy have been a constant manifestation of this. Now the quantitative homogenization of the cosmos makes it possible for man to realize both of these tendencies simultaneously. The mathematization of nature means something far deeper than an attempt to escape from the anthropomorphism involved in the subjectivity of sensibility. It is really an attempt on the part of the intellect to shake itself loose from the senses. This is in a way a natural movement, since intellect in its perfection is independent of sense. To construct the universe out of a minimum of experience is the next thing to positing the universe. To a certain extent the mind is successful in this attempt. But by an ironical paradox this success involves a falling back upon something similar to the very lowest form of sense life -- pure taction. It is a conception of the universe in terms of the homogeneous exteriority of pure materiality.

All this explains why the goal towards which science is ever striving is to reconstruct the universe out of sameness. "The aim of the analysis employed in physics," writes Eddington, "is to resolve the universe into structural units which are <u>precisely</u> like one another."(47) The analysis of matter has gone far in this direction; it has succeeded in resolving cosmic reality into protons which are all alike and electrons which are all alike. And when nature seems to present an irreducible dualism in the heterogeneity existing between protons and electrons, the theory of relativity will attempt to dissolve this heterogeneity by suggesting that "they are actually similar units of structure, and the difference arises in their relations to the general distribution of matter which forms the universe."(48)

The end towards which physical science is aiming is to reconstruct the whole universe, i.e. to conceive the universe in terms of structural knowledge determined with exactness by mathematical formulae. Knowledge of this kind

prescinds completely from the nature of the units which constitute the structure. In their place are substituted manipulatable mathematical symbols, which while they serve as admirable instruments for knowledge of structure, at the same time blot out all that lies beneath the structure. Mathematics is especially competent to express patterns, but incompetent to reveal the proper natures of entities and operations. Through group-structure mathematics is able to lay hold of realities which in themselves are not directly susceptible of mathematical conceptions.

All this explains the increasingly important place of mathematics in physics, for it is only in mathematical form that purely structural knowledge can be adequately expressed. In particular it explains the central role played by the Theory of Groups.

This structural knowledge is at once extremely objective and extremely subjective. It is objective in the sense that by prescinding from the proper determinations of things, the knowledge of which involves so many subjective elements, it is able to constitute a type of knowledge that is exactly communicable to all minds. It is at the same time subjective in the sense that the essential plasticity of the sameness out of which the structure is formed gives unlimited scope to the constructivity of the mind. In fact, this whole process must be looked upon as the mind's imposition of its engrained forms upon reality. This is a point that has been stressed by Eddington:

> Granting that the elementary units found in our analysis of the universe are precisely alike intrinsically, the question remains whether this is because we have to do with an objective universe built of such units, or whether it is because our form of thought is such as to recognise only systems of analysis which shall yield parts precisely like one another. Our previous discussion has committed us to the latter as the true explanation. We have claimed to be able to determine by <u>a priori</u> reasoning the properties of the elementary particles recognised in physics - - properties confirmed by observation. Accordingly we account for this <u>a priori</u> knowledge as purely subjective, revealing only the impress of the equipment through which we obtain knowledge of the universe and deducible from a study of the equipment. We now say more explicitly that it is the impress of our frame of thought on the knowledge forced into the frame . . . I want to show therefore that the concept of identical structural units expresses a very elementary and instinctive habit of thought, which has unconsciously directed the course of scientific development. Briefly, it is the habit of thought which regards variety always as a challenge to further analysis: so that the <u>ultimate</u> end- roduct of analysis can only be sameness. We keep on modifying our system of analysis until it is such as to yield the sameness which we insist on, rejecting earlier attempts (earlier physical theories) as insufficiently profound. The sameness of the ultimate entities of the physical universe is a foreseeable consequence of forcing our knowledge into this form of thought . . . I conclude therefore that our engrained form of thought is such that we shall not rest satisfied until we are able to represent all physical phenomena as an interplay of a vast number of structural units intrinsically alike. All the diversity of the phenomena will then be seen to correspond to different forms of relatedness of these units or, as we should say, different configurations. (49)

The foregoing analysis makes it clear that it is precisely through the source of homogeneity that the common matrix of quantity offers to the mind that it is possible for science to rationalize the cosmos. Much has been written on this point by modern philosophers of science. Professor Whitehead, for example has this to say in <u>Process and Reality</u>:

> It is by reason of this disclosure of ultimate system that an intellectual comprehension of the physical universe is possible. There is a systematic framework permeating all relevant fact. By reference to this framework the variant, various, vagrant, evanescent details of the abundant world can have their mutual relations exhibited by their correlation to the common terms of a universal system. Sounds differ qualitatively among themselves, sounds differ qualitatively from colours, colours differ qualitatively from the rhythmic throbs of emotion and of pain; yet all alike are periodic and have their spatial relations and their wave-lengths. The discovery of the true relevance of the mathematical relations disclosed in presentational immediacy was the first step in the intellectual conquest of nature. (50)

But perhaps the author who deserves particular attention in relation to this question is Emile Meyerson, for we are touching here upon the central theme which runs through all of his voluminous works. Meyerson has labored to show that the mind cannot understand reality except by reducing its diversity to some kind of identity, and that the identity in which it comes closest to realizing its ideal is that of undifferentiated spatiality. Unfortunately, there is usually a fairly thick penumbra surrounding his analyses because he fails to make a number of important and necessary distinctions. Like Parmenides and Anaxagoras, he confuses the noetic and the ontological problems of the one and the many; he does not seem to recognize the difference between what is more knowable for us and what is more knowable in se, between the rationality which things have for us and the rationality they have ontologically. From this arises a confusion between the different kinds of diversity and the different kinds of unity by which the mind seeks to triumph over the diversity. With regard to diversity, he fails to make the all important distinction between natural and noetic heterogeneity. And in his treatment of identity there is no attempt to distinguish clearly between the homogenization arising from the reduction of singularity to universality, from the coordination of laws in theories, from the relations of

causality, from the quantification of reality, and from the method of limits. It is especially important to keep this last type of unification distinct from all the others.

But in spite of these limitations, his fundamental tenets are quite correct. The following passage is a good expression of his central theme:

> Ce à quoi la science tend de la manière la plus immédiate, c'est à établir un rapport logique entre les phénomènes, à les déduire les uns des autres. Mais cette tendance n'est au fond, qu'une conséquence, une expression particulière du postulat de la rationalité du réel: c'est, en quelque sorte, de la même monnaie de rationalité. Il n'est donc point étonnant qu'en l'accumulant nous finissions par reconstituer, au moins partiellement, le capital primitif, c'est-à-dire qu'à force de déduire les phénomènes les uns des autres, la science finisse par faire crouler les murailles qui en divisaient le domaine en parcelles distinctes, privées de communication les unes avec les autres.
> Cette opération, cela est de toute évidence, ne peut s'accomplir qu'en renonçant à ce qui est qualitatif, au profit de la quantité. En effet, tout ce qui est affecté d'un indice qualitatif devient, par là même, spécifique, isolé . . .
> Mais ce qui apparaît certain, c'est que l'éclosion de la notion de quantité dans l'ensemble des conceptions du sens commun, tout en étant favorisée par des constatations des expériences sur les phénomènes... est cependant surtout conditionnée par ce souci de l'explication, de la rationalisation, qui constitue le ressort fondamental de notre pensée tout entière. (51)

If the ideal of science could be adequately achieved, the entire universe would be reduced to an immense tautology and would thus collapse and vanish completely. "La raison, en cherchant à expliquer, à rendre rationelle la réalité extérieure, la fait disparaître finalement dans le tout indistinct de l'espace et du néant." (52) According to Meyerson, this collapse will not occur because the cosmos will ever remain propped up, so to speak, by irrational elements which are essentially refractory to the mind's process of homogenisation. As we have already suggested, Meyerson fails to make it clear that from a more fundamental point of view these props are rational elements, in the sense that they derive from natural heterogeneity. It is because of them that our attempts at rationalization are kept from issuing into the utter irrationality of a purely homogeneous and amorphous universe which would correspond to the original irrationality of the human intellect in its state of <u>tabula rasa</u>. It is a striking and highly significant paradox that if our attempts at rationalization could succeed the universe would be rendered completely irrational.

Better than any one statement of Meyerson himself, the following passage of Prince Louis de Broglie sums up the essence of this doctrine:

Selon lui (Meyerson) dans la recherche scientifique

comme dans la vie quotidienne, notre raison ne croit vraiment avoir compris que si elle est parvenue à dégager dans la réalité mouvante du monde physique des identités, et des permanences. Ainsi s'explique en particulier la structure commune des théories physiques qui tentent de grouper des catégories de phénomènes par un réseau d'égalités, d'équations, cherchant toujours, autant que faire se peut, à éliminer la diversité et le changement réel et à montrer que le conséquent était en quelque sorte contenu dans l'antécédent. La réalisation complète de l'idéal poursuivi par la raison apparaît alors comme chimérique, puisqu'elle consisterait à résorber toute la diversité qualitative et toutes les variations progressives de l'univers physique en une identité et une permanence absolues. Mais si cette réalisation complète est impossible, la nature du monde physique se prête néanmoins à un succès partiel de nos tentatives de rationalisation. Il existe, en effet, dans le monde physique non seulement des objets qui persistent à peu près semblables à eux-mêmes dans le temps, mais des catégories d'objets assez semblables entre eux pour que nous puissions les identifier en les réunissant dans un concept commun. Ce sont ces 'fibres' de la réalité, comme dit M. Meyerson, que notre raison saisit dans l'expérience de la vie quotidienne pour constituer avec elles notre représentation habituelle du monde extérieur; ce sont ces fibres également et d'autres plus subtiles, revélées à notre connaissance par les méthodes raffinées de la recherche expérimentale, dont la raison du savant s'empare pour chercher à extraire de la réalité variée et mouvante le part d'identique et de permanent qu'elle renferme. Aussi, grâce à l'existence de ces fibres bien que l'idéal de la science soit en toute rigueur irréalisable, quelque science est possible: c'est là la grande merveille. Cette situation se trouve resumée par une phrase de M. Paul Valery, phrase sans doute inspirée par la lecture même des ouvrages de M. Meyerson: L'esprit humain est absurde par ce qu'il recherche; il est grand par ce qu'il trouve. Mais comme en définitive l'univers ne peut pas se réduire à une vaste tautologie, nous devons forcément nous heurter çà et là dans notre description scientifique de la nature à des éléments 'irrationnels' qui résistent à nos tentatives d'identification, l'effort

> jamais lassé de la raison humaine s'acharnant à circonscrire ses éléments et à en réduire le domaine. (53)

It is clear, then, how the mind through the homogenization of the cosmos succeeds in triumphing over the irrationality that arises out of the pure givenness of the deliverances of the senses. Unlike the isolated perceptions of sense experience, the quantities with which mathematical physics deals lend themselves to the mind's desire for deduction: they can be both the conclusions and the principles of deduction. And to the highly integrative value of quantities which makes them derivable from each other is added the advantage of the wide scope of relational possibilities which arises from the extension of the quantitative system to include zero values, negative values, infinite values, etc.

But what is the price which the mind must pay for this triumph? From what has been said about the movement of science towards tautology, one might be led to suspect that the price is rather high, and to wonder what has actually been gained by abandoning the logical homogeneity of generality in which the specific distinctions of things are swallowed up. It might seem that the homogenization of experimental science is contrary to the very nature of that science,

which seeks to get at things in their specific natures and consequently in their heterogeneity. To put the question quite bluntly: does not the quantitative homogenization of the cosmos destroy the specific concretion of things and thus turn science back from its essential aim?

The answer is: yes and no. There is an essential difference between the logical homogeneity of generality and the ontological homogeneity of quantity. In the first case there is a complete renunciation of specific differences; in the second case the renunciation is only partial. For as we explained above, by locating its object in the realm of common sensibles, mathematical physics does not deal with pure quantity; it deals with the quantitative modes or, to use the expression of Meinong, the "quantified surrogates" of the specific determinations of nature. And because mathematics is not only a science of great generality, but also a science of great exactness, mathematical physics can, through a process of rigorous physical measurement, get at these specific determinations with far greater concrete precision than sensibility can. All of the qualitative aspects of nature have their quantitative modes and their variations involve quantitative mutations. And we pointed out above that there can be a direct correlation between the transcendental quantity, that is intrinsic to qualities and forms, and the predicamental quantity that is measured by physical processes. That is why the homogeneity of mathematical physics is not a complete renunciation of the heterogeneity of nature. From one point of view it is a means of knowing it better, and in this sense there is a distant resemblance here of the perfection of cognition found in the separated substances in which it is precisely through the homogeneity that the heterogeneity is known. And even though in its superstructures mathematical physics moves towards undifferentiated spatiality and tautology, it always starts out from, and must inevitably lead back to, the heterogeneity of nature. This makes it essentially different science based on logical homogeneity.

Thus the mind is able to enjoy an experience remotely analogous to the combination of sight and touch in sense experience. It is able to get at nature with something that resembles the certitude that is derived from touch, and with something that resembles the distinctness that comes from sight. But it is extremely important to recognize that in both cases it is a question of a mere substitute. Mathematical method affords a kind of exactness and certitude in dealing with nature, but from all that was said above about the essen-

tially dialectical character of experimental science it should be clear that it cannot provide true objective certitude. The same must be said of distinctness. For, with whatever extreme precision we get to know the quantified surrogates of the qualities and forms in nature, it is always with a substitution that we are dealing and never with the qualities and forms in their own proper, specific nature. Exact knowledge is not the same as specific knowledge. Moreover, a surrogate is always ambivalent; at the same time that it unites us with the object for which it substitutes, it separates us from it.

To attempt to get at the proper nature of the qualitative through purely quantitative methods is to accept one of the fundamental principles of Hegelism and Marxist dialectics: every quantity if sufficiently increased turns into a quality.[54]

That many have actually been led to identify the qualitative with the quantitative is well known. Speier, for example, holds that our physical experiments succeed in measuring quality directly.[55] For him quantity is not something that exists objectively in the physical structure of reality, but a conceptual construction which results from our process of measurement.[56] But ordinarily this identification h s been approached fro t e opposite direction by a sacrifice of quality to quantity. The evident dependence of the sense qualities upon the organic structure of the sense faculties, and the immense success of quantitative methods in science have led some to deny an objective status to all qualities and to conceive of the cosmos as a purely quantitative structure. Such a position is completely gratuitous. We have already shown that even though conditioned by the instruments of perception, the sensible qualities are not psychical, but physical and hence existing objectively in nature. And the fact that they do not exist in the distant object in exactly the same way as they are perceived, is no argument that the object is deprived of all qualitative determinations.[57] Moreover, the success of quantitative methods cannot be adduced as a demonstration of the non-existence of qualities without transforming a methodology into an ontology.[58]

As a matter of fact, the existence of an infinitely homogeneous reality is hardly conceivable. And even if it were a possibil ty, it could never be a source of knowledge.[59] It could not even be measured. For, as

Professor Thompson has remarked, "quantity, per se, in other words, pure undetermined quantity, is as unmeasurable as quality. It is measurable only when bounded, stamped, or permeated with quality. The quantitative picture of Nature, in spite of its satisfying accuracy is not self-supporting : it is executed in a framework of qualities, with which the savant must maintain contact."[60] It is worth while pointing out, moreover, that the numbers out of which the structure of mathematical physics is erected are concrete measure-numbers. This means that they involve something more than pure quantity. For even though they do not necessarily have a direct and immediate relation with our qualitatively different sensations or with the ontological qualities of reality, they are the results of qualitatively different processes of measurement.

All this enables us to see what is actually involved in the scientific homogenization of the cosmos. The barriers isolating the specific properties of nature are broken down; the pure givenness of these properties are mastered; nature is transformed into a deductive system; reality is rationalized; the most profound aspect of the cosmos: the order of the whole, is in a sense, revealed to the mind. At the same time contact is maintained with the specific properties through a process of correlation and substitution.

All this is a great achievement. But it is not without its price. For the determinant properties of things in their specific essences, the very inner natures of things have faded out of the picture. The hillside with its greenness and its softness of turf, the elephant in its own proper essence — all of the things in Nature which seem to be of the greatest significance for the other sciences of reality, for all the arts, and for human life itself, have slipped through the fingers of the physicist and have left in their wake only a series of pointer readings.(61)

This raises the question of the relative rationality of the qualitative and the quantitative determinations of reality. It has often been stated, that the latter are more rational than the former. That there is a sense in which this is true is evident from all we have been saying. But perhaps one might be tempted to question this superior rationality on the score that quantity is said to follow upon matter which is the source of irrationality, whereas quality is said to follow upon the form. John of St. Thomas gives us the answer in the following terms:

> Non est intelligendum, quod quantitas sequatur ad materiam nudam sine forma, cum constet sequi ad gradum corporeitatis qui praebetur a forma. Sed intelligitur sequi materiam, vel quia solum invenitur in rebus materialibus, qualitas autem sequitur actum, etiam si immaterialis sit, et sic proprium est qualitatis qualificare sicut et formae; tum etiam quia quantitas se habet in genere accidentium, sicut materia in genere substantiae, quia non est activa, sed medium receptivum aliorum accidentium et inter reliqua primum.(62)

Quantity has the great advantage of being the accident closest to substance. Material substance is a substance that can't contain itself, so to speak; it is dispersed, divided into parts; and quantity is the order of these parts. It is precisely because quantity consists in order that it can provide us with formal causality and not just with a kind of material causality, as one might be led to think because of the fact that it follows upon matter. Quantity is more abstractable than sensible qualities — not, however, because the latter are qualities, but because they are sensible. Mathematical beings are more perfect than sensible beings from the point of view of exactitude and certitude. Their very homogeneity is the source of precision. Moreover, their very emptiness makes them more manipulatable by us. Finally, quantity provides the common matrix which, as we have just seen, is so necessary for the rationalization of the cosmos. For all of these reasons quantity has a source of rationality which the specific properties of reality do not possess. And it is a type of rationality that is perticularly amenable to the methods of physical science.

On the other hand, the specific properties of reality are far more rational from another point of view. They reveal the proper natures of things. Consequently, it is in philosophical science that their rationality is particularly relevant. As we explained in Chapter I, the rationalities proper to physics and to philosophy are related to each other in inverse proportion. In the last analysis, it all comes down to a difference in the type of measurement proper to each science. In the following chapter we shall return to this point.

And now, having seen the way in which the mind triumphs over one of the sources of irrationality connected with sensible perceptions — their isolation and pure givenness, we must turn our attention to the other element of irrationality about which we spoke earlier in this chapter — the indefinibility of proper sensibles. By the same processes which we have been describing science succeeds in mastering this second irrational element. It succeeds in defining the indefinable. Through its quantitative methods, physics is able to define heat and colour in terms of movement of molecules, light waves, etc. A non-scientific person with the faculty of sight cannot define what he means by redness, but a blind physicist can.

the advantages of this definability are so obvious that they do not need to be mentioned.

But once again we must remain critically aware of what is actually involved in this defining of the indefinable. From what we have said about the impossibility of attaining the qualitative in its proper, specific nature by means of the quantitative it is obvious that the scientific definitions of heat, colour, etc. do not give us the <u>quod quid est</u> of these properties. There is a world of ambiguity in such expressions as "heat <u>is</u> a movement of molecules." All that they actually mean is: there is a correlation between the movement of molecules and heat. And science cannot even tell why there is such a correlation.

> The scientist does not seek a derivative measure for qualities which are incapable of direct measurement in order to find what those qualities really are. The measure of an object, whether fundamental or derived, does not express what the object <u>is</u>; it expresses how the object, as an instance of a certain character, is related to another object chosen as a standard for that character or for a correlated character. (65)

The following lines of Dewey, in spite of their obvious instrumentalistic bias, bring out rather accurately the point we are trying to make:

> The resolution of objects and nature as a whole into facts stated exclusively in terms of quan-

tities which may be handled in calculation, such as saying that red is such a number of changes while green is another, seems strange and puzzling only when we fail to appreciate what it signifies. In reality, it is a declaration that this is the effective way to think things; the effective mode in which to frame ideas of them, to formulate their meanings. The procedure does not vary in principle from that by which it is stated that an article is worth so many dollars and cents. The latter statement does not say that the article is literally or in its ultimate 'reality' so many dollars and cents; it says that for purpose of exchange that is the way to think of it, to judge it. It has many other meanings and these others are usually more important inherently. But with respect to trade, it is what it is worth, what it will sell for, and the price value put upon it expresses the relation it bears to other things in exchange... The formulation of ideas of experienced objects in terms of measured quantities, as these are established by an intentional art or technique, does not say that this is the way they must be thought, the only valid way of thinking them. It states that for the purpose of generalized, indefinitely extensive translation from one idea to another, this is the way to think them. . .
There is something both ridiculous and disconcerting in the way in which men have let themselves be imposed upon, so as to infer that scientific ways of thinking of objects give the inner reality of things, and that they put a mark of spuriousness upon all other ways of thinking of them, and of perceiving and enjoying them. It is ludicrous because these scientific conceptions, like other instruments, are hand-made by man in pursuit of realization of a certain interest — that of the maximum convertibility of every object of thought into any and every other. (64) [made by man in pursuit of realization of a cert-]

It is clear then that mathematical physics does

not succeed in actually defining the specific properties of nature, but merely something that is correlated with them. But even with regard to this correlation a further important qualification must be made. For, since scientific definitions are necessarily operational, the definitions of physics do not give us an absolute, objective, quantitative element that is in correlation with the specific properties; they necessarily involve the whole operational procedure by which this quantitative element has come to be known by us. This obviously removes them still further from a direct rendition of the quod quid est of the sensible properties. And in this connection it is necessary to point out that though the pointer readings which issue from our processes of measurement are not abstract but concrete numbers, they are not concrete in the sense that they directly correspond to certain sensations, but only in the sense that they are produced by concrete processes of measurement into which a multiplicity of concrete determinations have entered.

