legitimate to say that "real" space is spherical or elliptical, or that the geometry of nature is not Euclidian but Reimanian.

But for the philosopher, the geometry of "real" space is not merely the geometry which best "saves" a collection of measure-numbers. It is the geometry which is realized (though not of course in the abstract state proper to the mathematical world) in the quantity of the objective world condition. There is a vast difference between this objective realization and an explanatory saving of a collection measure-numbers, and that is something that more than one modern scientist and philosopher of science have overlooked. Theoretical continuity between a geometrical system and a collection of measure-numbers does not constitute an experimental proof of the objective character of that system. In fact, it seems necessary to insist that as long as it remains true to its proper method mathematical physics can neither prove nor disprove that the absolute world condition is either Euclidian or non-Euclidian. Nor does the theoretical continuity just mentioned prove as some contemporary scientists have claimed, that the distinction between geometry and physics has been wiped out, in such a way that

the former must be considered an experimental science.

If the ability of a mathematical system to provide numerical values which coincide with those derived from physical measurement were a sufficient proof of the ontological character of the space proper to that system, then any and all fictitious constructs which could be put into continuity with measure numbers would have to have objective existence. Some modern scientists seem to have recognized this fact and have consequently felt the necessity of attempting to establish the possibility of some connection between non-Euclidian space and sensory perception. The results of these attempts have only served to show their utter futility.[33] Sir James Jeans, for example, while admitting the obvious difficulty encountered in trying to imagine "spherical space" believes that this difficulty derives merely from its unfamiliarity. He holds that our intuitive belief that space is Euclidian is similar to the "common sense" belief that the earth is flat, and compares the difficulty of imagining non-Euclidian space with the difficulty that a child has in imagining people existing on the other side of the earth without falling off.[34] A moment's reflection will show that there is no parity between these two cases. It is possible for the imagination to cope with the sphericity of the earth, but it is utterly impossible for it to cope with the concepts of non-Euclidian space.

Consequently, when "real space" is understood in the philosophical sense of the term it becomes necessary to say that the geometry proper to it can be nothing but Euclidian. The modern non-Euclidian geometries are purely dialectical structures and that they cannot be applied to real quantity without a contradiction. Only the entities of Euclidian geometry are capable of construction in the imaginative intuition, and this capability is necessary for realization in the objective world, since this realization means to exist with sensible existence. The entities of non-Euclidian geometry require Euclidian geometry as a foundation of their conceptual existence; consequently, their objective existence would involve a contradiction since it would deprive them of this foundation. For this reason the non-Euclidian constructions which have proved so fruitful in modern physical theory cannot be considered to have actual physical counterparts in nature; they must be looked upon not as something which directly reveal the quantitative nature of the objective world, but as pure geometrical symbols of this objective world. And the same

must be said, mutatis mutandis, of the mathematical constructions of quantum physics: they are pure mathematical symbols, which, without possessing any direct physical counterparts in the objective world, provide the best theoretical schema to explain and synthesize the results of our measuring processes.

It must be pointed out however that there is a sense in which the mathematical constructions which constitute modern physical theory have objective significance. Though without direct physical counterpart, they do, nevertheless, succeed in a certain fashion in seizing upon the structure of the objective world. By providing an intelligible scheme of relationship which establishes continuous connection between the members of the manifold which constitutes nature, they succeed in reflecting the interrelatedness of cosmic reality and the harmonious order that prevails in it. Were this not so, the value of modern science would be extremely dubious. For it would have gained very little for having condemned decadent Scholasticism for transforming facts into mere names, if in the end it resulted in nothing more than the transformation of facts into symbols. As a matter of fact, however, the sacrifice which mathematization has imposed upon it of renouncing the inner natures of things is repaid by a reflection of the all-inclusive structure of nature.

> All that the 'thing' of the popular view of the world loses in properties it gains in relations; for it no longer remains isolated and dependent on itself alone, but is connected inseparably by logical threads with the totality of experience. Each particular concept is, as it were, one of these threads, on which we string real experiences and connect them with future possible experiences. (35)

By reducing nature's manifold to a rational unity through relatedness in a number system, mathematical physics provides a quasi solution for the problem of the one and the many. By creating an order of pure homogeneous relatedness it affords a quasi sapiential view of the universe which enables the mind to derive the manifold from the one, even though the one be a pure substitute and the manifold be reached only in its purely material and numerical diversity and not in its proper specific nature. This explains why it is so easy for the mind to mistake mathematical physics for true wisdom.

All this is a great achievement. But it is paid for with a great price. Perhaps nowhere does the adage traddutore - - traditore obtain with greater force than in the mathematical "translation" of physical science. And that is what we must now try to see by considering

the transformation that this translation produces in the reflection of nature that is found in physical science.

2. The Transformation of Nature.

It would be virtually an endless task to attempt to bring out in full detail the profound metamorphosis that the mathematization of physics produces in the scientific view of nature, and we must limit ourselves to touching briefly upon a few of the most characteristic and significant points. And a moment's reflection will suggest that the pivotal point of this whole transformation is found in the concept of motion, which, as St. Thomas says,(36) is, so to speak, the very "life" of the world. In Chapter II we went to considerable lengths to show that physical science is essentially a study of mobility. For nature is necessarily defined in terms of motion(37) and that is why Aristotle could say that he who is ignorant of motion is ignorant of nature.(38) On the other hand, we explained in Chapter VI that mathematics essentially excludes motion. The mathematical world is a world of immobility. It is true that mathematicians speak of a kind of motion, but as we pointed out, this motion is only a dialectical, imaginary, intramental thing, which does not involve true becoming. As we mentioned in Chapter I, this opposition between the mobility of nature and the immobility of mathematics was traditionally one of the stumbling blocks for those who wished to mathematicize the cosmos, and provided one of the bases for Aristotle's criticisms of the Platonists and the Pythagoreans. We must now try to see in what sense mathematics may be applied to motion and what is the effect of this application.

Aristotle and St. Thomas explain that motion may be considered under two different aspects. In the first place, it may be considered in its proper and specific essence, and in this sense it signifies a coming into being. Considered in this way, it is something profoundly obscure, since it lacks the determination and actuality of being. Consequently, it can be correctly defined only in a way which will bring out this profound obscurity. Aristotle has given us the essential definition of motion in the third book of the Physics: the act of a thing in potency in so far as it is in potency. This coming into being is realized both in the substantial and in the accidental order, and in the latter case (which is the strictest meaning of the Thomistic term "motus") it is found in the three categories of quantity (growth in living beings), quality and place

(local motion). All substantial change involves accidental changes, and motion in the predicaments of quantity and quality always involve local motion of some sort. (39) This, in a sense all motion may be reduced to local motion. This kind of motion is the most superficial and the one which realizes the least the concept of becoming. It involves essentially an extrinsic denomination. To say, therefore, that all the motion in the universe may be reduced to local motion, is to say that it may be reduced to a system of extrinsic relations.

The second aspect under which motion may be considered is brought out by St. Thomas in his <u>Commentary on the Fifth Book of the Metaphysics.</u> (40) In analyzing the motion of quantity, he tells us that there are various kinds of quantitative modes. Some things are quantitative <u>per se</u>, such as "line", others are quantitative <u>per accidens</u>. Among those which are quantitative <u>per accidens</u> some are such by the fact that they are accidents inhering in a quantified subject; others however are quantitative by the fact that they are divisible according to quantity. In this category are found motion and time. St. Thomas writes:

> Alio modo dicuntur aliqua quanta per accidens non ratione subjecti, in quo sunt, sed eo quod dividuntur secundum quantitatem ad divisionem alicuius quantiatis; sicut motus et tempus, quae

> dicuntur quaedam quanta et continua, propterea quod ea, quorum sunt, sunt divisibilia et ipsa dividuntur ad divisionem eorum. Tempus enim est divisibile et continuum propter motum; motus autem propter magnitudinem; non quidem propter magnitudinem eius quod movetur, sed propter magnitudinem eius in quo aliquid movetur. Ex eo enim quod illa magnitudo est quanta, et motus est quantus. Et propter hoc quod motus est quantus, sequitur tempus esse quantum. Unde haec non solum per accidens quantitates dici possunt, sed magis per posterius, inquantum quantitatis divisionem ab aliquo priori sortiuntur.(41)

In his <u>Commentary on the De Trinitate</u> he shows how this quantitative aspect makes it possible for mathematics to enter into the study of motion:

> Ad quintum dicendum, quod motus secundum naturam suam non pertinet ad genus quantitatis, sed participat aliquid de natura quantitatis aliunde, secundum quod divisio motus sumitur ex divisione spatii vel ex divisione mobilis: et ideo considerare motum non pertinet ad mathematicum, sed tamen principia mathematica ad motum applicari possunt: et ideo secundum hoc quod principia quantitatis ad motum applicantur, naturalis considerare debet de divisione et continui, et motus, ut patet in VI Physicorum. <u>Et in scientiis mediis inter mathematicam et naturalem tractatur de mensuris motuum,</u> sicut in scientiis de sphaera mota, et in astrologia.(42)

These distinctions make it clear how it becomes possible for mathematics to be applied to the motion in the universe. By reducing all motion to local motion or movement in space, by considering this local motion not as a

coming into being but as a pure extrinsic relation, and by considering this extrinsic relation purely in terms of its quantitative aspect, mathematics is able in some way to seize upon motion. But in doing so it transforms it into the only sense in which it can have meaning for a mathematician — the simple variation of the relations of a point with coordinated axes. And thus in mathematical physics movement becomes nothing more than a variation of spatial relationship between two or more bodies which remain intrinsically unchanged. Lenzen, for example defines it as a "change of position in space with time."(43) A continual series of spatial points are united with a continuous series of temporal points and the four dimensional curve which results becomes the model of motion.

It should be immediately evident that such a notion of motion empties it of its proper physical essence. It is no longer a true change, but a mere displacement of a point, no longer a process but a relation, no longer a becoming, but a state which has a certain determined value than can be measured.(44)

Things do not <u>come into existence</u> at a certain place and at a certain instant of time — they simply <u>exist</u> at a certain point in a continuum.

That physico-mathematical motion is emptied of all becoming is clearly brought out by Sir Arthur Eddington:

> Events do not happen; they are just there, and we come across them. "The formality of taking place" is merely the indication that the observer has on his voyage of exploration passed into the absolute future of the event in question; and it has no important significance.(45)

It is clear, then, that there is no true becoming in the physico-mathematical world, and consequently no rest.(46) A good analogy of the difference between the physico-mathematical world and the real world may be found in the difference between a piece of music played by a symphony orchestra and a record made of the piece. There is something on the static record to correspond to all the movements and nuances of the piece, but the movements and nuances themselves have been lost. They have all been spatialized.

Because mathemataticized motion is not a coming into being but a pure relation, it is perfectly reciprocal. That is why Descartes who had identified real motion with mathematicized motion could say that it is perfectly indifferent whether we say that we are moving towards a goal or that the goal is moving towards us, since in both cases the variation of the relations of distance remains exactly the same.(47)

It is easy to see what has happened in this mathematization of motion. Nothing is so irrational, so refractory to the intellect as potentiality. That is why the mind in its attempt to rationalize the universe as completely as possible is inevitably led to the attempt to wipe out potentiality and to reduce everything to the plane of actuality. From this point of view Bergson is correct in maintaining that experimental science deals only with the "tout fait."(48) But in wiping out potentiality it destroys all true mobility. It thus succeeds in explaining nature only at the expense of destroying it. It reduces motion to something that is perfectly clear and intelligible, but in so doing it sacrifices its very essence, for motion, as we said, is something essentially obscure. That is why mechanism taken as a philosophy of nature involves an intrinsic contradiction. For in attempting to give an adequate account of reality by means of motion and extension it empties motion itself of its reality. It was because Descartes failed to realize that his mathematicized motion was not true motion that he heaped such supercilious scorn upon Aristotle's definition:

> At vero nonne videntur illi verba magica proferre, quae vim habeant occultam supra captum humani ingenii, qui dicunt motum, rem unicuique notissimam, esse actum entis in potentia, prout est in potentia?