This brings us to another significant question. One of the important reasons given above for the adoption of quantitative methods in physics was the attempt to overcome the subjectivity and anthropomorphism of sensibility. We pointed out how through a substitution of inorganic instruments of measurement for organic instruments of perception science has been able to triumph over the subjectivity of sense cognition. But just how complete is this triumph? Do our measuring instruments provide us with a perfectly objective rendition of reality? Until fairly recently, it was not uncommon for scientists to think so. (65) Yet a greater error could hardly be imagined. In the next Chapter when we come to analyze the process of measurement we shall try to show just how much subjectivity this process involves, and for the moment it will suffice to merely mention the more important sources of this subjectivity. In the first place, there is the mental operation involved in the conception and method of application of the measuring instrument; all instruments are constructed and applied in accordance with certain scientific theories, and hence participate in the subjectivity of these theories. In the second place, there is the physical operation involved in the actual process of measurement; the instruments of measurement enter intrinsically into the process of measurement in such a way that the results are not independent of them.

The measuring instruments are not merely pas-

> sive recipients simply registering the rays impinging upon them: they play an active part in the event of measuring and exert a causal influence upon its result. The physical system under consideration forms a totality subject to law only if the process of measuring is treated as forming part of it. (66)
> In principle a physical event is inseparable from the measuring instrument or the organ of sense that perceives it; and similarly a science cannot be separated in principle from the investigators who pursue it. (67)

In attempting to get away from a mixture of senses and objects we succeed only in arriving at a mixture of instruments and object.

While considering all the advantages that have accrued to science from the substitution of inorganic instruments of measurement for organic instruments of perception, it is important to realize that our senses are also instruments of measurement, and that from this point of view there is no essential difference between the two:

> Perception is a kind of crude physical measurement. . . There is no essential distinction between scientific measures and the measures of the senses. In either case our acquaintance with the external world comes to us through material channels; the observer's body can be regarded as part of his laboratory equipment, and so far as we know, it obeys the same laws. We therefore group together perceptions and scientific measures, and in speaking of a'particular observer' we include all his measuring appliances. (68)

The greater objectivity that comes to us by means of impersonal instruments differs from the objectivity that comes to us through the senses only by degree; there is no qualitative difference between the two cases. The sense of touch perceives differences of temperature, and it may be said that it is only by accident that one's finger is a poor thermometer. If it were possible to know the physiological state of the finger with great accuracy one could by means of it arrive at the degree of temperature with as great precision as that achieved by a thermometer. In general it must be kept in mind that in our perception of the common sensibles, even without the aid of impersonal instruments, we already have a comparison.

In connection with what was said above about the advantages of the homogenization of the universe deriving from the greatly extended range which measurement adds to our limited powers of perception, a reservation must also be made. For while it is true that there is much in nature which cannot be sensed but which can be measured, it is likewise true that there is a great deal which can be sensed and cannot be measured.

This analysis of the relation between science and

sensibility would not be complete if, before concluding, some attempt were not made to determine how closely the scientific world remains linked to the sense world. From one point of view the bond seems to have grown extremely tenuous. As has already been said, mathematical physics is based upon a minimum of experience. The only kind of sensibility that is directly required for the scientist to carry on his work is that which is necessary to recognize objects and instruments and to perceive the coincidence of a fixed line on a scale with another variable line. All that this demands is the ability to perceive a spatio-temporal exteriority that is qualitatively differentiated. It makes little difference just what the nature of this qualitative differentiation is, provided it affords a sufficient means for making necessary distinctions. In other words, science has come as close as possible to the lowest form of all sense experience — the quantitative contact of pure taction. (69)

But it is important to keep in mind that in spite of its tenuity the bond between the scientific world and the senses remains essential

What I mean is this: we rig up some delicate physical experiment with galvanometers, micrometers, etc., specially designed to eliminate the fallibility of human perceptions;

> but in the end we must trust to our perceptions to tell us the result of the experiment. Even if the apparatus is self-recording we employ our senses to read the records. (70)

The desensibilized processes of physics are not self-supporting. Independent of the whole background which they have in the sensible world they are meaningless. Moreover, it must not be forgotten that by the very fact that mathematical physics is physics, it must realize the <u>reductio ad sensum</u> mentioned in Chapter II, which is characteristic of every science of nature. It must both take its origin in the sense world and terminate in it. Planck explains this very clearly in <u>The Universe in the Light of Modern Physics:</u>

> In my opinion, the teaching of mechanics will still have to begin with Newtonian force, just as optics begins with the sensation of colour, and thermodynamics with the sensation of warmth, despite the fact that a more precise basis is substituted later on. Again, it must not be forgotten that the significance of all physical concepts and propositions ultimately does depend on their relation to the human senses. This is indeed characteristic of the peculiar methods employed in physical research. If we wish to [redacted] *form concepts and hypotheses applicable to physics, we must begin by having* to our powers of imagination; and these depend upon our specific sensations, which are the only source of all our ideas. But to obtain physical laws we must abstract exhaustively from the images in-

> troduced, and remove the definitions set up all irrelevant elements and all imagery which do not stand in a logical connection with the measurements obtained. Once we have formulated physical laws, and reached definite conclusions by mathematical processes, the results which we have obtained must be translated back into the language of the world of our senses if they are to be of any use to us. In a manner this method is circular; but it is essential, for the simplicity and universality of the laws of Physics are revealed only after all anthropomorphic additions have been eliminated. (71)

As physics progresses it inevitably becomes more abstract and more highly symbolic. But to even its most abstract symbolism there always remains attached a dictionery which links up the symbols with concrete entities. And these concrete entities ultimately lead back to the world of sense. Thus modern physics presents the paradox of an ever increasing detachment from the sense world, and at the same time an essential attachment to it. And this paradox is comprehensible only in terms of another paradox: modern physics is at the same time physics and not physics; that is to say, it is a hybrid science, an intermedir science. It is for- [redacted] *distinct* pure natural science, but at the same time it is a valid study of nature. Because it is formally mathematical it must in its development draw ever farther and farther away from the world of sense; but because it is terminatively physical it must <u>inevitably</u> lead back to it.

This brings us to the final point that must be touched upon before we leave the general question which has formed the subject of this Chapter. In setting up the problem which has been occupying us we mentioned that some authors see in the recent developments of physics an abandonment of the common sensibles similar to the former abandonment of the proper sensibles and complementary to it. We do not believe that this is the correct interpretation of the newer scientific constructions. It is true that they are not susceptible of direct imaginative representation. But this does not mean that science has removed its object from the realm of the common sensibles as earlier it had removed it from the realm of the proper sensibles. It probably means several things. For one thing, in so far as these recent constructions have to do with the microcosmic world, it means that science is beginning to discover that phenomena on this microcosmic level may not be capable of direct representation in terms of phenomena on the microscopic level. DeBroglie points this out in <u>Matière et Lumière</u>:

Plus nous descendons

> Plus nous descendons dans les structures infimes de la matière plus nous nous aperçevons que les concepts forgés par notre esprit au cours de l'expérience quotidienne, et tout particulièrement ceux d'espace et de temps, deviennent impuissants à nous permettre de décrire les mondes nouveaus où nous pénétrons. On dirait que le contour de nos concepts doit, si l'on peut s'exprimer ainsi, s'estomper progressivement pour leur permettre de s'appliquer encore un peu aux réalités des échelles subatomiques. (72)

But in general, the most fundamental significance of these developments seems to be that science, by using as its instruments mathematical entities, which, as we saw in the last Chapter, can stretch their connection with the imagination to the extremes of tenuity, has so intellectualized its subject as to place it outside of any immediate relation to the sensible. There is no reason why it should not do this, provided all of its intellectual constructions can be made to lead back ultimately to verification in the sensible world. In this way that can be said to "explain" the sensible world. But this does not mean that these constructions give us a direct and immediate revelation of things as they exist in the real world or that they prove the common sensibles to be illusory.

And now, having seen the basis for mathematization that exists in nature, we must see how science, by laying hold of this basis through the instrumentality of measurement, succeeds in transforming nature into a new world of symbolism. This Chapter has attempted to show that in mathematical physics the mind's ambition is to transform the universe into a purely rational system in which multiplicity and differences will be constructed out of unity and sameness. It is in measurement that the mind finds a road towards its goal. For measurement consists in the repeated application to reality of the same unity - - a unity which the mind has determined.

CHAPTER EIGHT

AN ANALYSIS OF MEASUREMENT.

1. Science and Measurement.

This Chapter is in a sense the pivotal point of our whole study. For the central idea in mathematical physics is that of a scientia media involving a union of the physical and mathematical worlds, and it is precisely through measurement that these two worlds are brought into contact. This was already recognised by John of St. Thomas, for in speaking of the mathematical physics of his time, he writes: "Astrologus non agit de coelo et planetis, ut sunt entia mobilia, sed ut mensurabiles sunt eorum motus."(1) The reason why measurement is able to achieve this union between a science that is essentially experimental and one that prescinds from experiment is that, while remaining a physical instrument of experiment, it is not an instrument which merely reveals physical phenomena; it both reveals them and transforms them into numerical values. "Ce qui distingue notre science," writes Bergson, "ce n'est pas qu'elle expérimente, mais qu'elle n'expérimente et plus généralement ne travaille qu'en vue de mesurer." (2) It is significant that the names of practically all of our modern experimental apparatus end in "meter" whereas formerly they ended in "scope".

In other words, there is something both physical and mathematical about measurement. It is, as it were, a transforming machine into which physical determinations enter and from which numbers emerge. And even though the concrete measure-numbers which issue from our pointer-readings are not in themselves a mathematization of the physical in the full sense of the word, they are the incohation of this mathematization. They are the stuff out of which all the mathematical elaborations of physical science evolve. Although still directly linked with the physical, they already have something of the idealization, the absolute character, the necessity, etc. that belong to the mathematical world. And just as the whole mathematical interpretation of nature arises out of the physical through processes of measurement, so it must ultimately lead back again to the physical through [illegible] which answers with the processes of measurement. For no mathematical theory in physics has any value if it cannot be verified in concrete pointer-readings.

This explains why the whole progress of physical science is directly bound up with the refinement of measurement. (3) And, as Norman Campbell has pointed out, it is to the fact that it is a science of measurement that physics owes its ascendency over the other natural sciences. (4) All this explains why nothing has any meaning in physical science except in terms of measurement. (5) For a physicist a thing is real only to the extent in which it is measurable and everything that falls outside the scope of measurement is irrational. To define a body by its physical properties means simply to enumerate the operational processes of measurement to which this body can be subjected, and to list the series of numbers which the instruments used in these processes render. For example, what meaning for a mathematical physicist can hydrogen have, with its various properties: colorless, of a certain density, liquifying at a certain temperature, etc.? It can have no meaning except the following: a body will be called hydrogen if when subjected to the instruments which define fludity, viscosity, compressibility, temperature, refraction, etc., it produces a collection of pointer-readings which square with the numbers cited in the definition of hydrogen.

Among modern philosophers of science no one has

labore
unders
the ad
Chapte

T o c t v s i c t v 1 1 ((.

ke this point generally
(6)
ngton. In connection with
ı we discussed in the last

of exact science consists
imilar indications. We
a definition of what are
indications. The obser-
ncidence of the pointer
generally be extended to
f any kind of coincidence -
essed in the language of
eory, an intersection of
al point is that, although
nite conceptions of objects,
ose conceptions do not
and are not in any way
exact science can begin
y must be replaced by
he results of physical

that although only the
nto the actual calculation
f the problem to leave out
g else. The problem
s kind of connecting back-
pointer reading of the
id down the hill: And yet
f exact science the thing
the hill can only be des-
ointer readings. (It should
hill also has been replaced
d the sliding down is no
re but a functional
ge measures.) The word
ain association of mental
lear that mental impressions
bject handled in the physical

ysicist com rises a number
. an le, velocity, force,
., which we call "physical
ecognized as essential
ined according to the

in which we actually recognize them when confronted with them, and not according to the metaphysica significance which we may have anticipated for them. In the old textbooks mass was defined as 'quantity of matter'; but when it came to an actual determination of mass, an experimental method was prescribed which had no bearing on this definition. The belief that the quantity determined by the accepted method of measurement represented the quantity of matter in the object was merely a pious opinion. At the present day there is no sense in which the quantity of matter in a pound of lead can be said to be equal to the quantity in a pound of sugar. Einstein's theory makes a clean sweep of these pious opinions, and insists that each physical quantity should be defined as the result of certain operations of measurement and calculation. You may if you like think of mass as something of inscrutable nature to which the pointer reading has a kind of relevance. But in physics at least there is nothing much to be gained by this mystification, because it is the pointer reading itself which is handled in exact science; and if you embed it in something of a more transcendental nature, you have only the extra trouble of digging it out again . . .
Whenever we state the properties of a body in terms of physical quantities we are imparting knowledge as to the response of various metrical indicators to its presence, and nothing more . . .
The recognition that our knowledge of the objects treated in physics consists solely of readings of pointers and other indicators transforms our view of the status of physical knowledge in a fundamental way. Until recently it was taken for granted that we had knowledge of a much more intimate kind of the entities of the external world. (7)

The nature of the Physical World 252-58

Perhaps a word of explanation should be immediately appended to this passage lest confusion arise. When we say that mathematical physics deals only with pointer readings, we do not mean that it begins and ends in numbers labored with greater zeal to make this point generally understood than Sir Arthur Eddington.[(6)] In connection with the adventure of elephant which we discussed in the last Chapter, he writes:

> The whole subject-matter of exact science consists of pointer readings and similar indications. We cannot enter here into the definition of what are to be classed as similar indications. The observation of approximate coincidence of the pointer with a scale-division can generally be extended to include the observation of any kind of coincidence - or, as it is usually expressed in the language of the general relativity theory, an intersection of world-lines. The essential point is that, although we seem to have very definite conceptions of objects, in the external world, those conceptions do not enter into exact science and are not in any way confirmed by it. Before exact science can begin to handle the problem they must be replaced by quantities representing the results of physical measurement.
>
> Perhaps you will object that although only the pointer readings enter into the actual calculation it would make nonsense of the problem to leave out all reference to anything else. The problem necessarily involves some kind of connecting background. It was not the pointer reading of the weighing-machine that slid down the hill. And yet from the point of view of exact science the thing that really did descend the hill can only be described as a bundle of pointer readings. (It should be remembered that the hill also has been replaced by pointer readings, and the sliding down is no longer an active adventure but a functional relation of space and time measures.) The word elephant calls up a certain association of mental impressions, but it is clear that mental impressions as such cannot be the subject handled in the physical problem . . .
>
> The vocabulary of the physicist comprises a number of words such as length, angle, velocity, force, potential, current, etc., which we call "physical quantities.' It is now recognized as essential that these should be <u>defined</u> according to the way

alone. If this were the case it would be mathematics and not mathematical physics. The numbers it deals with are measure numbers. In other words, the experience which gives rise to these numbers has something more than a pre-scientific function as in mathematics. The physical process of measuring the quantitative determinations of nature is an integral part of mathematical physics. Consequently, even though the numbers dealt with do not represent <u>things</u> in the objective cosmos, as we shall see, they are always tied up with objective determinations of the physical universe out of which they have issued through measurement. In this sense there is a physical background in which they are embedded. Yet the mathematical physicist cannot get at this background in any other way than by measurement, and that is why as long as he remains true to the nature of his science this background will always elude him. Of course it is possible for him to go out beyond the limitations of his science and embed the measure-numbers in a background of his own choosing, but, as Eddington remarks, in so far as mathematical physics is concerned, there is nothing to be gained by doing so.

We shall return later to discuss the nature of knowledge which grasps reality only through measurement.

For the present we merely wish to emphasize the fact that this is the only type of knowledge that is had in mathematical physics. Of course, in actual practice, scientists never restrict themselves completely to measure-numbers. [8] As Poincaré has remarked, they cannot be denied the liberty of using metaphors any more than poets can. [9] But in the last analysis their grasp of the cosmos is restricted to metric knowledge. It is because this is not always recognized that much of the confusion about the meaning of modern science has arisen. This is particularly true of many of the abortive criticisms of the Theory of Relativity. Einstein's great merit is to have recognized clearly the complete dependence of mathematical physics upon measurement, and to have seen the implications and limitations of this dependence.

That mathematical physics is essentially a science of measurement is now becoming generally recognized. But what is not generally recognized is that every science of reality is essentially a science of measurement. This statement is, of course, ambiguous, for obviously the term "measurement" cannot be understood in the same sense in which we have been employing it in relation to physics. And yet it is not an equivocation; in both cases the term is used in its strictly formal sense. And in order to understand accurately the part that measurement plays in physics it is extremely important to see how the other sciences, and particularly the philosophical sciences are related to measurement.

Taken in its general sense, measurement implies an effort on the part of the intellect to see a certain complexity in the light of a principle of simplicity. This principle is provided by a standard, and the attempt of the mind to reduce complexity to simplicity will be more or less successful in proportion to the degree of simplicity possessed by the standard. This explains why in physics there is a continual search for a minimum measure. But it is not only in physics that there is an attempt to see the complexity of reality in terms of the simplicity of a standard. This is found in the philosophical sciences as well, although the nature of the standard and hence the nature of the measurement is something quite different from what is found in physics.

so far as He is the cause of being. It is by comparison

St. Thomas defines measure as "that by which the quantity of a thing is made known." (10) But as we saw in the last Chapter, there are two kinds of quantity: predicamental and transcendental. The former consists in homogeneous

exteriority and the latter in interiority, that is, in perfection of being. Now whereas in physics it is predicamental quantity that is made known through measurement, in the philosophical sciences it is transcendental quantity. In both metaphysics and philosophy of nature it is the principal subject of the science which provides the ultimate principle of simplicity in relation to which every other subject in the science is measured. For, as John of St. Thomas remarks: "mensura importat perfectionem, cum semper accipiatur pro mensura id quod perfectissimum est in unoquoque genere; nec requiritur quod sit notificativum rei mensuratae, ut fundans imperfectam cognitionem; sed per modum alicuius magis simplicis et perfecti quo res mensurata magis ad unitatem et uniformitatem reducitur." (11)

In every order in which a relation of more or less is possible there is measure, and the "maxime tale" is always the measure of everything that is found in the order. (12) In metaphysics the principal subject which plays the part of the standard is God, known extrinsécally, in so far as He is the cause of being. It is by comparison with God, Pure Act, that the transcendental quantity of all metaphysical beings is measured and their intrinsic perfection revealed. In philosophy of nature the principal subject is man, and it is in relation to him that the transcendental quantity of all natural beings is determined. In this sense Protagoras was right in making man the measure of all things in the universe. Whereas from the point of view of the physicist man is the most complex being in the cosmos and the one that is the farthest removed from his standard of measurement and hence the one that is least amenable to his processes of measurement, from the point of view of the philosopher of nature he is the most simple being in the cosmos precisely because he possesses the highest degree of interiority. It is extremely significant that the measurements of physics and the philosophy of nature lead in opposite directions: the one determines things in relation to the simplicity of purely homogeneous exteriority, the other in terms of the simplicity of interiority. For physics interiority is irrational. That is why the experimental science which deals with man - - experimental psychology - - is the most irrational of all the experimental sciences. For the philosophy of nature it is homogeneous exteriority that is irrational and that explains why for the philosopher natural things become more obscure as one descends the scale of perfection. No one, perhaps, has handled this question with greater skill than Professor DeKoninck:

Toute science s'efforce de réduire le complexe au plus simple et de l'expliquer en fonction de lui. Mais il faut s'entendre sur la signification du terme 'simple'. La nature de la simplicité à laquelle on doit tout ramener différenciera profondément les savoirs. Or il est facile de montrer que ce que nous appelons simple en science expérimentale est tout opposé à ce que nous disons simple en philosophie. En science expérimentale une pierre est infiniment plus simple qu'une cellule; le va-et-vient d'un piston est beaucoup plus simple que le bond d'une panthère qui se jette sur sa proie; de tous les êtres qu'étudie la science expérimentale, l'homme est incontestablement le plus complexe. Or en philosophie c'est tout le contraire qui est vrai. L'animal est plus simple que la plante, et de tous les êtres qu'étudie la philosophie de la nature, c'est l'homme qui est le plus simple; de même qu'en métaphysique la mesure et la cause de tout être est la simplicité absolue de l'acte pur. En physique on mesure par la <u>minima mesura</u> - le temps par le temps atomique par exemple; en philosophie la mesure est toujours riche et compréhensive - - le temps est mesuré par l'éviternité, et tous les deux par l'éternité. En d'autres termes, la simplicité expérimentale est inversement proportionnelle à la simplicité ontologique. Le philosophe dira que le savant explique le supérieur par l'inférieur, le parfait par l'imparfait. Ainsi nous pouvons dire par avance que dans la mesure où une explication expérimentale de l'homme est possible, elle consistera à l'étudier dans la perspective de ce qui est expérimentalement plus simple que lui, non pas pour identifier entre eux le complexe et l'élémentaire, mais pour dériver l'un de l'autre. Il est donc tout naturel que le savant cherche à dériver l'homme de l'animal, celui-ci de la plante et à voir toute la hiérarchie des espèces naturelles s'ériger dans le sens d'une organisation toujours croissante et plus complexe. Le philosophe qui nie la possibilité même d'une théorie évolutioniste nie l'essence même de la méthode scientifique. S'il était logique il devrait nier aussi la valeur d'une mesure de longueur. (13)

This brings us back to what we saw in Chapter I in relation to the possible extent of the mathematization of nature.