> quis enim intelligit haec verba? quis ignorat quid sit motus? et quis non fateatur illos nodum in scirpo quaesivisse? Dicendum est igitur, nullis unquam definitionibus eiusmodi res esse explicandas, ne loco simplicium compositas apprehendamus; sed illas tantum, ab aliis omnibus secretas, attente ab unoquoque et pro lumine ingenii sui esse intuendas. (49)

It is to be noted that Descartes did not say: "quis ignorat <u>quod</u> sit motus," but "<u>quid</u> sit motus?" For Aristotle the existence of motion was perfectly clear; it had all the clarity of a direct intuition. Descartes thought that this perfect clarity of direct intuition could be extended to the very essence of motion. That is why to his question: "quis ignorat quid sit motus?" one is justified in answering: "Descartes." Pasteur's dictum: "Je plains les gens qui n'ont que des idées claires", is especially applicable to the realm of Nature where things are essentially obscure. (50)

All this helps us to understand the solution to the antinomy mentioned in Chapter I between the ancient and modern concepts of motion. For Aristotle, as we saw, it was evident that the continuance of a body in motion demanded a cause and without this cause the body would come to rest. For Descartes, on the other hand, the principle of inertia was perfectly evident, (51) and according

to this principle the cessation of the motion of a body demands a cause, and without this cause the motion will continue <u>ad indefinitum</u>. The enigma of this striking paradox immediately vanishes when we call to mind that Aristotle and Descartes are talking about two different things. For Aristotle motion means a coming into being, and since nothing can bring itself into being, there must be a cause to explain the process of becoming: quidquid movetur ab alio movetur. For Descartes motion is a state, that is to say a kind of entity which will retain its existence until robbed of it by some cause. The principle of inertia has to do with mathematicized motion, that is to say with a motion that is infinitely uniform and rectilinear. This principle does not in any way involve the falsity of Aristotle's notion of motion. They belong to two different orders. Aristotle made no attempt to treat the mathematical aspect of local motion. It is extremely important to keep in mind that this mathematization is not a substitute for Aristotle's definition; it is a passing to an entirely different order. All too many historians make the mistake of treating Aristotle's <u>Physics</u> as though he were attempting to write a treatise on mathematical physics.

This question has an important corollary in the problem of prima via in St. Thomas' demonstration of the existence of God. In this demonstration motion is considered as a becoming and not as a state. And that is why it makes no sense to say that the argument is disproved by the principle of inertia. Obviously, if motion is conceived as a state there is no need to have recourse to an actio to explain it. This shows that the mathematization of the cosmos has a profound effect upon the problem of causality in the universe. But before turning to this question we must consider in a summary way how this mathematization effects a notion that is intimately connected with that of motion, namely time: "tempus habet fundamentum in motu." (52)

Contemporary Scholastics have insisted upon the difference between Aristotelian time and Einsteinian time to the extent of denying that they have anything more in common than the name. (53) They have furthermore claimed that what Einstein has to say about the impossibility of simultaneity at a distance has nothing to do with the time of which Aristotle speaks. We feel that this is extremely ambiguous. For the term "time" does not always have exactly the same meaning in Thomistic terminology.

In the first place, it signifies the duration of mobile beings, that is to say, the persistence in existence of beings whose existence is successive. But the "time" which Aristotle defines in the fourth book of the Physics does not exactly coincide with this primary notion, although it is essentially connected with it. For by defining time as the measure of motion according to a relation of priority and posteriority he makes it clear that he is speaking of an extrinsic determination of this duration in relation to a chosen standard, that is to say, of a measurement of this duration. Consequently, in so far as both Aristotelian time thus defined and Einsteinian time have to do with measurement they coincide. And we believe that what Einstein has to say about the impossibility of simultaneity at a distance applies to the time defined by Aristotle in the Physics. For we know of no way in which the measure of motion according to a relation of before and after can be determined so that distant events can be fixed as simultaneous. Of course, in so far as time is successive duration there is such a thing objectively as distant simultaneity even though that simultaneity cannot be determined by us.

But it would be illegitimate to conclude from this that the time defined by Aristotle in the Physics is same as the time of which Einstein speaks. For in the time of the Physics the notion of true physical motion is involved. Consequently, this time can truly be said to "flow" from past to future. In Relativity physics, on the contrary, the notion of motion has been emptied of its proper physical meaning. There is no true process, no becoming. Consequently, Einsteinian time does not really flow; it is a mere dimension. It is studied in terms of geometry.(54)

But even before the advent of the Theory of Relativity the notion of time had already undergone a profound transformation by the mathematization of nature.(55) We have already spoken of the symmetry of mathematical equations. The processes of classical dynamics are reversible, that is to say, if the velocities of the particles of a system should at any given moment be reversed the motion would proceed in accordance with the same equations in the reverse direction. In so far as the notion of time is concerned, this means that the equations of classical dynamics make no distinction between the positive and negative directions along the time axes. Professor Cunningham does not hesitate to say that in so far as time is determined mechanically, past and future are interchangeable.(56) And Lindsay and Margenau

write: "If equations predict future events they predict past ones as well."(57) Of course the physicist in order to discover "time's" arrow may have recourse to entropy-gradient, but even then the irreversibility is only highly improbable and never absolutely impossible.(58)

In Relativity physics the mathematical transformation of the notion of time becomes complete. It is assimilated to the notion of space - united with space as a dimension in the four dimensional continuum called space-time which, as we saw in the last Chapter, may be cut up in different ways according to the position and velocity of the individual observer. Time then becomes "the totality of possibilities of relative temporal position of events."(59) To quote Eddington once again:

> In the four-dimensional world ... the events past and future lie spread out before us as in a map. The events are there in their proper spatial and temporal relation; but there is no indication that they undergo what has been described as 'the formality of taking place,' and the question of their doing or undoing does not arise. We see in the map the path from past to future or from future to past; but there is no sign-board to indicate that it is a one-way street. Something must be added to the geometrical conceptions comprised in Minkowski's world before it becomes a complete picture of the world as we know it. We may appeal to consciousness to suffuse the whole - - to turn existence into happening, being into becoming. But first

let us note that the picture as it stands is entirely adequate to represent those primary laws of Nature which, as we have seen, are indifferent to a direction of time.(60)

In this spatialization of time, mathematical physics has achieved the goal at which it has aimed from the beginning - - the transformation of all sensuous and intuitive heterogeneity into pure homogeneity. The first step in this transformation was the homogenization which gradually emptied external experience of its proper and specific content. But even when this had been accomplished there still remained untouched the "form of the inner sense" - - the process of duration which is so intimately connected with internal experience. Through the spatialization of time this last barrier of specific experiential content was broken down. Speaking of this transformation Cassirer writes:

> This transformation of the time-value into an imaginary numerical value seems to annihilate all the 'reality' and qualitative determinateness, which time possesses as the 'form of the inner sense', as the form of immediate experience. The stream of process, which, psychologically, constitutes consciousness and distinguishes it as such, stands still; it has passed into the absolute rigidity of a mathematical cosmic formula. There remains in this formula nothing of that form of time, which belongs to all our experience as such and enters as an inseparable and necessary factor into all its content. But, paradoxical as this result seems from the standpoint of this experience, it expresses only the course of mathematical and physical objectification, for, to estimate it correctly from the epistemological standpoint, we must understand it not in its mere result, but as a process, a method. In the resolution of subjectively experienced qualities into pure objective numerical determinations, mathematical physics is bound to no fixed limit. It must go its way to the end; it can stop before no form of consciousness no matter how original and fundamental; for it is precisely its specific cognitive task to translate everything enumerable into pure number, all quality into quantity, all particular forms into a universal order and it only 'conceives' them scientifically by virtue of this transformation. Philosophy would seek in vain to bid this tendency halt at any point and to declare ne plus ultra. The task of philosophy must rather be limited to recognizing fully the logical meaning of the mathematical and physical concept of objectivity and thereby conceiving this meaning in its logical limitedness. (61)