But in order to understand the peculiar nature of the knowledge that is based completely on a measurement of things in terms of homogeneous exteriority we must try to analyze the nature of measurement.

2. The Nature of Measurement.

Measure, according to Aristotle and Saint Thomas, is that by which the quantity of a thing is made known. (14) This definition immediately gives rise to a difficulty. For quantity may be known independently of any measure. In fact, homogeneous exteriority is an immediate datum of cognition, and consequently does not depend upon any medium such as a measure. Moreover, we have already pointed out that quantity is known and studied both by the philosopher of nature and the metaphysician, and in neither case does the knowledge of it involve measurement. This difficulty did not escape Aristotle and St. Thomas. For after laying down the fundamental definition just cited, they proceed to

qualify its meaning by adding the phrase: inquantum quantitas. That is to say, measure is that by which the quantity of a thing is made known precisely in so far as it is quantity. At first glance this may not seem to help matters, for is not quantity known as quantity independently of measurement in the ways just mentioned?

St. Thomas throws light upon the question by writing: "Addit autem (Philosophus)'inquantum quantitas' ut hoc referatur ad mensuram quantitatis. Nam proprietates et alia accidentia quantitatis alio modo cognoscuntur." (15) In other words, there are two fundamental aspects to quantity. In the first place, in so far as it is one of the nine accidents it is a certain essence and consequently can be known in the same way that all the essences of reality are known. In so far as it orders the parts of a material substance by contributing to it homogeneous exteriority, it is a primary and immediate datum of cognition. In so far as it is involved in the mobility of the cosmos, it can be studied by the philosopher of nature. In so far as it is one of the principles of being it can be studied by the metaphysician. In all of these cases it is a question of "quidditative" knowledge, that is, knowledge that answers the question: what is quantity? Now while this question "what" can be asked of all the categories of reality, there is a special question that can be asked only of quantity - - "how much" (quantum). And it is knowledge which answers this question that is revealed by measure. Since, then, the question "how much" (quantum) is proper to quantity alone, Aristotle and St. Thomas are justified in saying that measure is that by which the quantity of a thing is made known; and they are speaking with strict formality when they add the phrase: <u>in quantum quantitas</u>.

It is extremely important to insist upon the precise nature of the knowledge of quantity that is given to us through measurement. It is not "quidditative" knowledge; it does not in any way answer the question: what is quantity. It merely tells us how much quantity there is. This knowledge is mediate and derivative, since it comes to us through the medium of measure. But a measure is a very special kind of cognitive medium. Unlike a sign, it does not substitute for the thing known, nor does it in any way manifest its nature. And the practical conclusion to be drawn from these con-
siderations [illegible] in so far as science is based upon
siderations
measurement, not only does it not tell us the "whatness" of all the determinations of reality which fall outside the category of quantity, but it does not even tell us

the "whatness" of the quantitative determinations that are being dealt with. This point is frequently lost sight of.

But in order to understand more clearly the nature of this peculiar type of knowledge we must try to see just how quantity is revealed through measurement. A measure manifests the quantity of a thing not in any way whatsoever, but through the reduction of a certain type of complexity to simplicity, of indetermination to determination, of variability to uniformity - - in other words, of unintelligibility to intelligibility. When the determination of one thing manifests to us the determination of another thing, which without it would remain indetermined, we say that the first is the measure of the second. In this way the measure is a certification of the thing measured. From this it follows that there are two essential elements in measurement: a principle of perfection and uniformity and simplicity, which is the measure, and a process of re- (16)
duction of the complex and variable to this principle. This second element obviously involves some kind of union between the measure and the thing measured. In order to understand the nature of measurement it will be necessary to analyze each of these two elements.

With regard to the first it is clear that in order for a thing to be a measure it must be one and indivisible, for in no other way can it be simple and determined. That is why in the tenth book of the Metaphysics St. Thomas begins his explanation of measurement by saying: "cum ratio unius sit indivisibile esse; id autem quod est aliquo modo indivisibile in quolibet genere sit mensura; maxime dicetur in hoc quod est esse primam mensuram cuiuslibet generis." (17) But it must be pointed out that the "one" is not as such a measure. That is to say, indivisibility of itself does not necessarily constitute a measure; it must be indivisibility in a certain given order. The transcendental One is not a measure because it is not in a definite genus. Moreover, it does not possess strict unity. (18)

Aristotle and St. Thomas make it clear why indivisibility is one of the essential qualities of a measure:

> Assignat autem rationem quare mensuram oportet [illegible] esse aliquid indivisibile, quia scilicet hoc est certa mensura, a qua non potest aliquid auferri vel addi. Et ideo unum est mensura certissima; quia unum quod est principium numeri, est omnino indivisibile, nullamque additionem aut subtractionem suscipiens manet unum. (19)

A measure is a certification of the thing measured. But it can be a certification of something else only to the extent in which it is fixed in certainty itself. And it can be fixed in certainty only by being fixed in indivisibility.

A thing can be a measure, then, only to the extent in which it is indivisible. But as St. Thomas goes on to explain:

> Non similiter in omnibus invenitur indivisibile; sed quaedam sunt omnino indivisibilia, sicut unitas quae est principium numeri; quaedam vero non sunt omnino indivisibilia, sed indivisibilia secundum sensum, secundum quod voluit auctoritas instituentium tale aliquid pro mensura; sicut mensura pedalis, quae quidem indivisibilis est proportione, sed non natura. (20)

And elsewhere he writes: "Nec oportet, quod omnis mensura sit omnino infallibilis et certa, sed secundum quod est possibile in genere suo". (21) In so far as the measurement of predicamental quantity is concerned it is only the one which is the principle of number that has absolute indivisibility. That is why it alone is the perfect measure. "Esse mensuram est propria ratio unius secundum quod est principium numeri." (22) And just as all of our notions of measurement are derived from predicamental quantity, ([illegible]) so within the realm of predicamental quantity itself all our notions of measurement are derived from the measurement of discrete quantity:

> Primo ostendit quod ratio mensurae primo invenitur in discreta quantitate, quae est numerus dicens, quod id quo primo cognoscitur quantitas 'est ipsum unum', idest unitas, quae est principium numeri. Nam unum in aliis speciebus quantitatis non est ipsum unum, sed aliquid cui accidit unum; sicut dicimus unam manum, aut unam magnitudinem. Unde sequitur, quod ipsum unum, quod est prima mensura, sit principium numeri secundum quod est numerus.. Hinc scilicet ex numero et uno quod est principium numeri, dicitur mensura in aliis quantitatibus, id scilicet quo primo cognoscitur unumquodque eorum. Et id quod est mensura cuiuslibet generis quantitatis, dicitur unum in illo genere. (24)

For us the "one" which is the principle of number is the model for every measure. It is that by which quantity is <u>first</u> made known to us: "id quo primo cognoscitur quantitas."

In the measurement of other kinds of predicamental quantity only quasi indivisibility is possible. It is impossible, for example, to have a length which will be a universal measure for all lenghts as the one which is the principle of number is the universal measure for all numbers.

> Hoc modo derivatur ratio mensurae a numero ad alias quantitates, quod sicut unum quod est mensura numeri est indivisibile, ita in omnibus aliis generibus quantitatis aliquod unum

> indivisibile est mensura et principium. Sicut in mensuratione linearum utuntur homines <u>quasi indivisibile</u> 'mensura pedali,' idest unius pedis; ubique enim quaeritur pro mensura aliquid indivisibile, quod est aliquod simplex.(25)

And this quasi indivisibility is nothing but an imitation of the true indivisibility that is found in the "one" which is the principle of number. One inch, for example, is an imitation, for it cannot be by itself an absolute measure.

> Sed mensurae aliorum generum quantitatis imitantur hoc unum, quod est indivisibile, accipiens aliquid minimum pro mensura secundum quod possibile est. Quia si acciperetur aliquid magnum utpote stadium in longitudinibus, et talentum in ponderibus, lateret, si aliquod modicum substraheretur vel adderetur; et semper in majori mensura hoc magis lateret quam in minori.
> Et ideo omnes accipiunt hoc pro mensura tam in humidis, ut est oleum et vinum, quam in siccis, ut est granum et hordeum quam in ponderibus et dimensionibus, quae significantur per grave et magnitudinem; quod primo invenitur tale, ut ab eo non possit aliquid auferri sensibile vel addi quod lateat. Et tunc putant se cognoscere quantitatem rei certitudinaliter, quando cognoscunt per huiusmodi mensuram minimam.(26)

This attempt on the part of the measurement of magnitude to imitate the measurement of multitude must be considered in the light of what was said in Chapter II about the difference between arithmetic and geometry. We pointed out that the higher abstraction and superior intelligibility of arithmetic was based upon the superior rationality of number in comparison with magnitude. Number is in fact more immaterial, more determined, more actual than continuous quantity. The continuum is something essentially obscure, indetermined and potential because of its intrinsic divisibility into infinity. As a result, the measurement of discrete quantity is something clear and absolute, while that of continuous quantity is always something obscure and relative. In the latter there is always a background of irrationality.(27)

But since measurement is always a rationalization in the sense that it manifests the quantity of the thing measured, the mind can never rest satisfied with this background of irrationality. That is why there will inevitably be a constant attempt to assimilate as much as possible the measurement of continuous quantity to that of discrete quantity. "Omnis mensuratio quae est in quantitatibus continuis aliquo modo derivatur a numero. Et ideo relationes quae sunt secundum quantitatem continuam etiam attribuuntur numero."(28)

This process of assimilation will be at once both subjective and objective. In the first place, since a definite unit of measure is not given objectively for magnitude as it is for multitude, one must be constructed

by the mind, established by fiat.

> Quaedam vero non sunt omnino indivisibilia, sed indivisibilia secundum sensum, secundum quod voluit auctoritas instituentium tale aliquid pro mensura. (29)

> In gravitate ponderum accipitur ut unum indivisibile uncia, sive 'mna', idest quoddam minimum pondus; quod tamen non est simplex omnino, quia quodlibet pondus est divisibile in minora pondera, sed accipitur ut simplex per suppositionem. (30)

This point is of considerable importance for the philosophy of physical science. For the basic measurement in physics is that of magnitude. Though science employs a great variety of measurements, they are reducible in the last analysis to the measurement of length. It is clear, then, that the measurement out of which the whole structure of mathematical physics is erected, is not based on something absolute, something perfectly objective and given as such in nature, but upon a construction of the mind. Both the intellect and the will have to enter into the process of measurement to determine a standard and establish a unity that does not exist. Magnitude is lifted to a status of intelligibility that is not native to it. And all this obviously involves a separation of some sort from the real world. What is not by nature one and indivisible is considered by the mind as if it were. Once again, from this point of view, mathematical physics is a science of <u>als ob</u>.

However, this construction is not purely subjective and arbitrary. In order to assimilate the measurement of magnitude to that of multitudo it is not sufficient to declare by fiat something indivisible that is by nature divisible; it is necessary that what is declared indivisible approach as closely as possible to that which is objectively indivisible. In other words, the less extension the standard chosen possesses, the more perfectly will it be able to serve as a measure. That is why science is always searching for the smallest possible measure - - the <u>minima mensura</u>. And this is true of ancient as well as of modern science:

> Id quod est minimum in unoquoque genere, est mensura illius generis, sicut in melodia tonus, et in ponderibus uncia, et in numeris unitas; manifestum est autem quod minimus motus est qui est velocissimus, qui scilicet habet minimum de tempore, quod est mensura motus; omnium ergo motuum velocissimus est motus coeli. Et accipitur hic motus velocissimus, qui citius peragit cursum suum ex parte brevitatis temporis ... Unde ... attenditur secundum minimam magnitudinem. (31)

This choice of the speed of the movement of the heavens as the standard was based upon an hypothesis of ancient physics. As Saint Thomas points out: "Ponit (Aristoteles) hanc <u>suppositionem</u> quod motus coeli sit mensura omnium motuum." (32)

Today the standard has been changed and is now the speed of the movement of light. But whether the standard chosen be the speed of the movement of the heavens, or the speed of the propagation of light, or the wave-length of a red spectral line emitted by cadmium, the logical structure of the measurement of continuous quantity remains the same; it is always a question of a standard which is indivisible by fiat though not by nature, and which represents an attempt to come as closely as possible to the <u>minima mensura</u>.

It is clear, the, that there is something profoundly paradoxical about the measurement of continuous quantity. On the one hand, it is necessary for the scientist to search for the <u>minima mensura</u>, and the dialectical tendency towards certitude about which we spoke in Chapter V becomes in this field the search for an absolutely small measure. On the other hand, this infinitesimally small measure does not exist. "Sed in lineis non est invenire minimum secundum magnitudinem, ut sit scilicet aliqua linea minima; quia semper est dividere quamcumque lineam. Et similiter dicendum est de tempore." (33) An infinitesimally small measure would involve a contradiction, since it would consist in a contiuum without extension. It is then a purely dialectical limit that can be approached indefinitely;

it is not a limit given in nature that can ultimately be arrived at. And this impossibility of arrival is not due to any lack of precision on our part; it is due to the very nature of continuous quantity. We must then be satisfied to accept the minimum measure that is possible for us to have - - " accipere aliquid minimum pro mensura secundum quod possibile est."

How is it possible for the mind in spite of this paradox to succeed in some way in assimilating the measure of magnitude to that of multitude? In order to answer this question it is necessary to recall that it is possibile to know that two or more classes have the same number, without knowing what that number is. Thus, for example, if all the tickets to a certain theater have been sold, it is possible to know that there are as many people in the theater as there are seats without knowing in any definite way the number of the two classes involved. In the same way it is possible to know that two classes have different numbers, without knowing what these numbers are. Now something very similar is found in magnitude. By juxtaposing two rods x and y, I can discover that they are of equal or unequal length, even though I cannot say anything of the length of rod x or rod y taken separately.

If it happens that rod x can be placed twice along rod y, I can arrive at the formula: $y = 2x$. Yet once again, this does not reveal anything about the lengths of the two rods when they are taken separately. In other words, it is possible to arrive at the knowledge of continuous quantity by establishing ratios. (34) And since the structure of mathematical physics is based on the measurement of lengths, the knowledge that it gives us is reducible in the last analysis to a knowledge of ratios. When for example the wave-length of the line Hα in the spectrum of atomic hydrogen is indicated by the measure-number 0.000065628, this does not reveal any absolute property; it merely tells us the ratio existing between the length of a wave of Hα light to that of a centimeter, which is obviously an arbitrary standard. In like manner the whole of physics is built up out of ratios determined in relation to arbitrary standards. (35)

It is clear, then, how it is possible for the measurement of magnitude to imitate that of multitude. Just as I can know that two classes have the same number, so I can know that two rods have the same length. The two cases remain similar until I attempt to get at the meaning of the "same". In the case of multitude this meaning can be determined absolutely since it is based on cardinal number, and consequently it is possible to escape from mere knowledge of proportion. In the case of magnitude, the meaning of the "same" cannot be determined absolutely; it is impossible to escape from knowledge of proportion.[36]

From all that has been said thus far about the nature of measurement of magnitude it follows that from the point of view of the physicist the standard of length has no length. Sir Arthur Eddington has brought out this point very forcefully in the Prologue to <u>Space, Time, and Gravitation.</u>[37] But lest confusion arise it is necessary to make several distinctions. The term "length" is in fact extremely ambiguous and is susceptible of a great variety of meanings. It may be taken to mean: 1) dimension as such (and this is its most proper meaning); 2) a line, that is to say, a finite length;[38] 3) the measured magnitude of a finite length; 4) a geometrical line; 5) the measured magnitude of a finite line; 6) a sensible line taken as a dimension; 7) a sensible line as a finite magnitude; 8) the measured magnitude of a sensible line. Now, if the term be taken in the senses indicated under numbers six and seven it is obvious that the standard of length is a length. If it were not it could not be a standard: "oportet mensuram homogeneam esse mensurato." But when a physicist speaks of length it

is particularly the sense indicated by number eight that he has in mind. Then it is a question of a magnitude that is expressible by a measure-number which answers the question "what is the length of this line? In this sense it is true to say that the standard of length has no length. In so far as it is a standard it can be defined only by designation and in no other way. The same is true of the measurement of time. Understood in this way, the theory of Relativity is correct in maintaining that if an object could move with the velocity of light it would be outside of time, for the speed of the propagation of light is taken as the fundamental standard of the measurement of time.

It is possible, of course, to define a certain designated standard in terms of another standard, but then it is no longer being defined <u>qua</u> standard, since another standard has been substituted. For example, we can define a meter in terms of a hundred centimeters, and this gives us the illusion that we can know how long the standard meter is. But obviously in this definition the standard is ultimately known no longer the meter but the centimeter, and we are faced with the question: how long is a centimeter? There are just two ways by which one might attempt to answer this question: one is by saying that it is the hundredth part of a meter, and this obviously involves a vicious circle; the other is by having recourse to a still smaller standard, and this involves a process ad infinitum; by the time we have come to the Ånström as the standard we are still as far from the answer to our question as we were in the beginning.

The infinity of the vicious circle and the indefinite process is a sign of what is at the bottom of this whole question: the inexhaustible potentiality of the continuum. And most of the difficulties that arise in connection with this problem have their origin precisely in this that we attempt to confer upon the continuum a degree of intelligibility that belongs only to discrete quantity. It is extremely important to keep in mind that the measures of continuous quantity are essentially inadequate and imitative. They do not do away with the inherent unintelligibility of the continuum, for they cannot change its nature. Measurement consists in the juxtaposition of an unknown with a scale. It is usually taken for granted that this scale is something definitely known by itself. As a matter of fact, it is not. And as a consequence, measurement, in the last analysis is merely the juxtaposition of an unknown with an unknown. But perhaps this whole question will become clearer later on when

we take up the distinction between intrinsic and extrinsic measure. In the meantime it is worth while noting that this point is obviously of extreme significance for the whole question of mathematical physics and particularly for the theory of Relativity.

Indivisibility is then the primary quality of the first element of measurement mentioned above: the principle of perfection and simplicity. But there is another extremely important quality that is closely allied to it: the measure must possess uniformity. In order for measurement to be able to reduce complexity to simplicity, indetermination to determination, and variability to invariability it is not sufficient that the measure be one and indivisible; it is also necessary that it be uniform. In no other way can it provide objective certification in respect to the thing measured. Consequently it is necessary to choose a standard that is controllable, precise, uniform and invariable.

> Perfectio mensurae consistit in uniformitate et simplicitate, qua aliquid de se est notificativum alicuius quantitatis; hoc enim exigitur ad rationem mensurae ex parte suae perfectionis, eo quod perfectissimum in aliquo genere est mensura ceterorum (36)

And obviously the uniformity required is uniformity with respect to the particular genus in which the measurement takes place.

> Sola uniformitas seu regularitas, sumpta in abstracto, est communis ad omnem mensuram...Ergo oportet quod determinetur ratio talis mensurae essentialiter et intrinsece, per hoc quod sit uniformitas talis vel talis quantitatis, vel generis...Ergo pertinet ad ipsam essentialem rationem mensurae non solum habere uniformitatem, sed uniformitatem talis vel talis conditionis seu generis: ratione cuius sit apta et habilis mensura ad mensurandum talia mensurata. (40)

The perfection of a measure of length, for example, requires that it be uniform <u>in the genus of length</u>, in other words, that its length be <u>objectively constant</u>. Here we are touching upon one of the most important problems of measurement in so far as it effects mathematical physics — the problem of the rigid rod. We shall have a great deal to say about this question later on, and at this point it will be sufficient to merely touch upon the fundamental issue. In every measure of continuous quantity there is from the point of view of uniformity and invariability an essential imperfection that parallels its imperfection from the point of view of indivisibility. For every measure of continuous quantity is an extended piece of matter which is an <u>ens mobile</u> and consequently subject to a continual state of flux. It is a part of an extremely complex and unstable cosmos. It is at every moment undergoing innumerable physical influences which necessarily produce changes in it. These physical influences cannot be eliminated completely without changing the nature of the material standard and without separating it completely from the cosmos. Of course, the changes produced can be controlled to some extent.

But in order to have perfect control, it would be necessary to know all of the laws of nature; it would be necessary to have an exhaustive knowledge of the cosmos. Once again it is evident that the perfectly uniform standard is only a dialectical limit that can be approached indefinitely, not a natural limit that is objectively capable of being reached. Once again the mind must step in and construct; it must provisionally declare to be uniform what is by nature lacking in uniformity.[41] And we may apply here what St. Thomas has to say about the fluidity of human law: "Mensura debet esse permanens, quantum est possible; sed in rebus mutabilibus non potest esse aliquid omnino immutabiliter permanens."[42]

In connection with the first essential element we have been discussing — the principle of perfection and simplicity there is one final point that must be touched upon. We have said that a measure is that by which the quantity of a thing is made known. But there are two ways in which one thing may manifest another. In the first place, a less perfect object may serve to manifest a more perfect object. It is in this way that creatures manifest their Creator, and this is in keeping with the limited nature of our human knowledge which in the order of generation progresses from the less perfect to the more perfect. But it is also possible for a more perfect object to manifest a less perfect object, and it is obviously in this way that a measure manifests the thing measured, since in relation to the latter the former is always a principle of perfection.

> Licet mensura de se ordinetur ad notificandam quantitatem formalem vel virtualem rei mensuratae, non tamen est de ratione mensurae quod notificet nobis quantitatem rei mensuratae modo imperfecto, seu juxta modum quo procedit nostra cognitio de imperfecto ad perfectum; sed requiritur quod ex ipsa ratione mensurandi notificet nobis mensuratum modo perfecto, seu procedendo a perfectiori ad minus perfectum seu minus nobis notificatum . . . Hoc enim modo mensura notificat, scilicet per modum perfectionis et simplicioris, quia perfectissimum in unoquoque genere est mensura ceterorum: unde per modum perfectionis, et non via generationis (sive processus de imperfecto ad perfectum), debet mensura notificare. (43)

Now it happens that in the type of measurement with which science is primarily concerned — the measurement of length — there is no objectively perfect standard, no absolutely perfect principle of simplicity, as is evident from all that has been said thus far. That is why science must ever remain in search of a more perfect standard to manifest the less perfect. And that is why its measurement will always remain imperfect and obscure.

And now, having analysed the first essential element of measurement we must consider the second: the union between

the measure and the thing measured. In order for this union to be possible, it is obviously necessary that there be some kind of compatibility between the two. And this prerequisite condition is expressed in the fundamental Thomistic principle: "mensuram oportet esse homogeneam mensurato."

> Mensura semper debet esse cognatum, scilicet eiusdem naturae vel mensurae cum mensurato: sicut mensura magnitudinis debet esse magnitudo: et non sufficit quod conveniat in natura communi, sicut omnes magnitudines conveniunt: sed oportet esse convenientiam mensurae ad mensuratum in natura speciali secundum unumquodque, sic quod longitudinis sit longitudo mensura, latitudinis latitudo, vox vocis, et gravitas gravitatis, et unitatum unitas. (44)

But this immediately gives rise to several difficulties. In the first place, number is measured by the "one", which is not a number. Consequently, in this case the measure and the measured do not seem to be in the same genus. St. Thomas answers this difficulty in the paragraph which follows the one just cited: "Unde nihil aliud est dicere unitatem esse mensuram numeri, quam unitatem esse mensuram unitatum." (45) In other words, even though the "one" is not a number, it belongs to the same genus in the sense of being the principle of number. Though not in itself discrete quantity it pertains to the order of discrete quantity in so far as it is its principle. A more serious difficulty arises from the fact that God is said to be the measure of all beings, and eternity is said to be the measure of time; yet in neither case does it seem possible to apply the principle: "mensuram oportet esse homogeneam mensurato." St. Thomas suggests the solution for this difficulty in the Summa: "Mensura proxima est homogenea mensurato, non autem mensura remota."[46] In other words, in order to have measure in the strict sense of the word it is not necessary that the measure and the thing measured be in the same genus in the strict sense of the word. This is required only of the immediately proximate measure. For every other measure it is sufficient that they be in the same general category as for example in the case of time and eternity which belong to the category of duration, or even in the same universal order of being as in the case of God and creatures.[47] It is in the realm of magnitude that the principle which requires the measure and the thing measured to be homogeneous is most perfectly realized. For the measure of a length is not a point but another length. That is why St. Thomas in his commentary on the fifth book of the Metaphysics in speaking of the difference between number and magnitude uses the phrase: "magnitudo sive mensura."[48] Magnitude is, in fact, a measure, whereas number is not.

But this basic compatibility between the measure and the thing measured is only the prerequisite condition for the fulfillment of the second essential element in measurement. In order for the indetermination of the thing measured to be effectively reduced to determination some kind of union between the two is necessary. Now there are two ways in which a measure can be united with the object measured. In the first place, it can be united to it extrinsically by means of some kind of application. This application need not be physical; it may consist in a purely intellectual juxtaposition or comparison, as when, for example the transcendental quantity of creatures is measured by the Supreme Being. In physical science the application is in one way or another physical; but it does not have to be direct or immediate, otherwise it would be impossible to measure objects in motion and objects at a distance. Yet it must be pointed out in passing that physical measurement acquires certitude and objectivity to the extent in which the application becomes more direct and immediate. Now whenever a measure and an object measured are united, by means of an application the measurement is extrinsic.

But there is another and more intimate way in which a measure can be united with a measured object: by identification.

And when this type of union is realized the measurement is known as intrinsic.

This brings us to the distinction between extrinsic and intrinsic measure which is of considerable importance for an understanding of the nature of measurement. St. Thomas touches upon this distinction in several places, (49) but perhaps the clearest and fullest explanation of it is found in John of St. Thomas:

> Oportet distinguere mensuram instrinsecam et extrinsecam. Extrinseca est quae mensurat aliquid extra se; et ideo per applicationem et continentiam illius dicitur mensurare, sicut duratio et motus coeli mensurat motus inferiores tamquam extrinseca mensura illorum, et ulna mensurat pannum, et libra pondus. Unde talis mensura terminat relationem realem sui mensurati. Intrinseca mensura est illa quae inest rei mensuratae; et ita non mensurat per applicationem, sed per informationem; unde habet perfectionem mensurae, licet non relationem realem et imperfectionem dependentiae qua mensuratum dependet a mensura; . . . et in unoquoque genere perfectissimum est mensura sui et ceterorum, sui quidem instrinseca, aliorum vero extrinseca. (50)

It is obvious that this distinction rests upon a difference in the kind of union existing between the measure and the object measured. Now just as a measure is more perfect to the extent in which its first essential element, that of simplicity and uniformity, is more perfectly realized, so likewise it is more perfect in proportion to the degree of intimacy found in the union with the measured object. This has

already been noted with regard to union by application in physical measurement: the certitude and objectivity of the measurement depends upon how direct and immediate the application is. But obviously union by identification is more perfect than any kind of union by application, no matter how direct or immediate it may be. That is why, speaking absolutely and objectively, intrinsic measure is more perfect than extrinsic measure. Thus John of St. Thomas writes:

> Quanto perfectior est mensura, tanto perfectius coniungitur suo mensurato, illudque magis ad se trahit quantum possible est. Et ita cum aeternitas sit mensura perfectissima, summe coniungitur suo proprio mensurato: ita quod habet identitatem cum illo. (51)

The difference, then, between extrinsic and intrinsic measure comes down to this that, whereas the former measures and manifests a certain object per applicationem, the latter measures and manifests per informationem. In the first case there is a real distinction between the measure and the thing measured; in the second case the distinction is only logical. That is the meaning of the principle "omnis mensura in suo genere seipsa mensuratur." In Thomistic terminology, extrinsic measure measures

an extrinsic measure measures the object ut quod, that is to say, per contactum rei ad rem. Intrinsic measure, on the other hand, measures its object ut quo, that is to say, it is the very form of the thing measured.

We are so accustomed to making measure coterminous with extrinsic measure that it is difficult to form a clear notion of intrinsic measure. And yet it is evident that the perfection, simplicity and uniformity of a thing can manifest another thing only by manifesting itself in some way. In this sense intrinsic measure is the very foundation of extrinsic measure. John of St. Thomas writes:

> Quando mensura est intrinseca, idem quod est mensura intrinseca, est etiam forma; alioquin non esset mensura intrinseca, id est, per informationem mensurans; cum tamen necesse sit ponere aliquas mensuras intrinsecas, quia id quod est mensura in aliquo subjecto esse debet, et non mensuratur per aliquid extrinsecum, alioquin de illo inquiremus per quid mensuratur: et sic vel erit processus in infinitum, vel deveniemus ad aliquam mensuram, quae respectu sui subjecti sit forma et mensura respectu vero aliorum extra se sit mensura tantum. Nec tamen sub eadem formalitate est forma et mensura; sed est forma ut constituit formaliter; est autem mensura ut respicit quantitatem aliquam virtualem vel formalem, uniformitate affectam, et sic mensuratam. (52)

In other words, by the very fact that a thing exists it has a certain perfection and simplicity, independently of any comparison with another object. Consequently, it possesses a measure intrinsic to itself. And since it is the form of a thing
which makes it both be and be known
[redacted] known, this intrinsic measure is the form which gives perfection, simplicity and uniformity to the thing it informs and by so doing manifests it. It is only because this perfection and uniformity is possessed in-

dependently of any comparison that there can be a basis for the comparison necessary for extrinsic measure.

> Esse mensuram homogeneam mensurato potest intelligi vel ut quo vel ut quod, et respectu subiecti recipientis est homogenea ut quo, scilicet id, quo tale subjectum redditur homogeneum et uniforme alteri extrinseco, respectu cuius est homogeneum ut quod, si mensurat illud per applicationem et contactum rei ad rem. (53)

But the relation between intrinsic and extrinsic measure must be rightly understood. It is extremely important to keep in mind that the extrinsic measure does not reveal the intrinsic measure, as some might be tempted to think.

With regard to the nature of intrinsic measure, two important questions suggest themselves: first, does it manifest the quantity of the thing in the sense of answering the question "how much", secondly, is it something absolute? These questions are connected, but we shall consider them separately. With regard to the first, it is difficult at first glance to see how intrinsic measure manifests the "how much" of the quantity measured, since whenever we wish to find out how much quantity there is in a thing we inevitably have to fall back on extrinsic measure. On the other hand, we have defined measure in general as that by which the amount of quantity that a thing possesses is made known, and if this definition is valid it should apply to intrinsic measure. Perhaps the best way of solving this problem is by considering the following passage of John of St. Thomas:

> Aliud est considerare mensuram et mensuratum, ex parte rei cognitae, aliud ex modo et ex parte cognoscentis. Ex parte quidem rei cognitae, semper mensura est perfectior mensurato, et notificativa illius, atque explicativa confusionis eius vi perfectionis et simplicitatis At vero ex modo cognoscentis non semper mens nostra, propter suam imperfectionem, attingit simplicitatem et uniformitatem rei mensurantis supra mensuratum: hoc tamen non tollit rationem mensurae ex parte ipsius rei cognitae, licet per accidens ob defectum cognoscentis non possit uti illa mensura ad cognoscendum per illam, tamquam per medium, rem mensuratam. (54)

Intrinsic measure does make the quantity of a thing known in the sense of manifesting the "how much," and therefore realizes the definition of measure. But this manifestation is dependent upon two factors. In the first place, it is dependent upon the nature of the subject to which the manifestation is being made. It is possible that an intrinsic measure may manifest the quantity of a thing in a clear and adequate way to a superior intellect but only in a vague and general way to an inferior intellect. In this case the inferior intellect will have recourse to extrinsic measure. This is true of the intrinsic measure of predicamental magnitude. The intrinsic measure of an isolated extended object manifests adequately the quantity of that object to the divine intellect. But to the

human intellect this manifestation is only vague and obscure. Before comparing one extended object with another we know the quantity of the first in a very loose and inadequate way. If we did not there would be no basis for comparison. To answer the question: how much quantity is there in an extended object, we can point to the object and say: that much. But the intrinsic measure does not give us any accurate and definite knowledge of the quantity. It does not give us the precision of knowledge that can be expressed in a measure-number. That is why recourse must be had to extrinsic measure.

In the second place, the manifestation deriving from intrinsic measure is dependent upon the nature of the object manifested. When the quantity of this object is something fixed and absolute, it can be manifested in a definite and absolute fashion. This is true of the transcendental quantity of immaterial things. But the extension of material objects is not fixed and absolute. For, as we have pointed out, all material objects are entia mobilia and are constantly in a state of flux. The extension of every material object is always in a state of becoming since it is forever undergoing the changes being produced in it by the innumerable physical influences to which it is subject. That is why even to the divine mind the intrinsic measure found in every material object cannot manifest the quantity of that object as something fixed and definite. If it did, becoming would be identified with being.

And this brings us to the answer to the second question: is the intrinsic measure of material objects something absolute? The answer is yes and no. It is absolute in the sense of not possessing the relativity that is proper to extrinsic measure and that derives from the comparison of one object with another. It is not absolute in the sense of manifesting a quantity that is fixed and definite. The partisans of absolute dimensions in the cosmos consistently overlook this second point. (55) To their argument: omne ens est aliquid, must be appended the qualification: in quantum est ens. To the extent in which a thing is becoming it is not a being and hence is not absolute. And from this point of view it is likewise true to say that the standard of length has no fixed length. Through a progressive refinement of scientific processes, physics is constantly drawing closer to the absolute world condition. But in so far as the process of measurement is concerned, it is important to keep in mind that though this absolute world condition is absolute in the sense of not being
absolute world condition is absolute
relative to our ways of knowing, it is not absolute in the sense of being fixed and immobile. We are not drawing close to a static cosmos.

We said above that extrinsic measure differs from intrinsic measure in that whereas in the latter the relation between the measure and the object measured is only logical, in the former it is something real. Since all scientific measurement has to do with extrinsic measure it might be well before finishing the discussion of this point to try to determine as exactly as possible the nature of the relation that arises out of physical measurement.

Scholastics traditionally distinguish two types of relation: transcendental and predicamental. The former does not constitute a special category of being and hence is realized in several categories. It is found wherever an entity, though something absolute in itself, has in its very intrinsic nature a necessary orientation toward something else. The relation of act and potency is always a relation of this kind. Predicamental relation, on the other hand, is a special accident that is superadded to the absolute entity which it relates to something else. As Aristotle and St. Thomas point out, (56) there are three species of predicamental relation: 1) those based on number and quantity; 2) those based on action and passion; 3) those based on measure. St. Thomas clarifies the meaning of the third species by explaining (57) that measure here means something distinct from the measure of number or magnitude, otherwise there would be no difference between the first and the third species. It has to do with the "measurement of being and truth." In this sense our knowledge of things is measured by the things known, that is to say, the truth of our speculative science is determined by objective reality.

These distinctions throw light upon the nature of our physical measurements. In the first place, there is a transcendental relation between the standards and the measuring instruments used and the reality that is measured, for neither standards nor measuring instruments have any intrinsic meaning except in relation to an object to be measured. In the second place, there is a real predicamental relation of the first species between our units of measurement and the quantity measured. Finally there is a predicamental relation of the third type between the knowledge that we gather from our measurements and the object measured. But here it is necessary to introduce a distinction. The knowledge that comes to us from physical measurement in science is at once both speculative and practical; from one point of view it [redacted] reveals to us objective reality while from another reveals an article which we have manufactured. Hence there would seem to be a double predicamental relation of the third type involved. From one point of view objective reality is

the measure of our knowledge; from another point of view our mind is the measure of the object. But of the two the first relation is the most fundamental, for the second has only a functional character in relation to it. That is to say, the only reason why we become the measure of the object is to make it possible for the object to become the measure of our knowledge in a more perfect and adequate way. It is true that we choose the standard by which the quantity of reality is revealed, but it is also true that the object measured determines the measure. Some idealistic physicists tend to overlook this point.

3. The Limitations of Measurement

"If only the schoolmen had measured instead of classifying," writes Whitehead, "how much they would have learnt."(58) For historical reasons indicated in Chapter I it is doubtful perhaps just how much the medieval schoolmen would have actually learned if they had devoted themselves to science based on measurement. But there can be no doubt about how much has been learned in modern times through the systematic processes of measurement. The magnificent structure of modern physics is an eloquent proof of the amazing fruitfulness of metrical method. Yet the epistemologist must not allow himself to become unduly impressed by this towering structure. He must strive to remain completely detached, and examine its foundations with as much objectivity as possible. His task is to assess its value, not from the point of view of practical success but from the point of view of pure knowledge.

This is the task we must now undertake. Having once recognized the amazing success and fruitfulness of the processes of measurement it is necessary to try to analyze their limitations. Many of these limitations have been more or less implicit in what we have been saying about the nature of measurement, but it is important to try to make them as explicit as possible. It is only in this way that we can come to see the true nature and value of the knowledge that is found in mathematical physics, since, as we have seen, all of this knowledge is in the last analysis derived from measurement.

In the first place, metric knowledge is able to come to grips only with the quantitative determinations of nature. As we explained in Chapter VII, it is utterly blind to all the determinant properties of things in their specific essences, to the very inner natures of things, to

all that seems to be of the highest significance for philosophy, for art, and for human life itself. The proper realm of metric knowledge is the homogeneous exteriority found in nature, and from the point of view of pure knowledge this is an extremely proverty stricken area, both because of the homogeneity and because of the exteriority.

Perhaps the following considerations may serve to make the outline of this important limitation more clear-cut. In the first place, it must be noted that measurement can reveal nature to us only in terms of its differences. This is in itself an extremely significant limitation, but it is only half of the story. Added to it is the further limitation that measurement can handle these differences only in terms of sameness. All this is but a corollary from the fact that the proper field of measurement is one that possesses exteriority and hence differences, and at the same time homogeneity and hence sameness. But perhaps we can make this point still clearer by rendering it more concrete and precise.

There are two types of variety in nature. Some objects differ in kind, as e.g. green differs from large and hot from hard. Other objects (or states of objects) though of the same kind, differ by the fact that they possess their common character in various degrees. In face of the first type of difference measurement is wholly incompetent for the simple reason that it is a question of difference without sameness. (59) Measurement can come to grips with these differences only in an indirect way by introducing sameness through an artificial construction. That is to say, if changes in the one object are functions of changes in the other, or if certain occurrences in the one determine in some way corresponding occurrences in the other, then a correlation can be established between them. But it need hardly be remarked how limited is the type of knowledge that results from such correlations.

Measurement has far greater competence in relation to objects or states of objects which differ by degree. But even here an important distinction must be made - - the distinction between what have become known as "intensive quantities" and "extensive quantities." Examples of the former are density, hardness, temperature. The most important examples of the latter are length, time and mass, but there are many other examples of less importance, such as volume, electric resistance, momentum, etc. The measurements of both of these types of "quantities"

have this in common that their differences can be determined by a serial arrangement which will be both asymmetric and transitive. This is possible because there is a sameness uniting the differences. But they are distinguished from each other by the fact that in the case of intensive quantities the serial arrangement is not additive, whereas in the case of extensive quantities it is. It makes sense to say that eighty feet of length are twice as large as forty feet; but it is utterly devoid of sense to say that eighty degrees of temperature are twice as hot as forty degrees. This distinction arises from the fact that though in the case of "intensive quantities" there is sufficient sameness to allow the differences to be determined by a serial arrangement, this sameness is not true homogeneity, and consequently the series is not additive.

We have explained that all measurement consists in an attempt to assimilate in some way the object measured to the status of pure numbers. From this point of view there is a vast difference between intensive and extensive "quantities." In the first case there is an approach to ordinal number. It is only an approach because of the artificial and arbitrary elements entering into the arrangement of the order. In the second case, there is something more: because of the additive quality there is an approach to cardinal number. But once again, it is only an approach, since, as we explained above, the measurement of magnitude can never escape the limitations of ratios.