Once again it is important to recognize this spatialization of time as an attempt of the mind to triumph over its greatest enemy: potentiality. Designated points in space are all actual, and when time is homogenized with space, t1, t2, t3, etc. become but a series of actual "nows". Perhaps it is legitimate to see in this spatialization of time a striving of the human intellect towards the duration of perfect actuality that is proper to pure Intellect.(62) But this attempt only results in the destruction of time:

> ... si le devenir doit se transformer en être (selon M. Einstein), au point que l'acte de se produire, pour un événement, devient une simple formalité dénuée d'importance (selon M. Eddington), si la succession n'est qu'une illusion (selon M.H. Marais) et si tout système physique constitue une entité privée de changement (selon M. Cunningham), cela ne peut signifier qu'une chose: l'abolition et la disparition du temps. Aussi M. Cunningham n'hésite-t-il point à parler de l'univers non-temporel de Minkowski. (63)

The destruction of mobility in the universe has many far-reaching consequences, but perhaps the most significant from the point of view of science, which is a knowledge of things in their causes, is its effect upon causality. In the second book of the <u>Physics</u> Aristotle and St. Thomas place considerable emphasis upon the fact that the science of nature must study its object from the point of view of all of the four fundamental types of causality: efficient, final, formal and material.

> Dicit ergo primo quod cum quatuor sint causae, sicut supra dictum est, ad naturalem pertinet et omnes cognoscere et per omnes naturaliter demonstrare, reducendo quaestionem propter quid in quamlibet dictarum quatuor causarum, scilicet formam, moventem, finem et materiam. (64)

The reason for this is fairly obvious: there is an analytical connection between mobility, the formal object of the science of nature, and quadruple causality.

> Necesse est autem quatuor esse causas. Quia cum causa sit ad quam sequitur esse alterius, esse eius quod habet causam, potest considerari dupliciter: uno modo absolute, et sic causa essendi est forma per quam aliquid est in actu; alio modo secundum quod de potentia ente fit actu ens. Et quia omne quod est in potentia, reducitur ad actum per id quod est actu ens; ex hoc necesse est esse duas alias causas, scilicet materiam, et agentem qui reducit materiam de potentia in actum. Actio autem agentis ad aliquid determinatum tendit, sicut ab aliquo determinato principio procedit: nam omne agens agit quod est sibi conveniens; id autem ad quod tendit actio agentis, dicitur causa finalis. Sic igitur necesse est esse causas quatuor. Sed quia forma est causa essendi absolute, aliae vero tres sunt causae essendi secundum quod aliquid accipit; inde est quod in immobilibus non considerantur aliae tres causae, sed solum causa formalis. (65)

The last lines of this passage throw great light upon the effect that the mathematical transformation of physics has upon causality. The student of nature as long as he stays within his own field is bound to reduce natural phenomena to all of their four causes: "In naturalibus reddendum est propter quid penitus."(66) But unable to discover any universal and necessary <u>propter quid</u> for experimental propositions he is forced to have recourse to mathematics. Since mathematics, however, is a world of immobility, the only type of <u>propter quid</u> he can borrow from it is the unique type that is proper to it: <u>propter quid</u> in the line of formal causality.

In a passage immediately preceding the one just quoted St. Thomas gives an example of what he means by formal causality:

> quandoque enim propter quid reducitur ultimo in quod quid est, idest in definitionem, ut patet in omnibus immobilibus, sicut sunt mathematica; in quibus propter quid reducitur ad definitionem recti vel commensurati vel alicuius alterius quod demonstratur in mathematicis. Cum enim definitio recti anguli sit, quod constituatur ex linea super aliam cadente, quae ex utraque parte faciat duos angulos aequales; si quaeratur propter quid iste angulus sit rectus, respondetur quia constituitur ex linea faciente duos angulos aequales ex utraque parte; et ita est in aliis. (67)

It is clear that the only type of causality that can be found in mathematical physics is a kind of formal causality consisting in an expression of the metric coherence of phenomena. This metric coherence constitutes what is known as the causal structure or world occurrences. It is true that physicists may speak in terms which seem to indicate other types of causality. They may for example, use the expression "efficient causality", but in doing so they merely refer to a relation between the states of physical systems at different points of time, which are connected in such a way that, given the determination of the state of the system at any one point of time, its state at any designated future point of time can be logically deduced.(68) St. Thomas brings out the incompetence of

mathematics in the field of efficient causality:

> Mathematica accipiuntur ut abstracta secundum rationem, cum tamen non sint abstracta secundum esse. Unicuique autem competit habere causam agentem, secundum quod habet esse. Licet igitur ea, quae sunt mathematica, habeant causam agentem; non tamen secundum habitudinem, quam habent ad causam agentem, cadunt sub consideratione mathematici. Et ideo in scientiis mathematicis non demonstratur aliquid per causam agentem.(69)

In pre-Relativity physics the mathematization of the cosmos had already resulted in the disappearance of true efficiency from the concept of efficient causality, but in Relativity physics this effacement is made even more complete. For now, the concept of force, for example, is completely absorbed into a system of determinations bound together by mathematical relations implemented by the differential and tensorial calculus, etc.

In somewhat the same way, physicists often speak of matter, but their matter is far from being the material cause of which Aristotle speaks. It is something that is completely actual and not a potential principle of becoming. In fact, in Relativity physics matter becomes so formalized that it is absorbed into isotropic space. On the other hand it must be noted that if matter is formalized, it is also true to say that the formal cause is materialized.