Through processes of correlation similar to those mentioned a moment ago, the measurement of intensive "quantities" can to some extent be assimilated to that of extensive "quantities". This is done when the serial order of an intensive "quantity" is found to correspond to the serial order of an extensive "quanity". The most common examples of this are the correlation established between degrees of heat and degrees of length of a mercury column between the degree of color of a light and the degree of its refraction, between the degree of intensity of a sound and the length of a wave. Measurement obtained in this way is called derivative, whereas direct measurement of additive "quantities" is called fundamental. Now the indirect, artificial and arbitrary character of derivative measurement is so evident that it is hardly necessary to call attention to it. And obviously the knowledge which results from this measurement is extremely limited.

But even in the field most proper to it metric knowledge cannot get at the quantitative determinations of

the cosmos in the sense of being able to tell us what these determinations are. Precisely because it is "quantitative" knowledge it is not "quiditative" knowledge. It cannot answer the question "what", it can only answer the question "How much"? This is a profound limitation that must not be lost sight of. It makes little difference to what extremes of refinement we succeed in pushing our measurements. In the end the nature of the thing being measured is just as inscrutable as it was in the beginning. (60)

But the metric knowledge that is found in physics cannot even tell us the "how much" of the quantitative determinations of nature in any absolute way. If mathematical physics were based upon the measurement of number, upon counting, it could tell us something absolute about nature. But as a matter of fact, it is based fundamentally upon the measurement of magnitude. And it is always a question of mere extrinsic measure, never of intrinsic measure. This means that it never tells us anything absolute of the object taken by itself independently of the standard. It only tells us how one object stands in comparison with another object under certain given circumstances. In other words metric knowledge in science gives us only ratios. This point is sometimes lost sight of. We tend to transform the ratios into absolute properties of the objects measured. When, for example, we hear it said that the density of gold is 19. 32, it is easy enough to look upon this measure-number as designating something absolute that belongs to gold <u>in se</u>. As a matter of fact, it merely indicates the ratio between the weight of any piece of gold and that of a volume of water of equal size. Sir Arthur Eddington has brought out this point with his usual clarity:

> So in any statement of physics we always have two objects in mind, the object we are primarily interested in and the object we are comparing it with. To simplify things we generally keep as far as possible to the same comparison object. Thus when we speak of size the comparison object is generally the standard metre or the yard. Since we habitually use the same standard we tend to forget about it and scarcely notice that a second object is involved. We talk about the properties of an electron when we really mean the properties of an electron and a yardstick -- properties which refer to experience in which the yard-stick was concerned just as much as the electron. If we remember the second object at all we forget that it is a physical object; for us it is not a yard-stick, but just a yard. (61)

From what has been said thus far it should be fairly clear that strictly speaking metric knowledge does not reveal <u>things</u> to us. As Professor DeKoninck has remarked, "les entites fondamentales de la physique ne symbolisent que des coupures métriques dans les choses dont elles ne répresentent qu'un aspect. Il est absurde

de considerer un atome comme une chose." (62) One of the most common errors in science is to reify provisional metrical segmentations and to attribute to them the status of ontological entities. In this connection the following lines of Cassirer are extremely pertinent:

> It seems almost the unavoidable fate of the scientific approach to the world that each new and fruitful concept of measurement, which it gains and establishes, should be transformed at once into a thing-concept. Ever does it believe that the truth and the meaning of the physical concepts of magnitude are assured only when it permits certain absolute realities to correspond to them. Each creative epoch of physics discovers and formulates new characteristic measures for the totality of being and natural process, but each stands in danger of taking these preliminary and relative measures, these temporarily ultimate intellectual instruments of measurement, as definitive expressions of the ontologically real. The history of the concept of matter, of the atom, of the concepts of the ether and of energy offer the typical proof and examples of this. All materialism -- and there is materialism not only of 'matter' but also of force, of energy, of the ether, etc., -- goes back from the standpoint of epistemology, to this one motive. The ultimate constants of physical calculation are not only taken as real, but they are ultimately raised to the rank of that which alone is real. (63)

The fact that metric knowledge in science gives us nothing more than the ratios between two objects brings to light further limitations that are intrinsic to it. If nature itself determined the standards, the resultant ratios would have a fixed and objective meaning. But as

Bergson has remarked, nature does not measure. And since the standards of measurement are not given in nature they must be established by convention. The intellect and will of man must enter into the process of measurement to determine the norm in relation to which the ratio must be established. Man becomes the legislator for nature. As Professor Benezé has remarked, "dire que le choix de l'unité est arbitraire, c'est dire que la volonté de l'opérateur va introduire dans la connaissance un élément sur lequel la sensibilité n'a plus aucune prise. Et cela ne signifie pas que le nombre qui va apparaître ne soit pas lié au sensible, mais il ne lui est lié que justement parce que la volonté de l'opérateur en a décidé ainsi." (64)

All this evidently introduces an element of subjectivity and to a certain extent of arbitrariness into our metric knowledge. As a matter of fact, most of our systems of measurements derive originally from extremely arbitrary sources. In the English system of weights, for example, the weight of an average grain from the center of a head of wheat was originally selected as the standard, and the pound was consequently defined as the weight of seven thousand of these grains. The block of metal preserved in the United States Bureau of Standards now provides

a much more uniform standard, but the basic relativity and arbitrariness of the measuring system has not been changed. The same is true of the measurement of length, as Eddington has shown in his own whimsical way:

> If report is to be trusted, King Henry I, about the year 1120, fixed the yard by stretching out his arm. King David of Scotland (c.1150) more democratically ordained that the inch should be the mean measure of the thumbs of three men, 'an merkle man' a man of measurable stature, and an 'lytell man', the thumbs being measured at the root of the nail. The meter less picturesquely embodies the mistakes of the early geodesists. Thus the result of all our careful measurement is to determine, for example, how many hydrogen atoms to the length of King Henry's arm or to the thumbs of three Scotchmen. That does not carry us very deeply into the mysteries of Nature. (65)

It is true that science does not rest content with the pure arbitrariness of the standards just mentioned. It has been possible to discover certain constants in the cosmos, such as Planck's constant, the velocity of light, the mass of a proton, etc. and these to some extent enable the scientist to measure nature with her own gauge, so to speak. But even these constants are determined in relation to the originally selected standards. And no matter to what extent science may go in its attempt to purify its processes of arbitrariness, in the last analysis the essential relativity intrinsic to the measurement of magnitude will remain untouched.

This essential relativity imposes an infinite limitation upon the metric knowledge that physics affords us. For no matter what extremes of refinement the progressive perfection of our processes of measurement may reach, the resultant measure-numbers are always an infinite distance from any absolute meaning. Sufficient attention is not always paid to this infinite limitation. The impression is often given that an absolute measure actually exists in nature, though profoundly hidden and extremely difficult to get at. This is, of course, an illusion.

> Il pense volontiers que le nombre exact est là, caché dans le sensible, et il l'y poursuit comme on poursuit un gibier que l'on sait difficile à attraper. Métaphore trompeuse: l'impossibilité de l'atteindre ne tient pas au fait que la mesure exacte serait profondément cachée, mais au fait que le nombre est le résultat de cette tentative du jugement d'imposer à la matière l'influence d'un élément, l'unité pure, qui lui est originairement étrangère. (66)

It should be clear why it is illegitimate to dismiss this question, as some authors do by merely stating that our measure-numbers are only approximative. For approximation implies a relation to a definite terminus and in this case no such terminus exists.

In order to make up in some way for this limitation science must seek to remain in a state of

tendency towards the dialectical limit of the minima mensura. The possibility of indefinite progress in this tendency, even though it would never succeed in triumphing over the limitation, would at least provide some compensation for it. But here we are brought up short before another restriction. For even though theoretically this indefinite progress is possible, practically it is not. There are, in fact, definite limits to the accuracy of our measurements in atomic physics. For no matter how highly refined our instruments of measurement become, they are in the last analysis made up of atoms themselves, and as Planck has remarked, "the accuracy of any measuring instrument is limited by its own sensitiveness." (67) Moreover, it is impossible for us to receive any message from nature of greater refinement than that brought to us by a complete photon. This is a very serious confinement, and at present at least there seems to be no way of evading it. As Sir James Jeans has said, "we have clumsy tools at best, and these can only make a blurred picture. It is like the picture a child might make by sticking indivisible wafers of colour on to a canvas." (68)

In relation to this question of the limitation of the accuracy of measurement in atomic physics, the much-discussed problem of indeterminism readily comes to mind.

So much has been written about this problem in recent years that it hardly seems necessary to go into detail in explaining its nature. It is well known that classical mechanics was rigourously deterministic. Its whole structure was built upon the assumption that every given state of universe was completely predetermined in its antecedent state, in such a way that if all the elements entering into this antecedent state had been known, it could have been mathematically deduced from it. And this applied not only to the universe as a whole but to every individual particle contained in it. The future state of each particle was already precontained in its present state. Past, present, and future were perfectly convertible. It is true that the existence of statistical laws was recognized, but this existence was attributed merely to subjective ignorance, and not to any objective indetermination in nature. That is why thermodynamics was for a long time considered to be the least scientific of all the branches of physics, and it was taken for granted that as science progressed the role played by statistical laws would inevitably decrease.

As a matter of fact, it is just the opposite that has taken place. Statistical laws now reign supreme in atomic physics, and classical physics' fond dream of determinism has been completely dissipated. Progress in science, in general,

and progress in the refinement of measurement in particular, has not provided us with greater power to predict future states of particles. On the contrary, it has demonstrated with increasing clarity our utter incapacity for making such predictions. It has now become generally recognized in physics that it is impossible to determine both the position and the velocity of a particle at the same time. It is possible to determine with great accuracy its position by prescinding from its velocity, or its velocity by prescinding from its position, but it is impossible to do both simultaneously. Not only that, but there is a constant proportion in our knowledge of these two facts; that is to say, in the precise measure in which our knowledge of the position increases in accuracy, our knowledge of the velocity decreases, and vice versa. And this proportion is equal to Planck's constant, h, the quantum of action.

All this has become known as Heisenberg's principle of indeterminacy, and a great deal has been written about how this principle should be interpreted. It would take us too far afield to attempt to analyze its philosophical significance here, but in so far as our present purpose is concerned, it is necessary to point out that there are two fundamental issues involved in this question, and both of them reveal an intrinsic limitation of the process of measurement.

In the first place, the velocity and position of a particle cannot be simultaneously measured with a high degree of accuracy simply because such a thing is a contradiction in terms. A particle in motion is not in place; it is passing from one place to another. And the higher the velocity, the less is it connected with any one definite place. At any given instant one can speak of its position only by prescinding from its velocity. It is true that by being satisfied with rough and inexact measurements we can determine both the position and the velocity at the same time, especially if the velocity is low. But as soon as we try to determine both of them with a high degree of accuracy, we shall find that they are necessarily mutually exclusive, for a thing is moving to the extent in which it is not in any one position, and it is in a definite position to the extent in which it is not moving. It is not surprising, then, that science finds it impossible to measure both the position and the velocity simultaneously with any great degree of accuracy. And all this shows how the process of

measurement, by the very fact of its being perfected, leads us inevitably into an impasse from which there is no escape.

But we are far from pretending that this is an adequate solution to the problem of indeterminacy. There is in fact a good deal more involved in the question. And the principal issue is, of course, whether the indeterminacy which science has discovered in its processes is a revelation of an objective indeterminacy actually existing in nature itself. One must always be extremely diffident about attempting to determine the philosophical significance of the teachings of experimental science, and it would be foolhardy to arrive at hasty conclusions. But we feel that at least this much can be said: in the measure in which scientific indeterminacy is a revelation of ontological indeterminacy it is in perfect conformity with Thomism — all the writings of contemporary Scholastics to the contrary notwithstanding. No one can read the works of Aristotle and St. Thomas without being impressed b the large measure of contingency and true objective indeterminism that they attribute to the material universe. It is something that is a pivotal point in the whole Thomistic system, since it is an immediate corollary of the doctrine of matter and form. To deny objective indeterminism to the material universe and to affirm at the same time that emp of the co-principles which constitutes the very essence of the things of the universe is a principle of pure indetermination — prime matter, is a contradiction in terms.

An adequate discussion of this question cannot be given here. That has already been accomplished with admirable skill by Professor DeKoninck. (69) We have introduced the problem only because it reveals another important source of limitation of the measuring process. For, as we pointed out at the beginning of this Chapter, there is something at once both physical and mathematical about the process of measurement. The mathematical character is revealed in its attempt to arrive at exact determination. If measurement were being carried on in a mathematical world from which all contingency is excluded, the refinement of its exactitude could go on ad infinitum, but as a matter of fact, scientific measurement is carried on in a cosmos that is filled with chance, and that consequently is refractory to the exact determination which measurement seeks to realize.

This discussion of the progressive refinement in the exactitude of measurement raises a question which cannot be overlooked. We have said that the definitions which result from measurement can never be anything more than operational: physical properties are defined in terms of the concrete processes by

which they are determined. And at first sight this seems to involve us in an insoluble problem. For since physical properties are defined by the processes through which they are measured; since every measuring process involves the use of a physical instrument; and since an instrument cannot be known or defined except in terms of its properties, it is difficult to see how we can escape an immediate vicious circle except through another vicious circle, which would consist in falling back upon the senses from whose limitations the whole process of measurement is intended to deliver us.

It is true, as we pointed out in Chapter VII, that all physical experimentation involves an ultimate dependence upon sense. But this does not mean a going back to the limitations of the senses which physical science encounters at its point of departure. And we can escape this without getting involved in a vicious circle. It is not a question of a circle, but of an ascending spiral. In the beginning, science, by making use of ordinary sense data, arrives at an elementary physical theory. The substitution of measuring instruments makes it possible to correct the primary theory; the new theory helps to reveal the deficiencies of the instruments employed and makes it possible to perfect them through the use of more perfect instruments science is able to arrive at a more perfect theory, and so on *ad infinitum.*

There are two things that must be noted about this process. In the first place, it never arrives at perfect exactitude. And this is an important point to keep in mind. For it means that from this point of view mathematical physics does not have an absolutely certain point of departure. Its primary data, the measure-numbers, are not truly certain. And the fundamental reason why they are not certain is that they aim at a kind of certainty that cannot be attained in the realm in which it is being sought. From this point of view the primary data of the parts of the study of nature that are not mathematicized have greater certitude. This is true above all of the philosophy of nature. But lest this limitation appear greater than it actually is, attention must be paid to two points. First of all, even though the measure-numbers are not certain, they are certainly an approximation, and science is often able to determine with great eracotitude the limits within which this approximation certainly falls. Secondly, because of its highly theoretical character, mathematical physics is not so essentially interested in the certainty of its point of departure as a purely inductive science must be. In a sense it is true to say that it is more interested in its point of arrival. It is satisfied

with any point of departure which will provide a sufficient basis for a theoretical structure which will eventually "save the phenomena."

The second thing to be noted about the process we have been discussing is that the more highly refined it gets, the more implicated it becomes in theory, and consequently the more deeply immersed in subjectivity. The use of the yard-stick does not depend upon very many theoretical assumptions. But the extremely elaborate and complicated instruments now employed by science are dependent upon a veritable mass of postulates and assumptions. As a matter of fast, does not our method of deciding that one process of measurement is more accurate than another consist in determining that it is more in accordance with our theories and with the laws which we have assumed to be true?

This brings us back to what we saw in Chapter IV about how the subjective logos is injected into nature through the processes of experimentation. Everything that was said in that connection applies with particular force to the processes of measurement. For measurement is an operation which we perform upon nature, and this operation has a double aspect. In the first place, it involves a mental procedure which gives the operation a meaning only by placing it in a highly complicated pattern of interwoven assumptions. In the second place, it involves the actual physical procedure of measurement. Both of these aspects implicate measurement in a manifold of complex limitations. But for the moment we are interested only in the mental procedure by which hypothetical elements enter into the operation.

Measurement has been considered by some as a purely empirical procedure, dependent only upon perception and its means, and completely free of hypothetical assumptions. (70) Nothing could be more false. Not even the simplest measuring operation has a purely empirical and immediately certain starting point. There is always a multiplicity of conceptual presuppositions lurking in the background, which, though subtly implicit, determine, nevertheless, the whole meaning of the procedure. If all the implicit assumptions upon which the ordinary process of measuring temperature by means of a column of mercury could be disengaged and laid bare the results would probably be startling. How much more is not the elaborate and complicated scientific processes of measurement dependent upon hypothesis. Innumerable theoretical assumptions go into the whole conceptual setting up of the experiment, into the construction of the instruments of measurement employed, into the precise way in which they are used, and, in

fact, into every operation that goes to make up the experimental procedure. (71) And every attempt to verify these assumptions only leads into a more complicated network of presuppositions.

Since a number of things have already been said about this general question in Chapter IV, we shall not attempt to develop it any further here. But we cannot refrain from quoting the following lines from Ernst Cassirer, who has laid considerable stress upon this point:

> For any, even the simplest, measurement must rest on certain theoretical presuppositions, on certain 'principles', 'hypotheses,' or 'axioms,' which it does not take from the world of sense, but which it brings to this world as postulates of thought. In this sense, the reality of the physicist stands over against the reality of immediate perception as something through and through mediated; as a system not of existing properties, but of abstract intellectual symbols, which serve to express certain relations of magnitude and measure, certain functional coordinations and dependencies of phenomena. . . .
> In this sense, each measurement contains a purely ideal element; it is not so much with the sensuous instruments of measurement that we measure natural processes as with our own thoughts. The instruments of measurement are, as it were, only the visible embodiments of these thoughts, for each of them involves its own theory and offers correct and useful results only in so far as this theory is assumed to be valid. It is not clocks and physical measuring-rods but principles and postulates that are the real instruments of measurement. For in the multiplicity and mutability of natural phenomena, the thought possesses

> a relatively fixed standpoint only by taking it. In the choice of this standpoint, however, it is not absolutely determined by the phenomena, but the choice remains its own deed for which ultimately it alone is responsible. (72)

But not only do innumerable limitations result from the mental operations which construct the processes of measurement, they also result from the physical operations involved in the actual concrete processes. This is an extremely important point and too much attention cannot be paid to it. It immediately reminds us of all that was said in Chapter IV about the operational character of the definitions of experimental science. But a few special considerations must be introduced here which apply in a particular way to the process of measurement.

In the first place, it is important to keep in mind the proper reason why definitions of magnitudes are necessarily operational: the measurement of magnitude can never give us more than a proportion between the object measured and the standard employed. Consequently the whole meaning of the result depends upon the way in which the standard is chosen and the precise manner in which it is employed, and all this involves innumerable arbitrary elements, as we have already suggested. That is why the knowledge

which the measurement of magnitude gives us is always essentially relative, even when it is a question of the determination of the proper length of an object. By proper length in physics is understood the length which results from a measurement in which a standard is applied to an object that is at rest in relation to it. Later on we shall see that a second kind of relativity enters in when measurement is made of an object in motion.

Because number is something absolute, counting is an absolute operation. No matter how many different ways of counting a certain given plurality may be devised, their results must coincide exactly if they are to be true. As a matter of fact, counting is not essentially an experimental process, for it does not necessarily involve a manipulation of bodies. It is true that physical manipulation may be used as an aid, but in itself counting is a purely mental operation. Magnitude, on the other hand, is not something absolute, nor can the operation by which it is determined be considered absolute. It is possible for a number of individuals to measure the same extension by means of different operations and all arrive at different results. And it is possible to consider all of these results as equally true. To conceive the results of a certain measurement of magnitude as the revelation of something absolute in nature to which all other operations must conform is to misconstrue the whole nature of magnitude. That is why such measurement can never have any meaning independently of the concrete operations involved.

And all this means several things. In the first place, it means that if we wish to get at the exact significance of a definition of a length we must be able to specify completely and with perfect precision all of the operations which have entered into its determination. Because of the extreme complexity of even the simplest kind of measurement this seems to be an impossible task, not only because of the innumerable elements involved, but also because the operations interfere with each other, and there is no way of fixing upon the exact nature of the different interferences. But even if one could specify the operations completely and with perfect precision, the results would be very meager. For in the last analysis this specification would consist in merely pointing out certain processes and certain material instruments. One does not reveal very much about the nature of man by merely pointing out an individual man.

The operational character of the definitions of length means that when the operations change, the significance of the definition changes. As Professor Bridgman has pointed out, "In principle the operation by which length is measured should be uniquely specified. If we have more than one set of operations, we have more than one concept, and strictly there should be a separate name to correspond to each different set of operations." (73) The primary meaning which measurement has in physics is that found in the determination of a length by the direct application or juxtaposition of a material standard to an object at rest in relation to it. But not all the measurements with which physics deals can be arrived at by the same operation, and when new types of operations are introduced, the meaning of the process changes. But lest confusion arise it might be well perhaps to point out that this does not mean that the results of the measurement depend solely upon the nature of the operations employed, for otherwise all objects measured in the same way would have the same length. We shall have a similar remark to make in connection with the second kind of relativity mentioned a moment ago: the results of the measurement of a body in motion do not depend solely upon the frame of reference in relation to which it is measured, for otherwise every body measured in relation to the same frame would have the same length.

This relativity of measurement is often lost sight of. One type of operation is constantly being substituted for another on the presumption that they are equivalent and interchangeable. An operation proper to one field is projected into another field where determinant factors are different, and it is tacitly assumed that the operation preserves its original meaning. How is it possible to have any assurance that operations which give similar results under certain circumstances will necessarily give similar results under any other circumstances?

Perhaps a few concrete illustrations will serve to bring out more closely this important limitation of the measuring process. (74) In the first place, a very simple example is found in the difference between fundamental and derivative measurements. All too often these two types of measurement are considered to be practically equivalent; yet there is a vast difference in the operations by which they are determined. A more important case is that of the measurement of a body in motion. Such a process involves

operations that are quite different from those involved in the measurement of a body at rest, and the higher the velocities of the motion, the more complicated do these operations become. As a result the meaning of the process undergoes a profound change. We shall have more to say about this case later on because of its capital importance in modern physics.

Another way in which the concept of length is extended beyond its original meaning is found in the measurement of extremely large objects. Here the "tactual" operations which are employed in measurements that fall within the range of ordinary experience, and which consist in the successive direct application of the standard rod to the object, can no longer be employed, and optical operations are substituted. This is already found to some extent in terrestrial measurements, but it is particularly true of solar and stellar distances, where the character of space is entirely optical, and where no opportunity is given of making even a partial comparison between tactual and optical operations. And the complexity of the operations increases in proportion to the remoteness of the distance measured. As Bridgman has remarked:

> At greater and greater distances not only does experimental accuracy become less, but the very nature of the operations by which length is to be determined becomes indefinite so that the distances of the most remote stellar objects as estimated by different observers or by different methods may be very divergent . . . We thus see that in the extension from terrestrial to great stellar distances the concept of length has changed completely in character. To say that a certain star is 10^{9} light years distant is actually and conceptually an entirely different <u>kind</u> of thing from saying that a certain goal post is 100 meters distant. (75).

Something similar to this occurs when measurement is extended in the direction of the infinitely small. The operations involved change; they become more indirect and more highly complicated. Consequently, the results of microscopic measurements have a different meaning than those of molar physics. In this connection it it interesting to note that though in the determination of the <u>number</u> of molecules in a certain piece of matter we are forced to use indirect and complicated mehtods, and though different methods may give results that are systematically different, there can be no doubt but that the number of molecules is something absolutely determined in nature; consequently the results do not depend for their meaning upon the operations employed. In so far as these methods are theoretically good and accurate they must all arrive at the

same absolute result. But it does not seem to make any sense to say that in the determination of length, mass, force and other quantities of this kind involved in atomic physics, we must arrive at something absolutely given in nature independently of the operations which enter into the determination.

Of course in all of these cases of the extension of measurement beyond its original meaning, the changes which result do not occur in a fortuitous and uncontrollable way. That is to say, the new operations are not chosen in a purely arbitrary fashion; they are selected by design in such a way that within the realm in which both the original and the new operations may be applied, they both give the same numerical results within the limits of experimental error. Yet there is never any assurance that when the new operations are applied outside this realm where new circumstances are involved, the original coincidence will be preserved.

It is possible for several divergent definitions of length to be employed in circumstances in which direct measurement is impossible, such as, for example, in intense electric and magnetic fields. This is quite legitimate, provided that, as the fields tend toward zero, they all converge towards the accepted definition. It is impossible to say that one of these definitions is right and the others wrong. For they will all be confirmed by observation, since the very observation will depend upon the theory that is originally accepted. But as Eddington has pointed out, it must be kept in mind that the distances thus measured will be pseudo-distances, "since they lack the most fundamental characteristic of the metrological conception of length, namely the correspondence between similarity of length and similarity of physical structure." (76)

The second thing that must be noted in regard to this operational character of the measurement of magnitude is that the operations in question are concrete, physical, material operations. No matter how completely mathematicized or how highly theoretical physics may become, the definitions of the quantities involved in it are never independent of singular, concrete, material operations, nor do they ever have any meaning except in relation to them. The definition of length of a Relativity physicist is the same as that of an ordinary metrologist.

> If, instead of length being defined observationally, its definition were left to the pure mathematician, all the other physical quantities would be infected with the virus of pure mathematics . . .
> In all orthodox physical theory, the metrological practice - - or more strictly the principle which it attempts to carry out - - supplies the theoretical definition. Thus it is secured that, when the experimenter checks the theorist, both are referring to the same thing.
> Accordingly, by length in relativity theory we mean what the metrologist means, not what the pure geometer means. In accepting relativity principles, the physicist puts aside his paramour pure mathematics, dismisses their go-between metaphysics, and enters into honourable marriage with metrology. (77)

From the point of view of the logical structure of science, the limitations which all this implies are simply enormous. No definitions in physics are detached and universal; they are all tied down to particular material operations. They have no significance independently of the concrete instruments of measurement employed.

All too often measuring instruments are looked upon almost as if they were immaterial cognitive faculties which register events in a purely trans-subjective fashion. But a moment's reflexion will show how far this is from the truth. In the processes of measurement the instruments employed do not remain purely passive; they enter into the experiment in an active way. For obviously a physical instrument can reveal an event to us only if there is a physical causal connection between the instrument and the event. And this causal connection inevitably involves an interference of the instrument in the event.

The seriousness of this interference depends upon several factors. In the first place, it is clear that the interference will ordinarily be greater in proportion to the greater imperfection of the instrument employed. And in this connection it is necessary to recall that perfect instruments exist only in the mind of the scientists; they do not exist in reality. Consequently, there is always something defective about every measurement made. Moreover, measuring instruments never remain the same; they are constantly in a state of flux. The very fact that instruments wear out is a sign that they are at all times subject to minute derangements. But even if measuring instruments were perfect there would still be considerable interference in the event that is measured. For purely material things cannot register objective events in a purely transsubjective fashion.

Another important factor upon which the seriousness of the disturbance depends is the degree of refinement demanded by the experiment in question. In molar physics the interference is relatively light, though even here it cannot be over-

looked. But in the microscopic world the interference is of the same magnitude as the quantities measured, and consequently the limitations of measurement in this realm are simply enormous. The degree of intimacy in the causal connection between the measuring instrument and the quantity measured has also much to do in determining the seriousness of the disturbance. In the measurement of microscopic phenomena the causal nexus is extremely close, and as a result the interference is of great magnitude. This magnitude decreases in proportion to the increase of causal distance between instrument and event, but it can never be reduced to zero, since, as Planck has remarked "if the causal distance is assumed to be infinitely great, i.e. if we completely sever the object from the measuring instrument, we learn nothing at all about the real event."[78] Nor must the fact be overlooked that when experiments depend upon a multiplicity of pointer-readings, there is necessarily mutual interference between them.

Perhaps one might be tempted to think that this limitation of measurement is not so serious as it appears at first sight, since it is possible for scientists to take account of the interferences in question and to make compensations for them in their computations. It must be admitted that certain possibilities of this kind lie open. But they are extremely meager in comparison with the problem in question — if for no other reason than that every attempt to account for a disturbance involved in a measurement demands another measurement for its verification, and this obviously starts us out on an infinite series. (79)

In our discussion of this limitation of measurement arising from the causal influence of the instrument upon the quantity measured we have been using the terms "interference" and "disturbance" because they are the expressions which have become current in the modern scientific literature which has treated this problem. But perhaps they do not bring out the most profound aspect of the question as accurately as could be desired. For they tend to give the impression that the causal influence of the instrument is a purely accidental and extrinsic thing, or, in other words, that the measure-number emerging from a process of measurement is essentially a revelation of the object measured, but this revelation has been accidentally and extrinsically modified by the instrument used. To conceive the problem in this light is to miss the main issue. For measure-numbers are essentially the product of both the object measured and the instrument employed. And here we have in mind something more than the

point brought out above about measure-numbers being mere ratios resulting from a comparison of an object with a standard of measurement. We have in mind here something that has to do with physical causation. We mean that the measure-numbers are works of art produced by the co-causality of both the object measured and the measuring instrument.

Perhaps this point can be clarified to some extent by a simple distinction. The influences which an instrument has upon the results of measurement are of two kinds. Some of them are causal, and in a sense extrinsic, and these the scientist may labor to correct, or at least, to account for. But there are other influences which are essential, since they result from the very nature of the instrument and from the very purpose it was designed to achieve, and these it would be nonsensical for a scientist to attempt to eliminate. (80)

Professor De Koninck has brought out with great exactness the fundamental issue involved in this question:

> Entre ces nombres-mesures repérés sur l'échelle graduée d'un instrument et le sujet matériel, il y a la fabrication dont on ne peut faire abstraction sans tomber dans le subjectivisme. Ne confondons pas la donnée prescientifique avec le nombre-mesure qui n'est pas une traduction immédiate et adéquate de cette donnee. Ce n'est pas l'objet sur le plateau de la balance qui sera le point de départ propre de l'élaboration scientifique, mais tel nombre sur l'échelle graduee auquel s'arrête l'aiguille. Une fois définie la propriete, je ne puis l'attribuer telle quelle à l'objet,

comme si la balance n'était qu'une espèce de rideau et que dans la pesée on épiait 'derrière' la balance pour surprendre l'objet tout nu. (Et c'est bien ce qu'on croyait faire avant la critique einsteinienne des mesures d'espace et de temps, oubliant que les circonstances mêmes de mensuration font partié d'une définition et que la différence de circonstances change qualitativement cette définition. Dire que des définitions de longueur qualitativement différentes doivent avoir la même valeur quantitative c'est tomber dans ce relativisme dont Einstein nous a libérés. (81)

One of the reasons why this point has often been lost sight of, at least to some extent, results from the innate and inevitable tendency of science to idealize the entities with which it deals. As we pointed out in Chapter IV, the physicist tends to substitute in his mind an ideal geometrical model for the physical apparatus with which he is working. He tends to de-materialize his instruments, in such a way that a concrete meter rod, for example is transformed into an immaterial meter. Speaking of this question Sir Arthur Eddington writes:

> Primarily we say yard rather then yard-stick because a great many equivalent substitutes for the yard-stick are possible. But we do not generally think of a yard as a general name for one of a large variety of physical objects or systems; we do not think of it as an object at all. I grant that another physical object may be an equivalent substitute for a yard-stick, but I do not grant that a de-materialized yard is an equivalent substitute for a yard-stick.

When the quantum physicist employs a standard of length in his theory, he does not treat it as an object; if he did, he would according to the principles of his theory have to assign a wave function to it, as he does to the other objects concerned in the phenomena. In my view he is wrong. Either he is using the standard length as a substitute for the second body concerned in the observed relation of size, in which case he ought to attribute to it a wave function, so that he can bring it into his equations in the same way that the second body would have been brought in; or he is treating size as though it were not an observable relation between one physical object and another, and the lengths referred to in his formulae are not the lengths which we try to observe. We have to recognize then that what are called the properties of an electron are the combined properties or relations of an electron and some other physical system which constitutes a comparison object. For an electron by itself has no properties. If it were absolutely alone, there would be nothing whatever to be said about it — not even that it was an electron. And we must not be misled by the fact that in current quantum theory the comparison is replaced by an abstraction, e.g. a metre, which does not enter into the equations in the way that an observable comparison object would do; for that is a point on which current quantum theory is clearly at fault. (82)

These considerations will serve to bring to light the position occupied by the instrument in the process of measurement. In some sense it is an ambiguous position, for the instrument belongs at the same time to the subject who is measuring, and to the object measured. For on the one hand, it is a kind of prolongation of the cognitive powers of the subject; it refines these powers and enables them to arrive at more exact and more sensitive discriminations. On the other hand, it is one with the object both because it is one term of the comparison which every measurement implies, and because of the physical causality it exercises in the measuring process.

In connection with this limitation of measurement arising out of the part played by the instrument, another closely associated source of limitation must be touched upon. We are referring to the various cosmic influences that enter into every concrete measuring process. Those influences are legion, and they have a very definite effect upon the results of the measurement. It is true that it is possible for scientists to cope with them to a certain extent. In every process of measurement there is an attempt to achieve an ideal state in which such influences as arise from electric and magnetic fields, unfavorable atmospheric conditions, strain, corrosion, flexure, etc. are either removed, or controlled, or accounted for theoretically. And through the method of successive approximation employed so extensively in physics [83] science is able to achieve an ever increasing degree of perfection in the control of these influences. But no matter how much progress may be made in this direction, the goal will ever remain at an infinite distance, for it is a purely dialectical limit. In order to

be able to account for all of the cosmic influences which play a part in the measuring process, one would have to be perfectly acquainted with these influences, and that would demand an exhaustive knowledge of Nature. And perhaps it is not superfluous to note that this involves much more than a perfect knowledge of all the laws of nature. For chance plays such an important part in the cosmos that many of the influences that actually bear upon concrete experiments are pure chance events which have no determined cause, and which are therefore outside the pale of all law. It seems safe to conclude, then, that our actual knowledge of the influences entering into our experiments will ever remain infinitesimally small. And in this sense there is a great deal of wisdom in Planck's remark that "measurement gives no immediate results which have a meaning of their own." [84]

What is it that we actually measure in our concrete processes? Perhaps it is not an exaggeration to say that even in such a trivial measurement as the weighing of a pound of meat, we are not merely measuring the weight of the meat -- we are actually measuring the whole cosmos. For the object measured and the instrument employed never constitute an isolated system. Nor can an isolated system ever be achieved through successive approximation in the control of known cosmic influences. A perfectly closed system, other than the entire cosmos, is a pure idealization. It exists nowhere but in the mind of the scientist. The following lines of Louis De Broglie have considerable relevance here:

> Le concept d'unité physique n'est donc vraiment clair et bien défini que si l'on envisage une unité complètement indépendante du reste du monde, mais, comme une pareille indépendance est évidemment irréalisable, le concept d'unité physique pris dans toute sa pureté apparaît à son tour comme une idéalisation, comme un cas qui jamais ne s'adapte rigoureusement à la réalité. Il en est de même, d'ailleurs, du concept de système. Le système, dans sa définition stricte, est un organisme entièrement fermé et sans relations avec l'extérieur; le concept n'est donc vraiment appliquable qu'à l'univers entier. (85)

These general considerations lead us inevitably to a question which constitutes one of the most central problems in any discussion of the significance of measurement - - the question of the rigid scale. It is immediately evident that rigidity, or what Whitehead calls self-congruence, is the primary requirement for any standard of measurement. Elastic tapes are never used as standards, nor are easily expansible metals ever employed in measuring devices. And the fundamental reason for this has been brought out in our analysis of the nature of measurement.

But to what extent is self-congruence possible? Or, to put the question more pointedly, does the concept of self-congruence even have any meaning? If it is impossible to arrive at any definite determination of rigidity, and if the very notion of self-congruence is without meaning, then to say the least the validity and significance of the whole measuring process will be extremely questionable. And at first sight it might seem that we must be lead to this conclusion. For if the statement which we made a moment ago, that a length must be measured with a rigid scale, is to have any meaning for us, we must be able to define what we mean by a rigid scale. And the definition which naturally suggests itself to us is: a rigid scale is one that preserves the same length. But this immediately involves us in a vicious circle, for we have defined length in terms of a rigid scale, and a rigid scale in terms of length. (86) And as long as we cling to these two definitions we shall be confronted by an impasse. For, obviously, if length is a quantity obtained by means of measurement with a rigid scale, it will be necessary to have recourse to another rigid scale to decide whether or not the length of the first scale changes, and this sets us on an infinite series. The only possible way of surmounting this impasse is to revise one of the two definitions. And a moment's reflection will show that the definition of length cannot be the one revised, since length can have no definite meaning except in terms of the self-congruence of a standard. We must then attempt a solution of the problem by seeking for a determination of rigidity independently of the notion of length. At first sight this may seem an impossibility, for it is difficult to see how one can decide whether an extension has increased, or decreased, or remained the same, except by means of measurement. And if measurement is employed, a vicious circle is inevitable.

Fortunately there is a way out of this impasse. And the way is suggested by a remark made earlier in this analysis: the standard of length has no length. Since we cannot speak of length in relation to a standard of length, it is illegitimate, and even nonsensical, to attempt to determine the rigidity of a standard in terms of length. Some might be tempted to object immediately that, far from leading us out of our impasse, this only complicates the problem all the more. For if the standard of length has no length, what sense is there in speaking of self-congruence or rigidity? No matter how much an elastic meter tape measure may be stretched, everything that is measured with it will

always be a meter in length. As a result the whole process of measurement loses its significance.

A moment's reflection will show that this objection arises from a confusion over the meaning of the term "length". As we have already pointed out, this term is susceptible of a multitude of meanings. But since we are dealing with physical science, we have been using it, and shall continue to use it, in the sense in which it is employed in physics: the measured magnitude of a sensible line. No standard has length in this sense. That is why we cannot employ measurement to determine the rigidity, for then the standard would be a measured magnitude. But obviously every standard has length in the sense that it is an object with a definite extension. And it is possible independently of any process of measurement and merely by having recourse to identity and non-identity (87) to determine the constancy or inconstancy of this extension.

A number of bars of different material may be taken and their identical extension determined by noting the coincidence of extremities. These bars may then be subjected to a variety of influences such as pressure, temperature, atmosphric conditions, etc., and by comparison their coefficients of expansion or contraction observed.

The bar which comes closest to identity with the original extension is chosen as the standard. A special room is prepared in which conditions considered to be ideal are kept as constant as possible, and every effort is made to exclude disturbing influences. The chosen bar is then placed in this room, and at last a rigid scale has been achieved. This is, in substance, the way in which the international legal standard of length was arrived at - - the Mètre des Archives, which is a bar of platinum preserved in Paris at the temperature of melting ice and under atmospheric pressure.

This process of determining self-congruence may appear extremely dubious, and one might be tempted to ask, "Is this rigid scale actually rigid?" A question of this kind contains considerable ambiguity, and it is difficult to know how it should be answered. If its meaning is: "Can this meter rod ever be longer or shorter than a meter?", the answer must obviously be in the negative. Once a standard has been chosen, it is impossible for it to change qua standard. The question might also mean; does the scale remain absolutely rigid as far as science is concerned? and it is possible to answer such a question in the affirmative, in the sense that the whole structure of science is based upon the assumption that the scale is rigid.

Perhaps the word "assumption" will be immediately seized upon and the question pressed home: "But is it really rigid?" The answer to this question depends upon what is meant by "really". If it means that there is existing somewhere in the cosmos an ultimate and absolutely immobile ideal standard in relation to which the constancy or inconstancy of the chosen standard may be objectively determined, it is extremely doubtful just how much sense a question like that can have. It certainly has no sense from the point of view of physical science.[88] We do not see how it can even have sense from the point of view of philosophy. But if the question means: does the scale possess absolute objective immobility, then a definite answer can be given. And the answer is: certainly not, for the very notion of an absolutely immobile material object is a contradiction in terms.

And this brings us to the central point towards which most of this discussion has been directed: the whole significance of the measuring process depends upon the rigidity of the scale that is employed as a standard, and it is impossible to arrive at an absolutely rigid scale. The rigidity that is spoken of in science is one that is determined by fiat; it is a convention. And this obviously introduces a profound limitation into the process of meaning. But it is impossible to have a clear notion of the nature of this limitation except by pointing out that, while it is meaningless to ask whether this convention is true or false, it is extremely important to determine to what extent it is arbitrary. It is obvious that like every convention, the determination of the rigid rod is in some measure arbitrary. But it is likewise obvious from what has been said that it is far from being purely arbitrary. In other words, it is something that is at once both subjective and objective. And though it will always remain impossible to determine the relative degrees of subjectivity and objectivity, it is important to note that purely objective rigidity is a dialectical limit to which science may draw constantly closer and closer, by means of its usual method of successive approximation through an ascending spiral similar to the one described above. When we stated that once a rigid scale has been chosen, it cannot change, we do not mean of course that science can never reject a chosen standard in favor of one that seems more perfect. In fact, it is of the very nature of physical science to be constantly in search for a more perfect standard. It is probable that the Paris meter will eventually be supplanted by another standard, such as, for example, the

grating space of a calcite crystal, whose latice structure has the advantage of associating the standard with pure numbers. It is likewise probable that science will gradually achieve greater and greater rigidity in its standard. The only important point to keep in mind, as far as our present discussion is concerned, is that no matter what degree of rigidity may be attained, there will always be in the standard an indeterminable margin of subjectivity deriving from the free intervention of the human intellect and will.