That is to say, the formal cause that is treated of in mathematical physics is not the proper specific formal cause which reveals the nature of things in their heterogeneous interiority, but a homogenized formal cause of spatial relations.(70)

In insisting upon the necessity of studying nature in terms of all four causes, Aristotle and St.Thomas place special emphasis upon the importance of final cause. "Et haec species causae potissima est inter alias causas: est enim causa finalis aliarum causarum causa."(71) In fact, after explaining in a general way how nature involves all four types of causality, they single out only final causality for particular attention. The whole last part of the second book of the <u>Physics</u> is devoted to a study of it, and to an insistence of its prime importance in the study of nature. Yet of all the causes that disappear in the mathematization of the cosmos, this is perhaps the type that is most efficaciously and most completely effaced. One looks in vain for anything that even remotedly corresponds to finality in mathematical physics. And the fundamental reason for this has already been pointed out in Chapter VI: since there is no good in mathematics, there can be no final causality:

> Ex hoc enim quod finis non potest esse in rebus immobilibus, videtur procedere quod in scientiis mathematicis quae abstrahunt a materia et motu, nihil probatur per hanc causam, sicut probatur in scientia naturali, quae est de rebus mobilibus, aliquid per rationem boni. Sicut cum assignamus causam quare homo habet manus, quia per eas melius potest exequi conceptiones rationis. In mathematicis autem nulla demonstratio fit hoc modo, quod hoc modo sit quia melius est sic esse, aut deterius si ita non esset. Puta si diceretur quod angulus in semicirculo est rectus, quia melius est quod sic sit quam quod sit acutus vel obtusus. Et quia posset forte aliquis esse alius modus demonstrandi per causam finalem, puta si diceretur, si finis erit, necesse est id quod est ad finem praecedere: ideo subjungit, quod nullus omnino in mathematicis facit mentionem alicuius talium pertinentium ad bonum vel ad causam finalem. Propter quod quidam sophistae, ut Aristippus, qui fuit de secta Epicureorum, omnino neglexit demonstrationes quae sunt per causas finales, reputans eas viles ex hoc quod in artibus illiberalibus sive mechanicis, ut in arte 'tectonica,' idest aedificatoria, et 'coriaria,' omnium rationes assignantur ex hoc quod est aliquid melius vel deterius. In mathematicis vero, quae sunt nobilissimae et certissimae scientiae, nulla fit mentio de bonis et malis.(72).

From all this it follows that it is entirely illegitimate for critics to reproach scientists as some modern Scholastics have done, for failing to take all types of causality into consideration. The very nature of his science makes it impossible for the mathematical physicist to consider anything but formal causality. And it is important for the scientist to be aware of his own limitations, so that he will not, for example, confuse his substitute for efficient causality with true efficient causality. There is particular danger of this happening with this type of causality since it is the best known and the most manifest to the mind.

At first sight it might appear that this banishment of causality from the cosmos might make the physico-mathematical world like Malebranche's world of occasionalism. As a matter of fact, there is only a surface likeness between the two. In a deeper sense they are opposed. For in the world of Malebranche it is necessary to have constant recourse to God, since every event is the occasion of His action. In the physico-mathematical world, on the other hand, God is completely dispensed with; there is no need to go to Him at all; nor is it even possible to go to Him. Because of its rationality, its ever increasing unity and its immutability, the physico-mathematical world is more like the Parmenedian sphere.

This analysis of the effects of the mathematical transformation of the cosmos might go on interminably. We might for example show that it destroys not only the becoming of the universe, but in a certain sense even its

being. For as we saw in Chapter VI, mathematics prescinds from existence , and the only meaning that being has in the physico-mathematical world is the occupation of a "place" in a certain order, in a space-time scheme. In this sense, Bergson is correct in saying that in modern science " l'existence concrète des phénomènes de la nature tend à s'évanouir . . en fumée algébrique." (73) We might also show how the concept of substance is transformed into the notion of persistent system. (74) But we feel that enough has already been said to show that the nature of which the mathematical physicist speaks is not the nature that is defined by Aristotle and St. Thomas in the second book of the Physics as a principle of motion and of rest and as a "ratio" or rational principle put into things which directs them in their striving for ends. (75) The nature of the mathematical physicist is, as Eddington has remarked, "only an empty shell." (76) In other words, as we have already remarked, in order to explain nature the physicist has found it necessary to destroy it.

> Obéissant aux deux tendances, nous avons, de théorie en théorie, et d'identification en identification, fait complètement disparaître le monde réel. Nous avons d'abord expliqué, c'est-à-dire nié le changement, identifiant l'antécédent et le conséquent, et la marche du monde s'est arrêtée. Il nous restait un espace rempli de corps. Nous avons constitué

> les corps avec de l'espace, ramené les corps à l'espace, et les corps se sont évanouis à leur tour. C'est le vide, 'rien du tout', comme dit Maxwell, le néant. Car le temps et l'espace se sont dissous. Le temps, dont le cours n'implique plus de changement, est indiscernable, inexistant; et l'espace, vide de corps, n'étant plus marqué par rien, disparaît aussi. (77)

It need hardly be pointed out, of course, that the great loss resulting from this destruction of nature has rich compensations that are daily becoming more apparent. For even though in destroying nature we destroy the intelligence that Aristotle saw in it and rob it of its seeking for ends, at the same time we make nature more intelligible than it is by injecting our own intelligence into it. The mathematical representation of nature is an improvement of it, in the sense in which a mathematical line is an improvement of a physical line. We construct a model for nature, and this construction forces nature to yield up its secrets. (78)

From all that has been said about the nature of this rationally constructed physico-mathematical world it is clear why it should inevitably appear to Sir James Jeans as a world consisting of pure thought, the thought of a mathematical thinker. (79) But it should also be clear why it is illegitimate for him to conclude that the objective

universe, that is to say, the absolute world condition, is nothing but pure thought and the product of a pure mathematician acting as a pure mathematician. For even though a physico-mathematical world may tend towards the absolute world condition as though towards its asymptote, a pure mathematician acting purely as such, neither would nor could create a physical universe. As Bridgman has remarked, "What Jeans might have said is that man is a mathematician, and reflected that it is no accident that he forms nature in his own image." (80).

A SHADOW WORLD OF SYMBOLS

1. The Nature of Symbolism.

Having seen how the mathematician transforms the physical universe into a new world of his own making, we must now try to analyse briefly the nature of this new world. All the best philosophers of science are now unanimous in characterizing the physico-mathematical world as a symbolic universe. Sir Arthur Eddington, for example, has, as is well-known, repeatedly described it as "a shadow world of symbols." (1) We believe that if this phrase be rightly understood, it brings out with great accuracy the true nature of the universe constructed by mathematical physics. Let us try to determine what precise meaning must be given to it.