This discussion of the rigidity of the measuring rod may perhaps bring to mind the question of the Fitzgerald contraction, first postulated to account for the absence of any indication of aether drag in the Michelson-Morley experiment and later confirmed by the electromagnetic researches of Larmor and Lorentz. According to the postulate of Fitzgerald, a material rod moving at high speed contracts in the direction of the line of motion. The consequences of this postulate for the problem of measurement are immediately apparent. What determined meaning can measurement have if the standard scale expands and contracts according to the velocity at which it is moving and according to the direction in which it is turned -- especially if (as is the case) it is impossible to know in any absolute way the velocity of the scale. In ordinary circumstances this contraction is negligible; for example, the diameter of the earth contracts two and a half inches, or one part in two hundred million, in the velocity of nineteen miles a second of its movement around the sun. But at the speed of one hundred and sixty one thousand miles a second the contraction would be one half. And is there any way of knowing whether in relation to some point of reference in the cosmos, the whole solar system is not moving in a manner that approaches this velocity? What is worse, is there any way of knowing whether the whole frame of reference in relation to which we make our measurements is not moving in relation to other frames of reference in different directions and at different velocities, which perhaps do not remain constant?

It becomes immediately evident that all of our determinations of length (and of time also, as we shall see presently) are dependent upon the particular frame of reference within which they are made. And here we are touching upon the profound difference between Classical and Relativity physics. But the point is not that Classical physics failed to realize that different velocities and

different frames of reference have an influence upon the process of measurement. In fact, it provided formulae by which each observer could apply "corrections" to reduce his "fictitious" length to the "unique" Newtonian length. The whole crux of the matter lies in the meaning of the words "corrections", "fictitious", and "unique". In other words, Newtonian physics realized that measurements made by different observers will give different results. But it took it for granted that there was an absolute observer who occupied a priviledged position -- a position that was Nature's own position. And from this supposition stemmed two implicit postulates: 1) that spatial relations determined by the measurement of length could be reduced to an absolute meaning; 2) that temporal relations had an absolute and independent character. Einstein was astute enough to see that both of these postulates were perfectly gratuitous, and he proposed to do without them. But in order to understand the significance of his doctrine for the question of measurement, it is necessary to return for a moment to the Fitzgerald contraction and try to fix upon its exact meaning.

At first sight, this contraction might seem to be in the same category with the changes in the standard scale, discussed in connection with the problem of rigidity, but as a matter of fact, it constitutes an entirely different problem. Indeed, it is true to say that, paradoxical as it may seem, the Fitzgerald contraction has nothing to do with rigidity. The meaning of this statement will be fully explained in a few moments, and for the present it is sufficient to point out that the contraction is determined completely by the velocity of the motion and not by the specific nature of the rod in question. All rods moving at the same velocity undergo exactly the same contraction, no matter what degree of rigidity they may possess in relation to such influences as temperature, stress, etc. The contractions of a rod of platinum and a rod of rubber moving at the same speed are identical. Hence this contraction must not be looked upon as an imperfection of the rod. It must not be considered a defiency in relation to an absolute rod. Such a rod does not exist, nor can it exist.

In order to come to understand how the problem of the Fitzgerald contraction differs radically from the problem of rigidity, it is important to note that the length of an object measured is in a sense completely independent of the difference between its temperature and that of the measuring rod. A cold scale may be brought into direct

contact with an extremely hot body and determine its length with precision. But the length of an object measured is not independent of the difference between its motion and that of the standard scale. In fact, it is, in a sense, completely dependent upon it.

When a scale and an object can be brought into immediate contact, or when their motions are correlated in such a way that they are moving with the same velocity and are thus at rest in relation to each other, the measurement gives us the <u>proper length</u> of the object. From one point of view, physics would be immensely simplified if it were always possible to arrive at the proper length of the objects measured. But, as Eddington has remarked, "it is not convenient to send your apparatus hurling through the laboratory - - after a pair of α-particles, for example." (69)

Perhaps at first sight the difference between the determination of the proper length of an object and the determination of the length of an object in motion in relation to the scale may not seem to constitute any serious problem, since it appears to be a fairly easy matter to reduce the one to the other. Let us suppose, for example, that a straight rod is moving with uniform velocity with respect to a certain frame of reference. It is possible to mark on the frame the simultaneous positions of the extremities of the rod, and then measure the distance between the two positions marked on the frame. Will the results correspond to the proper length of the object in motion? One might be tempted to answer in the affirmative, since the two positions were marked simultaneously. But then he will be obliged to tell us what he means by simultaneity. And therein lies the whole crux of the matter.

As we have already suggested, Classical physics attributed to the notion of simultaneity an absolute meaning. But Einstein pointed out that this attribution was based on an implicit assumption which was utterly incapable of being verified experimentally, since this verification would presuppose that signals announcing distant events could come to all observers instantaneously, that is, with an infinite velocity. Concepts have no meaning in physics unless they can be defined operationally, and Einstein made it very clear that every attempt to define simultaneity operationally inevitably results in making it something relative to the frame of reference in which the operation was carried on. In other words, the only kind of definition of simultaneity that has any meaning is such that if two

events verify it in one system they will not verify it in another system that is in motion with respect to the first. The measurement of time, then, becomes essentially relative to a given system. And since the determination of the length of a body in motion necessarily involves the notion of simultaneity, every determination of such a length is essentially relative to a certain frame of reference. Thus Einstein was able to arrive at the following statement: "If a body has the length ℓ with respect to a system in which it is at rest, then with respect to a system in which it is moving with the velocity v it will have the length $\ell' = \ell\sqrt{1-\frac{c^2}{v^2}}$ where c is the velocity of light. That is, length of a body has in each system a different value, depending on the velocity v of the body with respect to the system in question." This difference of value is equal to the Fitzgerald contraction. And since the determination of the other quantities which enter into physics is bound up with the reckoning of length, it follows that mass, periods of vibration, electric and magnetic fields, etc. become relative to a certain frame of space.

Because of the way in which simultaneity is involved in our determination of length, it is clear that not only space, but time as well is implied in all our measurements. In other words, to quote Eddington, "the fundamental measurement is not the interval between two points of space, but between two points of space associated with instants of time." (90) Events in nature are exterior to each other in four different ways, of which three are spatial and one temporal, and the order of these events constitutes one indissoluble four-dimensional space-time order. It is the purpose of the laws of physics to express this order in the form of numerical relations, and this can be done without ambiguity only by having recourse to a system of reference of four coordinates. That is why non-Euclidian geometry has become the instrument of Relativity physics.

Science has been led to reconstruct the world in this four-dimensional order, not by any arbitrary choice, but by the very nature of extrinsic measurement upon which its whole method is founded. Because the bodies which constitute the cosmos are in motion with respect to each other length can be measured only in relation to time, and time only in relation to length. Consequently, observers with different motions will have different reckonings of space and time, and each observer by merely changing his motion will make a different division of the four-dimensional

order into space and time. In other words, each observer, according to the different operational definition he gives of simultaneity, will cut up the space-time continuum into space and time in different ways. But while the determinations of length and time are relative, the space-time continuum which they constitute has an absolute character. And it was Einstein's chief aim to attempt, by a comparison of measures made with respect to different systems, to arrive at elements which would be independent of particular observers.

But, to get back to the main purpose of this analysis, it follows from what has been said that the same body may have any number of different lengths, depending upon the frame of reference in relation to which the length is determined. It makes no sense to say that one of these lengths is true and all the others are false. They are simply different. Nor is it legitimate to give to one of them a special meaning by attributing to its frame of reference a privileged position in the cosmos. Nature has not revealed any privileged frame. And the profound significance of the theory of Relativity is not that it discovered that the frame of reference used by Classical physics was wrong or that it involved experimental inconsistencies, for such a discovery would not have produced any great revolution; but rather that it brought to light the fact that neither this frame nor any other frame that might be chosen can be considered unique.

It is clear, then, that there is a double relativity involved in every determination of length. In the first place, every length is essentially relative to the chosen standard, and this standard is arbitrary. Secondly, it is essentially relative to the particular frame of reference by which it is determined, and this frame is also arbitrary. That is why length has no absolute meaning. All this refers, of course, to extrinsic measure which alone has significance for physical science.

But what about intrinsic measure? Is that also essentially relative? This question has already been solved, at least in a general way, earlier in our analysis. But perhaps it will be worth while to bring it back into focus again in relation to what we have been saying about Fitzgerald contraction. Scientists are often asked: does the Fitzgerald contraction actually take place? Such a question is extremely ambiguous, and no definite answer can be given until several important distinctions are made. In the first

place, the question might be taken to mean: do measurements of a body in motion with respect to a given frame of reference give results which differ from those obtained by the measurement of a body at rest, and if so, is this difference equal to the Fitzgerald contraction? Taken in this sense, the question will receive an affirmative answer from scientists. And this seems to be the only sense in which the question can have any significance for them. For in physics the phrase "actually takes place" can only refer to what actually takes place in measuring instruments.

But perhaps one might be tempted to push the question further and ask: But does velocity make the length of a rod contract in the same way that a change in temperature does? This question is still ambiguous, since it attempts to establish a comparison between "lengths" which have entirely different meanings. But perhaps the issue can be clarified by putting the problem in these terms: does the motion of a body decrease its intrinsic measure? And then the answer is: first, the Fitzgerald contraction actually does not imply such a change, since it has nothing at all to do with intrinsic measure; secondly, there is no way of knowing what actually happens to the intrinsic measure of a body in motion, for in order to determine the dimensions of such a body we are forced to have recourse to extrinsic measurement made in relation to a particular frame of reference.

In this discussion of the limitations of measurement it has been necessary to restrict ourselves to rather general and superficial considerations. A more refined analysis of particular processes of measurement, such as those which have to do with time, for example, would throw fuller and more definite light upon the extremely limited character of the knowledge which measurement affords us. But perhaps enough has been said to show how highly artificial and subjective this knowledge is. There is, indeed great wisdom in Bergson's remark that nature does not measure. It is man that measures. And he cannot measure without projecting his own logos into nature. At every step in the measuring process there is a projection of the human intellect and will. And the more perfect this process becomes, the greater becomes the part played by the subjective elements. In a very true sense, measure-numbers are not found in nature. They are imposed upon nature by man.

But lest all this seem to give too much aid and comfort to idealism it is worth while pointing out, as we

bring this question to a close, that measurement is after all a real physical operation which comes to grips with the real world. And the relations which arise out of it are basically real relations, in spite of the large margin of subjectivism. Moreover, the subjective element is purely functional; it exists only to enable us to come into more intimate relation with the objective world.

CHAPTER NINE

THE MATHEMATICAL TRANSFORMATION OF NATURE

1. The Transformation of Natural Science.

"The mathematician," Goethe once remarked, is like a Frenchman: if you speak to him, he translates it into his own language, and at once it becomes something altogether different." In this Chapter we must endeavor to see at least in a summary and schematic way, how the mathematician who is called in to assist the physicist in the study of nature translates the world of the physicist into his own language and makes of it something altogether different. And we shall consider this transformation from two points of view. First we shall try to see the way in which the introduction of mathematics into physics effects the very structure of physical science itself; and secondly, we shall attempt to bring out the change that this produces in the reflection of nature that is found in physical science.

In the last Chapter we considered the preliminary step in the mathematical transformation of physical science. In order for science to be mathematicised, all of its processes of experiment must be transformed into processes of measurement; all of the phenomena with which it deals must be translated into pointer readings. This preliminary step provides the scientist with a collection of measure-numbers, by which are determined various properties of bodies such as mass, volume, temperature, pressure, viscosity, valence, molecular weight, various optical, electrical and magnetic properties, etc. But just as physics is not a collection of phenomena, so mathematical physics is not a collection of measure-numbers. In order for science to emerge, the unifying process described in Chapter IV must undertake, by using measure-numbers as materials, to construct out of them an integrated and coordinated system. And the first step in this process is the establishment of law.

Since the only materials of construction available are numbers, laws in mathematical physics can be nothing but the expression of relations between numbers. Since a law must be universal, that is to say formulate a constant relation, a physico-mathematical law will express a relation between variable magnitudes, and consequently will not be algebraic and not arithmetical (in the restricted sense of the term "arithmetic"). The uniformity of association which constitutes the essence of experimental law finds its best expression the language of numbers because it is at once both exact and universal. This expression usually takes the form of differential equations.

"A physical law", writes Planck, "is any proposition enunciating a fixed and absolutely valid connection between measurable physical quantities — a connection which permits us to calculate one of these quantities if the others have been discovered by measurement."(1) In other words, a physical law is a constant relation between variable quantities; it takes the form of an algebraic equation which expresses a functional relationship indicating the precise value of any one of the measures that corresponds to any given value of the other measures. Once the concrete measure-numbers are absorbed into mathematical equations they become susceptible of all the pliancy of mathematical manipulation. The mathematician is free to have recourse to all of the resources at his disposal: powers, roots, divisors, dividends, sines, cosines, vectors, etc. There is nothing to prevent him from squaring the symbol for time, for example. These manipulations, obviously, do not effect the concrete properties from which the original measure-numbers have arisen,

but they may lead to the discovery of new properties.

It is extremely important to grasp the true nature of the functional relationship of physico-mathematical law. As is evident from our analysis of the nature of mathematical abstraction, mathematics prescinds from all causality except a type of formal causality that is found in formal relationships. For example, the geometric "law" B = S/H: the base of a rectangle is equal to the surface divided by the height does not mean that a surface can actually be divided by a length. And if B varies it is not because (in the sense of true causality) S varies, or vice versa. The law merely states that <u>if</u> the base is changed, the nature of a rectangle is such that the survace will undergo a proportional change. The "if" makes all efficient causality extrinsic to the law, and the phrase "the nature of a rectangle is such that" shows that the law deals with formal causality, since it is the form of a thing which determines its nature.(2) Consequently, in the measure in which physical laws are expressed in mathematical equations they are stripped of all true causality. Genuine causal statements are irreversible, that is to say they always involve ontological symmetry and usually temporal assymetry. The effect depends upon the cause for its being and not vice versa. Formulae of covariation and purely functional statements, on the other hand, are essentially symmetrical. Any one of the variables may be arbitrarily considered as independent or dependent.

When a mathematical physicist states that the movement of the planets is in accordance with the following law: the force of attraction between bodies is directly proportional to the product of their masses and inversely proportional to the square of the distance between them, he is not expressing the cause of planetary movement. He cannot treat force as a true cause since for him it is reduced to a measure-number which is a product of the multiplication of the numbers derived from the measurement of mass and acceleration. He is merely expressing a formal interrelatedness emerging from a comparison of the mass, distance, and acceleration of planets. Force and movement, then, are not related as cause and effect. They are simply two data which are mutually dependent in somewhat the same way as the diameter and circumference of a circle. (3)

Poincaré has insisted upon this point in La Science et l'Hypothèse:

> Qu'est-ce que la masse? C'est, répond Newton, le produit du volume par la densité. Il vaudrait mieux dire, répond Thompson, que la densité est le quotient de la masse par le volume. Qu'est-ce que la force? C'est, répond Lagrange, une cause qui produit le mouvement d'un corps ou tend à le reproduire. C'est, dira Kirchoff, le produit de la masse par l'accélération. Mais, alors, pourquoi ne pas dire que la masse est le quotient de la force par l'accélération?
> Ces difficultés sont inextricables. Quand on dit que la force est la cause d'un mouvement, on fait de la métaphysique, et cette définition, si on devait s'en contenter, serait absolument stérile. Pour qu'une définition puisse servir à quelque chose, il faut qu'elle nous apprenne à 'mesurer' la force, cela suffit d'ailleurs, il n'est nullement nécessaire qu'elle nous apprenne ce que c'est que la force 'en soi', ni si elle est la cause ou l'effet du mouvement. (4)

Ohm's law merely signifies that the numbers obtained by the measurement of the intensity of an electric current, the electromotive force, and the resistances are so related that they always verify the equation: $I = E/R$, whatever be the numerical values of the symbols in individual cases. The law of Mariotte is likewise stripped of causality when it is transformed into a mathematical equation. It does not mean that the pressure is the cause of the increased volume; in so far as the mathematical physicist is concerned. Both the pressure and the volume may be considered either as the independent ("cause") or the dependent ("effect") variable. The law merely states that when all other measures are equal, if the measure of temperature increases, there is a definite corresponding increase in the measure of the volume. Or to put it in other words which will bring out the assimilation of a physical law to a geometrical law, and show what type of causality is in question; the law states that if a cause should increase the temperature of a gas, the nature of the gas is such that there will be a proportional increase in its volume.

The same is true of all the laws of mathematical physics: they do not declare that A is the cause of B; they merely state that one set of events B is a function of another set of events A. If the mathematical formulation of the law expressed causality, the causality would have to be reversible. Perhaps one might be tempted to think that the intervention of a time measure into a law might introduce causality since this measure will indicate which of the variables is the antecedent and which the consequent. But a moment's reflection will show that this is not true. This intervention of a time measure merely expresses the fact that the other measures vary in relation to the time measure. An expression of antecedence does not involve causality.

It is clear, then, that the mathematical formulation of physical laws empties them of all true efficient causality. And the same must be said of final causality. Just how profound this change is becomes evident when one stops to consider that all law essentially involves finality. By its very nature law means an inclination, an ordination to an end. We shall return to this question later on.

From all that was said in the last Chapter on the

nature of measurement it follows that, in spite of the exact mathematical formulation by which they are expressed, and in a certain sense precisely because of it, the laws of physics do not have exact and absolute validity.(5) In fact any mathematical expression of physical constancy is only one of an infinite number of slightly different expressions which might possibly be employed to formulate the same phenomenon. All physical laws are essentially provisional. And they are provisional for two reasons in particular; first because they are merely approximative, and in this sense neither true nor false; secondly because they are schematic. They are approximative because the measures whose relations they express are never made with absolute exactitude. That is why they must ever remain open to successive corrections, for progress in the refinement of measurement will continually introduce slight changes in the numerical coefficients, and there is no limit to this process of refinement.

Laws are schematic because they include only a small fraction of the possible measures that could have been made; that is to say, they express a relation between certain chosen properties, independently of all the other properties which may be connected with the ones chosen. Consequently, as science progresses its la s must be constantly modified in such a way as to take into consideration attributes previously omitted. Physical properties are defined by the description of their process of measurement, and as we noted in the last Chapter all of the circumstances entering nto t is process ce never be enumerated. Progress in experimentation reveals an increasing multiplicity of circumstances which have a definite influence upon the results of the measuring processes, but which were neglected in the original formulation of a general law. That is why all laws must remain forever open to a progressive modification by which these newly discovered influences are integrated into its structure. This modification does not change the form of the law or its numerical coefficients, as does the modification occasioned by its approximative character. The newly discovered circumstances can be introduced only by the introduction of new measures and consequently new properties. Thus progress in experimentation with gases revealed the fact that in order to determine with precision the relation between pressure and volume attention must be paid to the mutual attraction of the molecules and their proper volume. A determination of these additional circumstances results in the transformation of the law of

important than this is the simplification resulting from the discovery that the results of certain processes of measurement coincide with mathematical combinations of other processes. Laws reveal constant relations between the measures of different properties. These constant relations make it evident that certain measures can be replaced by a combination of other measures. In this way it is possible to reduce a vast multiplicity of measures to a few fundamental measures. In fact science has been able to push this process of simplification to the extent of reducing all physical measures to combination of the fundamental measures of length, mass and time, in such a way that the former may be considered as functions of the latter. It thus becomes possible to define the multiplicity of physical properties in terms of combinations of a few irreducible properties. This does not mean that bodies have no other properties but the three that are measured b a rule, a balance and a clock. It merely means that when the variety of physical properties are measured by different measuring processes the results are numerically the same as certain mathematical combinations of the measure numbers provided by a rule, a balance and a clock. (6(By this simplification the scientist is able to synthesize his knowledge into a small number of propo-

Mariotte into that of van der Waals.

Thus, as science progresses its laws become increasingly complicated by the integration of newly discovered influences. This complication results in investing general laws with greater precision and accuracy. But as we saw in the last Chapter, even the simplest measuring process involves the whole universe. That is why a perfectly exact law would require an exhaustive descriptive of the entire cosmos.

But while this process of complication is taking place there is a concomitant process of simplification going on, which consists in the reduction of the ever increasing multiplicity of measures to a few fundamental measures. This is done in two ways. In the first place it is discovered that a number of different instruments give the same results. Since physical properties are defined by their processes of measurement it remains theoretically true that two different processes define two different properties. Nevertheless it sometimes becomes evident that the results of two or more different processes coincide, as for example when heat is measured by the expansion of a metal spring and by the expansion of a column of mercury. But even more

sitions into which only a few basic measures enter, and from the relations existing between the fundamental measures it becomes possible to deduce the multiplicity of relations existing between the particular measures.

All this shows how this process of simplification opens the way for the scientist to take the next step in the unification of his knowledge -- to ascend from laws to theories. But before passing on to an analysis of the nature of physical theory it is necessary to remark that because of the approximative and schematic character which we have been discussing, physical laws are always a simplification of the mind and in this sense a product of the mind. And their provisional nature cannot be lost sight of without undermining their objective significance. Casting physical reality in mathematical form has the advantage of providing it with great openness, that is to say, of opening it up to the unlimited reaches of mathematical speculation which affords such abundant sources of explanation. But at the same time, it has the disadvantage of imposing upon reality a frame which because of its exact determination is too closed. And in this connection it is worth while recalling the well-known remark of Einstein that in so far as the theses of mathematics are certain they do not refer to physical reality, and in so far as they are made to refer to physical reality they are not certain. (7)

But perhaps one might be tempted to object to the statement that all of the laws of physics are provisional on the grounds that there are certain fundamental laws known as principles which are not subjected to the successive change about which we have been speaking and which consequently seem to have an absolute and not a provisional character. The conservation laws, the law of inertia etc. are all laws of this kind. The answer to this objection is that the absolute character of these principles is a pure gift of the mind. The principles of experimental science are laws which have been merely suggested by nature, but which the mind has arbitrarily erected into fixed and absolute principles. The reason why progressive experimentation does not modify them is simple: the mind has accepted them as conventional definitions of the very objects to which they apply. Consequently it is impossible for these objects not to be in accord with them. And now, having examined the nature of physical laws we must take up the problem of physical theories.

For reasons explained earlier in this study, the

the mind cannot rest satisfied with an a posteriori possession of physical laws. It will never feel that it has assimilated them perfectly until it is able to possess them in an a priori fashion. Just as the formulation of laws makes it possible for the mind to arrive at the results which a certain measuring process would give without actually effecting the process, so the scientist instinctively seeks for a point of departure which will enable him to arrive at a certain law in a way that does not depend upon experience. In other words, having arrived at physical laws by induction, the scientist is led to attempt to arrive at them by deduction; having posited their existence, he must attempt to explain them; having arrived at universal functional relationships, he must try to show that these relationships are necessary. This is done by making the laws appear as logically necessary conclusions. (8) Since the laws themselves are numerical relations, the point of departure from which they are to be deduced must be general numerical relations. These general numerical relations constitute what is known as a mathematical theory. A theory has been defined by Duhem in the following terms: "un système de propositions mathématiques, deduites d'un petit nombre de principes, qui ont pour but de représenter aussi simplement, aussi complètement, et aussi exactement que possible un ensemble de lois expérimentales." (9)

Not only does a physical theory synthesize the laws which experience has suggested, but it tends to fill in the gaps which observation has left open by substituting what Cassirer has called "a continuous connection of intellectual consequences." (10) In this way science becomes a coordinated system. And this system is perfected by a continual simplification and reduction of the principles which form its point of departure and a continual increase of the experimental propositions which constitute its terminus. As Whyte has remarked, "the highest possible aim for science is the formulation of a self-consistent closed chain of concepts and principles permitting deductive argument in one direction at every point of the chain." (11) The dialectical limit of this movement would be a science in which the whole universe could be deduced from one mathematical formula. (12)

On more than one occasion in this study we have insisted upon the fact that the fundamental reason why physical science reaches out to mathematics is to discover an explanation which it finds itself unable to provide for physical phenomena, in other words, to discover a reason or propter quid for its experimental propositions. But perhaps what has been said thus far in the present Chapter about the

mathematical transformation of physical science may give rise to doubts as to whether this goal is actually achieved. As a matter of fact, a number of authors explicitly deny that a physico-mathematical theory is an explanation. Duhem, for example, writes: "Une théorie physique n'est pas une explication." (13) We believe that the difficulty here arises from the ambiguity of the word "explanation". As a matter of fact, it is a term that is susceptible of a variety of meanings. In its most fundamental sense it means to give the proper reason for a thing by presenting one or several of the four causes by which reality is constituted. This is the type of explanation that is employed in the philosophical sciences.

There is another sense in which the term explanation is used and which has long been associated with experimental science. It consists in presenting a model whose structure and functions reproduce the structure and function of the phenomena to be explained. We understand the term "model" here in the sense of a mechanical construct or at least of a pictorial image, and not in the sense in which it is now sometimes used and which includes mathematical "patterns" such as "tensors and metrices, manifolds and their curvature, differential forms and their invariants."[14] It is well known that mechanical models constructed out of pulleys, wires, rubber tubes, etc. were the favorite form of explanation employed by the classical physicists, particularly those of the English School, such as Lord Kelvin, Oliver Lodge, Faraday, Maxwell, etc. We have already quoted Kelvin's well-known remark that for him to understand reality meant to be able to construct a mechanical model of it and apart from such a model no explanation of reality could have any meaning for him.

But even when less emphasis was put upon concrete mechanical models and more upon abstract mathematical conceptualization, there was, until recently, always lurking in the background of mathematical theories physical models of some kind. For example in the background of the mathematical kinetic theory of gases there has always been a fairly definite physical model constructed of molecules which are so idealized and so simplified that they are susceptible of accurate mathematical treatment, even though spectrum analysis has given abundant evidence of a considerable gap between the idealized and simplified molecules and the actual molecules. These idealized and simplified physical models have served as a kind of bridge between actual physical reality and mathematical theory.

Because of their physical character they have been considered to be in contact with reality; at the same time their simplified and idealized state makes them directly amenable to mathematical manipulation. Recent physics has discovered however that it can get along without this bridge, that independently of any physical model it can set up a correspondence between the results of its mathematical constructions and the physical system. This has been particularly true of the quantum mechanics of Dirac.[15] Speaking of this significant change Professor Bridgman writes:

> What we now have is in effect mathematical models rather than physical models. This emancipation I feel to be a very important step forward toward greater theoretical power, because there is an enormously greater wealth of possibility among the structures of mathematics than in the physical models which we can visualize and which have a simple enough mathematical theory. It cannot be denied, however, that a mathematical model cannot be visualized in the same sense that a physical model can be. Although we may recognize with our intellect that the mathematical model is just as good as the physical model if it only enables us to answer any question that we may propose about the behaviour of the physical system, nevertheless we have an uncomfortable feeling that we have lost something.(16)

Professor Bridgman is correct in maintaining that this recent change in physical theory represents significant progress. As a matter of fact, the identification of scientific explanation with the construction of mechanical models, such as is found in the writings of Lord Kelvin, and the classical physicist's insistence upon physical models as the criterion of the value of theories, make the intellect the slave of the imagination. Moreover, they destroy the true notion of science, since they seem to make the sensible as such the formal object of science. In a word, they amount to a confusion of the material and the formal object of science.

It is true that this tendency to explain reality in terms of physical and mechanical models reveals a trait that is native to the mind in the sense that it is natural to man to want to reduce the unfamiliar to the level of the common and the familiar. But to tie science down to this type of explanation can only result in creating insurmountable obstacles in the path of progress. For reality is infinitely richer than any fixed frames that derive from ordinary experience. Moreover it is presuming a great deal to expect to find in familiar molar experience counterparts of microscopic reality. Scientists are coming to realize this more clearly every day, especially in the field of wave mechanics, and the work of Dirac, Schrodinger, etc. has put particular emphasis upon this point. But the most important aspect of this question is that true progress in

science, as we saw in Chapter IV, does not consist in transforming things into what is most knowable for us, but in approaching closer and closer to what is most knowable in se, though least knowable for us. In other words, it does not consist in imposing our measure upon reality, but in allowing reality impose its measure upon us. And if it becomes necessary to have recourse to art, the only reason is, as we have seen, to open up reality more and more as an object.

But in this question it is not necessary to be a purist. The remark made by Dirac to Schrodinger "Beware of forming models or pictures at all," must not be taken too literally.(17) Even though physics has recently taken a very definite step in asserting its emancipation from physical models, it is doubtful that this emancipation will ever be complete, or even that such a complete emancipation would be desirable. Imaginative construction inescapably accompanies intellectual activity. Moreover, this imaginative construction may often prove useful for the physicist, as Professor Bridgman has pointed out:

> I think that the ordinary physicist will want to keep his physical models as long as he can.. Unless one has supreme power as a mathematician, one may well find it useful to have at his command methods of reasoning by analogy that

> will give him an insight into the nature of the solution of special problems, and one may cheer from the sidelines any attempt to invent combinations of the elements of the mathematical analysis which may be handled somewhat like the elements of ordinary experience, and of which we may hope ultimately to acquire a more intuitive command. I suspect that Bohr's attempt to find a dualistic aspect of nature is an attempt of this sort.(18)

Even mathematical conceptualization is necessarily tied up with the imagination, as we saw in Chapter VI. The imaginative construction which accompanies this conceptualization, while on the one hand less free than that found in metaphysical knowledge, is freer and less determined than that found in physical knowledge. In mathematical theory it is of little importance what the nature of the imaginative construction is, provided that it prove useful and that it remain in continuity with the measure numbers out of which the theory is evolved.

Consequently, the physicist is free to employ any physical models that may prove helpful to him, provided he remain critically conscious of their true significance. He is free to conceive of light in terms of "waves" or "corpuscles" or both, provided he does not allow himself to slip into the delusion of thinking that the ontological nature of light is actually like waves of water or like

tiny pellets. The most important function that these models can play is to provide suggestive sources of mathematical manipulations and an imaginative support which will aid the mind in coordinating experimentally observed relations. The fruitfulness of Bohr's theory of the structure of the atom did not consist so much in the planet-like circulation of electrons around a nucleus as in the fact that this structure provided a basis for mathematical speculation. By considering seven electrons circulating in one atom and eight electrons in another, one is enabled in some way to seize upon the difference between nitrogen and oxygen. (19(

In La Science et l'Hypothèse Poincaré has brought out the true function played by models in physical theory and showed that they are essentially transitory while the mathematical relations which they suggest constitute the essential and permanent part of physical theory:

> ... ces équations expriment des rapports et, si les équations restent vraies, c'est que ces rapports conservent leur réalité. Elles nous apprennent, après comme avant, qu'il y a tel rapport entre quelque chose et quelque autre chose; seulement, ce quelque chose nous l'appelions autrefois mouvement, nous l'appelons maintenant courant électrique. Mais ces appellations n'étaient que des images substituées aux objets réels que la nature nous cachera éternellement. Les rapports véritables entre ces objets réels sont la seule réalité que nous puissions atteindre, et la seule condition,

> c'est qu'il y ait les mêmes rapports entre ces objets qu'entre les images que nous sommes forcés de mettre à leur place. Si ces rapports nous sont connus qu'importe si nous jugeons commode de remplacer une image par une autre. (20)

And he goes on to explain that the scientist may employ models that are mutually contradictory:

> Il peut se faire qu'elles expriment l'une et l'autre des rapports vrais et qu'il n'y ait de contradiction que dans les images dont nous avons habillé la réalité. Les hypothèses de ce genre n'ont donc qu'un sens métaphorique. Le savant ne doit pas plus se les interdire que le poète ne s'interdit les métaphores; mais il doit savoir ce qu'elles valent. (21)

We believe that this view of the meaning of scientific models is correct and that it fits in perfectly with the Thomistic doctrine of the nature of mathematical physics. For in a science which is formally mathematical and terminative physical, the explanatory constructions will be essentially mathematical. It will not be necessary that in these constructions there be physical re-embodiments of nature. All this is required is that the mathematical constructions be in the end verifiable in physical experiment.

But, to return to our original question: is a mathematical theory an explanation? Professor Bridgman,

after noting that the emancipation from physical models gives us an uncomfortable feeling that we have lost something, goes on to say: "I think that we discover on analysis that it is the explanation which we feel we have lost". It is certain that a mathematical theory is not an explanation in the sense of a reduction to familiar experience, nor does it provide an explanation of the type that philosophy affords. That is to say, the purpose of physical theory is not to give us the real foundation of the laws, but a logical foundation. For theories are mental constructs, and it must be kept in mind that mathematical physics is dialectics. Nevertheless, we feel that a physical theory may be called an explanation in a true sense of the term.

> Qu'est-ce donc qu'expliquer? C'est tout uniquement faire rentrer un fait dans une forme. Le fait est expliqué lorsqu'il apparaît identique à l'un des phénomènes qu'engendre un de ces sorites indéfinis que nous appelons théorie ou forme. (22)

Physical theory provides an explanation of reality in the sense of making it deducible and thus rational. It is an explanation in the line of formal causality, even though it is not a question of the proper ontological formal cause that is found in nature. It is a mere substitute formal causality - - and never more than provisional. Nevertheless by means of it mathematical physics truly achieves the aim of subalternation.

And it must be pointed out that the emancipation from physical models of which we have been speaking does not in any sense dissolve the intimate union between mathematics and physics that subalternation implies. To the question: what is the theory of Maxwell, Hertz is supposed to have replied: "The Theory of Maxwell is Maxwell's system of equations."[23] And Poincaré writes: "Une loi pour nous ... en un mot, c'est une équation différentielle."[24] There is obviously a sense in which these expressions are correct. And yet it would be false to suppose that Maxwell's Theory or any other theory in physics consisted merely of mathematical equations and nothing more. In so far as science remains materially physical, there must be a link binding these mathematical equations with physical reality - even when the bridge constituted by a physical model has been removed. This link is provided by what is known as a text or a dictionary. This text reveals the physical significance of the mathematical equations and shows how these equations are to be used in order for that significance to be maintained. For example, the formula $s = \frac{1}{2} gt^2 + v_0 t$ has no physical significance unless it be accompanied by a dictionary which explains that it is the formula for falling

bodies and that the symbols s,g,t,v refer to distance, gravitational attraction, time and original velocity, or, to be more accurate, to sets of concrete measuring operations whose resultant measure-numbers represent the properties of distance, attraction, etc.. To say that this is the equation for falling bodies means that the numbers obtained by the concrete processes of measurement determined by the text satisfy the equation when they are substituted into it. The text determines not only the nature of the measurements involved, but also the precise connection between the various symbols used in the equation. If for example the time and the distance must be obtained by simultaneous measurements, this must be specified by the text. It is clear that in the dictionary we shall find the way in which the multiplicity of individual measures are reduced to the fundamental measures and how particular measures, such as that of temperature, for example, become absorbed by the theory and lose themselves, so to speak, in combinations of the basic measures of length, time and mass.

It is easy to lose sight of the importance of the dictionary in physical theory. And yet its function is essential, for it maintains the intimate union between the mathematics and the physics. It is precisely by means of the text that the mathematical physicist is able to keep in mind that what he is dealing with directly is a physical element, and that the mathematical element enters into his object only by way of connotation. To quote Bridgman once again:

> It appears, therefore, that a complete mathematical formulation requires equations plus text, and the text may perform a variety of functions. The necessity for a text is almost always overlooked, but I think it must be recognized to be essential, and a study of what it must contain is as necessary for an adequate conception of the nature of the mathematical theory as is the study of the equations themselves. One of the functions of the text, we have seen, is to tell us how to set up the correspondence between the numbers given by the equation and the numbers obtained by manipulations of the physical system. The text cannot tell us what it is that the correspondence is to be set up with without going outside the system of the mathematical theory and assuming an intuitive knowledge of the language of ordinary experience. In classical mechanics, the geometrical variables in the equations of motion are the coordinates of massive particles, but unless we know intuitively what a massive particle is, we simply cannot make connection with equation or theory. Not only is the theory powerless to describe, either in text or equations, what the elements are to which correspondences are to be made, but all the more is it powerless to explain why the elements have the properties that they do. (25)

The truly great physicist never allows the symbolism of mathematics to make him lose intimate contact with physical reality. Of Einstein Langevin could write:

> Pour lui jamais le voile du symbole ne masque la réalité. Nombreux sont les esprits pour lesquels le signe cache souvent la chose signifiée; Einstein se meut à l'aise dans le monde des symboles, mais jamais ceux-ci ne lui dissimulent l'aspect physique des choses. (26)

But this union between mathematical construction and physical reality must be correctly understood. In the simple example cited above of the formula for falling bodies there is a one to one correspondence between the mathematical symbols and operationally defined physical properties. Must we expect this same correspondence to be found in all mathematical formulations and throughout the whole of physical theory? Such an expectation would misconstrue the proper function of mathematics in physics and would impose sterilizing restrictions upon the theoretical power of mathematical construction. (27)

There is no reason why each symbol in the mathematical equations, nor even each step in the structure of mathematical theory, should have a definite counterpart in the physical system. Nor is it necessary that all of the operations performed by the mathematician in his interpretation of nature should have a physical meaning, or that all of the quantities manipulated be accessible to experience.

It is true that all physico-mathematical theories must originate in measure-numbers produced by physical processes and must ultimately terminate in formulas which have direct physical relevance and which correspond to concrete measure-numbers. But between this point of departure and this terminus the theoretical physicist is free to create any auxiliary mathematical quantities which will help him to carry forward his task, even those whose realization in nature would involve a contradiction. Nor is there any contradiction in maintaining that fictitious entities can make a positive contribution to the explanation of reality. It is well known how the fictitious constructs of the Theory of Relativity both provided an explanation for phenomena previously inexplicable, such as the anomoly of Mercury, and led to the new discovery of the deviation of luminous rays in the neighborhood of the sun. And if pure logical entities and fictitious constructs can be efficaciously used to solve practical problems, as in the rather well-known case of Steinmetz's use of the mathematical surd, $\sqrt{-1}$, to solve the problem of getting electrical locomotives over the Continental Divide, they can <u>a fortiori</u> serve as efficacious explanatory devices to solve theoretical problems.(28)

Modern physics has exercised wide freedom in this regard. It has felt free to push the theoretical power and the creativity of mathematics to the limit, provided only that in the end there result formulae that can be given a physical meaning. Weyl has claimed for physics the right to make use of every possible resource no matter how strange the results may appear.(29) In this connection Eddington writes:

> The pure mathematician, at first called in as a servant, presently likes to assert himself as matter; the connexus of mathematical propositions becomes for him the main subject, and he does not ask permission from Nature when he wishes to vary or generalise the original premises. Thus he can arrive at a geometry unhampered by any restriction from actual space measures; a potential theory unhampered by any question as to how gravitational and electrical potentials really behave; a hydrodynamics of perfect fluids doing things which it would be contrary to the nature of any material fluid to do.(30)

We see in this exercise of freedom a confirmation of the Aristotelian and Thomistic interpretation of mathematical physics as opposed to that of the ancient and neo-Pythagoreans. As Bridgman has remarked, the feeling that all the steps in the structure of mathematical theory must have their counterpart in physical reality derives from the Pythagorean belief that the mathematical interpretation of nature means a discovery of mathematics in nature,(31) which is in the last analysis a mathematical construction. In the doctrine of Aristotle and St. Thomas the mathematical world is extrinsic to the physical world (in the sense already explained) and consequently the use of mathematics in the study of the physical world is not a discovery; it is an application. As a result the theorist in making this extrinsic application is granted all of the freedom that is native to the world of mathematics. It took the genius of Einstein to fully realize that geometrical conceptions must be manipulated with the utmost freedom in order to provide an explanation of physical phenomena.

The following lines of Cassirer are relevant here:

> For it is precisely the complex mathematical concepts, such as possess no possibility of direct sensuous realization that are continually used in the construction of mechanics and physics. Conceptions, which are completely alien to intuition in their origin and logical properties, and transcend it in principle, lead to fruitful applications within intuition. This relation finds its most pregnant expression in the analysis of the infinite, yet is not limited to the latter.(32)

This brings us to the mooted question of the geometrical structure of "real" space. It is a question that has been rendered obscure by the ambiguity of the terms employed. As a matter of fact, the word "real" can have

more than one meaning. For the physicist, if he so desires, is entitled to consider a space as "real" when the geometry to which it corresponds provides the greatest theoretical power in explaining (in the sense determined above) the concrete measure-numbers derived by actual experimentation with the physical world, and has the greatest success in synthesizing in an exact, simple, coherent and complete fashion all of the experimental laws. The only meaning that reality can have for the mathematical physicist is the numbers that are the results of concrete measuring processes. That is why the geometry that best "explains" these results is for him the geometry of reality.

When the question is understood in this sense, it is clear that no particular type of space, and no particular system of geometry is privileged. Any geometry which at a given stage in the development of physics provides the greatest explanatory power for all of the discoveries that have been made up to that point may be considered to be the geometry of real space. Abd just as soon as any other system of geometry provides greater explanatory power or is better able to meet the problems arising from newly discovered phenomena, it must supplant its predecessor and become the geometry of "real" space. In this sense it is perfectly