In the first place, it is necessary to fix upon the meaning of the word "symbol". And here we come upon a great lack of unanimity. All will agree that in its primitive meaning the term "Symbolon" signifies a mark or emblem or index employed to designate something, and that consequently every symbol is a sign. But is every sign a symbol? Not a few authors seem to think so. Thus R. B. Perry writes: "Any datum may be a symbol if it means something or operates

as a sign." (2) And he goes on to explain that such data may include:

> ... conspicuous features of nature, monuments, written or spoken words, small images or familiar objects easily duplicated or distributed. Any of these is a symbol provided it directs expectation or interest to something other than itself. Symbolism is, then, the study of the part played in human affairs by all these signs and symbols, especially their influence on thought. Symbols direct and organize, record and communicate. For words, arrangements of words, images, gestures, and such representations as drawings or mimetic sounds we use the term symbols.

To make the sign and the symbol coterminous in this way is to rob symbolism of all precise meaning. (3) And the ordinary usage of the term seems to insist upon a precise meaning. Clouds are considered to be signs of rain, and smoke a sign of fire, but they are never referred to as symbols. It is necessary, therefore, to try to press the meaning of the term a bit closer.

In the first place, the examples just referred to make it clear that purely natural signs (i.e. those which have a natural and real connection with the thing signified, prior to any connection established by the mind) must be excluded from the notion of symbolism. To apply the term "symbol" to a natural sign is actually a distortion of language. (4) In other words, symbols are necessarily arbitrary or conventional signs, i.e. signs in which the connection with the thing signified is not found in nature as such, but created by the mind. This does not, however, exclude the possibility of there being in nature a foundation for the connection established by the mind.

Having made this important distinction we are faced with this problem: are all conventional signs necessarily symbols? Once again, a good many authors seem to think so — at least if it be question of the most important type of conventional signs, namely those which make up language. Miss Stebbing, for example, tells us that "a word is a special kind of sign called a symbol." (5) And again she writes: "A sign consciously designed to stand for something will be called a symbol." This opinion seems to be shared by Professor Whitehead: "The word symbolizes the thing. Language almost exclusively refers to presentational immediacy as interpreted by symbolic reference." (6) This tendency to make all language and even all thought symbolic (7) makes it difficult to attach any precise and proper meaning to the term.

Since the word is currently employed in such a loose way it is necessary for us to try to fix upon the particular meaning it is to have for us in this discussion of the symbolism of science. Its etymology provides us with a helpful suggestion. The Greek words συν and βαλλειν mean "to throw together". Now, whatever may have been the original

historical usage of these words which gave rise to the term we are analyzing, it is clear that they suggest a collection of things among which there is no strict natural unity — an aggregate whose principle of unification is purely extrinsic. If we keep this in mind we shall be able to see why Saint Thomas, in his Commentary on the Sentences (8) gives this description of the symbol: "...nomen symboli similitudinem et collectionem importat." It would seem that a symbol must be defined as an artificial sign established to signify a determined object that is one only according to the mind. In order to bring out the meaning of this definition, it is necessary to see the difference between a symbol and a name. (9)

In his Commentary on the Perihermeneias, St. Thomas explains the important distinction between the name and the infinite name:

> Deinde cum dicit (Aristoteles) "non homo vero non est nomen" etc., excludit quaedam a nominis ratione. Et primo, nomen infinitum; secundo casus nominum; ibi: "Catonis autem vel Catoni" etc. Dicit ergo primo quod "non homo" non est nomen. Omne enim nomen significat aliquam naturam determinatam, ut "homo"; aut personam determinatam, ut pronomen; aut utrumque determinatum, ut Sortes. Sed hoc quod dico "non homo", neque determinatam naturam neque determinatam personam significat. Imponitur enim a negatione hominis, quae aequaliter dicitur de "ente" et "non ente". Unde "non homo" potest dici indifferenter, et de eo quod non est in rerum natura; ut si dicamus, "chimaera est non homo", et de eo quod est in rerum natura; sicut cum dicitur, "equus est non homo". Si autem imponeretur a privatione, requireret subiectum ad minus existens: sed quid imponitur a negatione, potest dici de ente et de non ente, ut Boethius et Ammonius dicunt. Quia tamen significat per modum nominis,

quod potest subiici et praedicari requiritur ad minus suppositum in apprehensione. Non autem erat nomen positum tempore Aristotelis sub quo huiusmodi dictiones concluderentur. Non enim est oratio, quia pars eius non significat aliquid separata, sicut nec in nominibus compositis; similiter autem non est negatio, id est oratio negativa, quia huiusmodi oratio superaddit negationem affirmationi, quod non contingit hic. Et ideo novum nomen imponit huiusmodi dictioni, vocans eam nomen "infinitum" propter indeterminationem significationis, ut dictum est. (10)

It is clear from this passage that the name must signify something that is one by nature. Because of its indetermination the infinite name does not signify something that is one by nature. Because it is a pure negation, it does not even have the determination of privation which must always be in the same genus as the thing of which it is the negation.

Nevertheless, in spite of the indetermination of the infinite name, it has a significance; in some way it signifies something that is one. St. Thomas explains this in his Commentary on the Second Book of the Perihermeneias:

...nomen infinitum quodam modo significat unum. Non enim significat simpliciter unum, sicut nomen finitum, quod significat unam formam generis vel speciei aut etiam individui, sed in quantum significat negationem formae alicuius, in qua negatione multa conveniunt, sicut in quodam uno secundum rationem. "Unum" enim eodem modo dicitur aliquid, sicut et "ens"; unde sicut ipsum "non ens" dicitur "ens", non quidem simpliciter, sed secundum quid, idest secundum rationem, ut patet in IV Metaphysicae, ita etiam negatio est unum secundum quid, scilicet secundum rationem. Introducit autem hoc, ne aliquis dicat quod affirmatio, in qua subiicitur nomen infinitum, non significat unum de uno, quasi nomen infinitum non significet unum. (11)

There is, then, a unity in the infinite name — a unity that is founded upon the unity of the thing negated. It is possible to predicate the infinite name of anything except the thing negated. But it is important to note that even though the infinite name can be applied to any one of the things that fall within the class which includes everything except the thing negated, it does not properly signify any one of them. Nor does it signify the class of all those things, as a genus signifies everything that falls within it. The infinite name is not a collective noun; there is a class of things to which it may be applied, but it does not express any of them.

Now all this has a very important bearing upon the nature of the symbol. For we believe that the symbol falls somewhere between the name and the infinite name. The name may signify a collection, but it never signifies a collection qua collection, i.e. as a mere accidental union. The infinite name on the other hand, though it may be applied to a collection, does not formally signify a collection, because of its indetermination. The symbol alone signifies a collection formally as a collection. Unlike the universal name, the symbol does not abstract from multiplicity; in fact, it is precisely the multiplicity that it signifies. Like the name and unlike the infinite name, the symbol signifies a determined object; but unlike the name and like the infinite name it does not signify anything that is one by nature.

A simple example will serve to clarify the issue. Is the sign "3" a symbol? That depends upon what it is taken to signify. If it represents the three which is a numbering number, a pure aggregate, a collection of 1+1+1, it is a symbol in the strict sense of the term. If however, it is employed to signify numbered number, or predicamental number, which is not three ones, but one three, because the three have a common physical genus and constitute an unum per se, it is not a symbol in the strict sense, but merely a convenient substitute for the name "three". In other words, in order for a sign to be a symbol it must signify something that possesses only logical unity; it must signify a collection in its pure collectivity. If Russell's definition of number as "the class of all classes that are similar to it" were correct, all numbers would be nothing but symbols.(12)

The transcendent terms of logic used so extensively in the Priora Analytica are illustrations of the symbol, for they signify at the same time everything and nothing. Of them St.

Albert the Great writes: "Ideo terminis utimur transcendentibus, nihil et omnia significantibus. Nihil dico, quia nullam determinant materiam. Omnia vero dico significantibus: quia omnibus materiis sunt applicabiles, sicut sunt a, b, c."(13)

It is clear that a symbol is something quite different from a mere abbreviation. An abbreviation has only the outward appearance of a symbol, and is in reality nothing but a convenient substitute for a name. Vossler's remark that the language of mathematics is pronominal, must be rightly understood. If it means that the language of mathematics consists in signs that substitute for names, it is true of traditional mathematics. If it means that mathematical signs stand in the place of names in the sense of signifying collections which names cannot signify, it is true only of the dialectical part of modern mathematics.

Nominalism is at bottom nothing but a denial of the important distinction we have just drawn between name and symbol. By a strange paradox, it is a rejection of the name in the true sense of the term, for if all names signify nothing but a collection of singulars, if "being" for example, means nothing but the whole collection of beings,(14) all names can be nothing but symbols.

If names in the last analysis were only symbols, and if reality were such that it could be represented and expressed only by means of symbols, then there would be no true natures in existence and all things would constitute nothing more than an accidental collection without any intrinsic or essential unity. Universal mobilism which denies all determined natures must necessarily conceive all language in terms of pure symbolism. That is why Whitehead, for whom reality is a process, is logical in holding that all names are symbols. And in this connection it is interesting to note that Cratylus, who pushed universal mobilism to its absolute extreme, held that words should not be employed at all, and had recourse to the movement of a finger in order to express himself. (14e)

And now, having fixed upon the precise meaning to be attached to the term "symbol" let us try to see in what sense the physico-mathematical world can be truly called a world of symbols.

2. Symbolism and Mathematical Physics.

It has long been customary for scientists with a penchant towards scientism to ridicule the philosophical sciences for their "verbalism". This attitude has been based upon the assumption that philosophy deals essentially with vague

and shadowy concepts which have no definite counterparts in reality, and that only in experimental science are things laid hold of in their true objective natures. The new self-revelation that has occurred in the realm of experimental science has done much to mitigate this naive view. It has become increasingly evident that experimental science, in so far as it attempts to employ names, is the most verbalistic of all the sciences. The philosopher can define with precision the fundamental concepts which he employs such as substance, accident, motion, time, etc., he can set forth the nature, the quod quid est of things. The physicist, on the other hand, is hard put to it to define what he means by even the simplest and most basic notions that enter into his science, such as body, energy, matter, mass. As we shall see presently, every attempt to define these notions involves him in an endless circle from which there is no exit.

The fact of the matter is that experimental science is essentially nominalistic in the sense defined above. By its very nature it is committed to the use of symbols rather than of names. And nothing could be more striking than the contrast between the vagueness of scientific language when interpreted in terms of names, and its precision when interpreted in terms of symbols.

It has taken science a long time to realize this. Because experimental science necessarily tends towards the condition of science in the strict sense of the term, it was only natural that in its origin and development it should aspire towards a state in which its language could consist of names in the proper sense of the word. The great mistake of scientists has been to believe that this state was already a fait accompli. This was characteristic of classical physics. It was particularly characteristic of a view that was current in the nineteenth century, especially among such men as T.H. Huxley, which held that science is nothing but organized and refined common sense, and that its language is only the ordinary language of common sense rendered more precise and accurate.

This view is no longer popular. The cleavage between science and common sense has become so profound that it has caused dismay not only in the minds of laymen who are interested in trying to find out what science is about, but even in the minds of the scientists themselves who desire to comprehend the meaning of their science. How for example, can Schrodinger's oscillations signs operating in multi-dimensional space be expressed in the ordinary language of common sense? We believe that this state of affairs can be understood only by

becoming conscious of the fact that experimental science is essentially symbolic, that is language is not a language of names, but a language of symbols. Let us try to see why this is so.

As Saint Thomas points out in the lines cited above from the Commentary on the Perihermeneias, a name in the strict sense of the term always stands for a definite nature (or person); it indicates something that is an unum per se — a quod quid est. Now we have seen that though experimental science tends towards laying hold of natures, it necessarily falls short of its goal. Pure induction by enumeration can never of itself disclose a nature that is strictly one. That is why from the very start, experimental science is doomed to deal with collections, no matter how it may strive to rise above their multiplicity and arrive at the unity of a strict nature. What the nominalists taught about knowledge is perfectly correct when applied to experimental science. "Science, writes Weyl, "concedes to idealism that this its objective world is not given but only propounded (like a problem to be solved) and that it can be constructed only by symbols." (15) But that is not all.

In this striving to rise above multiplicity, it is forced to operate upon nature. This operation, as we saw in

Chapter IV, never reveals the objective nature of things; its results depend essentially upon the whole collection of concrete elements which entered into it. Since, then, the definitions of physics can be nothing but operational, none of its notions can stand for a strictly unified objective nature. They can mean nothing more than the whole collection of elements entering into the operations from which they derive; they can signify only a collection qua collection, that is to say an accidental aggregate of nature plus a multiplicity of operational elements, all of which have a unity that comes from the mind alone. Symbols alone, and not names can stand for collections of this kind. That is why all of the language which physics uses, whether it consist of words or any other type of signs, is necessarily symbolic. As a consequence, when the physical world is identified with the world in se it is impossible to escape transcendental symbolism. Likewise to look upon these signs as names is to confuse art with nature, subjective construction with objective reality what is one only in the mind with what is one by nature; it is to fall into a very pernicious type of idealism, as we shall point out in a later context.

It should be evident from the foregoing that science is symbolic not merely in its more theoretical superstructures

(16)

but in the very results of its primary contact with nature. Lindsay and Margenau bring this out in the following passage:

> It thus appears that the symbol here is but a shorthand expression for the results of a given operation leading to the assignment of a number value to the symbol. Instead of describing in words the entire series of acts involved in the setting of the tubes and the reading of the scale, the whole matter is summed up in the one phrase: measurement of P. Is this then all that there is to the meaning of symbolism? If it were necessary to associate a symbol with the results of every single physical operation the description of these operations might indeed be simplified but it would not constitute what we now consider theoretical physics. The real power of symbolism in physics first becomes clear when we envisage the possibility of letting a symbol stand for a concept which is, so to speak, the synthesis of the results of a whole set of operations which may appear to be superficially dissimilar, but are assumed by the physicist to have a common element. (17)

It should also be evident from the foregoing that the symbolic character of science does not consist in its abstractness, as some seem inclined to believe. The language of the philosophical sciences is abstract, but it is not essentially symbolic. There is, as we observed earlier in this Chapter, a profound difference between symbols and names which stand for abstract natures. Duhem has endeavored to clarify this distinction in La Théorie Physique:

Prenons une loi de sens commun, une des plus simples comme une des plus certaines: Tout homme est mortel. Cette loi, assurément relie entre eux des termes abstraits, l'idée abstraite d'homme en général, et non l'idée concrète de tel ou tel homme en particulier; l'idée abstraite de la mort et non l'idée concrète de telle ou telle forme de la mort; c'est en effet à cette seule condition de relier des termes abstraits qu'elle peut être générale. Mais ces abstractions ne sont nullement des symboles théoriques; elles extraient simplement ce qu'il y a d'universel dans chacun des cas particuliers auxquels la loi s'applique; aussi, dans chacun des cas particuliers où nous appliquons la loi, trouverons-nous des objets concrets où seront réalisées ces idées abstraites; chaque fois que nous aurons à constater que tout homme est mortel, nous nous trouverons en présence d'un certain homme particulier incarnant l'idée générale d'homme, d'une certaine mort particulière impliquant l'idée générale de mort. . .

Il n'en est plus de même pour les lois de la Physique. Prenons une de ces lois, la loi de Mariotte, et examinons-en l'énoncé, sans nous soucier, pour le moment, de l'exactitude de cette loi. A une même temperature, les volumes occupés par une même masse de gaz sont en raison inverse des pressions qu'elle supporte; tel est l'énoncé de la loi de Mariotte. Les termes qu'elle fait intervenir, les idées de masse, de température, de pression, sont encore des idées abstraites; mais ces idées ne sont pas seulement abstraites, elles sont, de plus, symboliques, et les symboles qu'elles constituent ne prennent un sens que grâce aux théories physiques. Plaçons-nous en face d'un cas réel, concret, auquel nous voulons appliquer la loi de Mariotte; nous n'aurons pas affaire à une certaine température concrète réalisant l'idée générale de température, mais du gaz plus ou moins chaud; nous n'aurons pas devant nous une certaine pression, mais une certaine pompe sur laquelle on a pesé d'une certaine manière. Sans doute, à ce gaz plus ou moins chaud correspond une certaine température, à cet effort exercé sur la pompe correspond une certaine pression; mais cette correspondence est celle d'une chose signifiée au signe qui la remplace, d'une réalité au symbole qui la représente. Cette correspondance n'est nullement immédiate; elle s'établit au moyen d'instruments, par l'intermédiaire souvent très long et très compliqué des mesures; pour attribuer une température déterminée à ce gaz plus ou moins chaud, il faut recourir au thermomètre; pour évaluer sous forme de pression l'effort exercé par la pompe il faut se servir du manomètre et l'usage du thermomètre, l'usage du manomètre, impliquent, nous l'avons vu au chapitre précédent, l'usage des théories physiques.(18)

The symbolism of experimental science may take on various forms. In the first place, it may take the form of words. But words serve the purpose of symbolism very inadequately. For they are primarily designed to signify natures. That is why their use as symbols presents the constant danger of their being mistaken for names, and it is well known how many scientists and philosophers have fallen prey to this danger. It is a sign of extreme naiveté on the part of philosophers to rejoice over the fact that certain terms, such as "substance", "matter", "body", etc. are shared in common by both philosophy and science, and to believe that it is legitimate for them to incorporate into their philosophical system these notions as they are understood in science. Moreover, there is an isolation about words which makes them incompetent to express the interconnectedness that science tries to achieve. Because therefore experimental science must necessarily speak in symbols and because words serve this purpose so inadequately, there is a natural tendency, especially in mathematical physics, to draw away as completely as possible from words, to have recourse to other signs, and to construct a language of its own which defies all translation into the ordinary language of common sense — much to the discomfiture of the popularizers of science.

A second form which scientific symbolism may take is that of models. These serve the purpose of symbolism somewhat more effectively than mere words. The danger of their being mistaken for natures in the strict sense of the word is to some extent diminished. Besides this they have the advantage of giving a direct and immediate expression of interconnectedness. But they are still extremely inadequate. For one thing, because of their direct connection with intuition they all too easily give the impression that they represent nature in its pure objectivity, independently of the manufacturing processes of the scientist who works upon nature. This easily leads to the delusion that they are direct and immediate copies, or pictures, or at least schemas of objective natures. That the classical physicists labored under this delusion constantly is a matter of history, and it is now generally recognized how great an obstacle this delusion placed in the path of scientific progress. Models are not well adapted to symbolize the true collections that are involved in the notions of experimental science. Moreover, their immediate connection with intuitive schemas makes their capacity for expres-

sing interconnectedness extremely limited. For these reasons science has in recent years tended to free itself more and more from the restrictions of these models. As we intimated in the last Chapter, however, since experimental science deals with the realm of the physical, it is doubtful if it will ever be able to dispense entirely with the sensible support that such sensible constructs provide. But it is extremely important to remain conscious of the fact that they are mere constructs, mere symbols, and to be aware of what they actually signify.

The next step in science's search for adequate symbolic forms has been the use of what have sometimes been called pseudo-sensible constructs. (19) These constructs include such entities as atoms, electrons, etc. Though some of these constructs may be said to be closer to nature than others, none of them has any immediate correspondence with anything in reality. As Professor Margenau points out, their value has no relation to their mode of existence. There is less resemblance between them and objective entities than there is between clues and criminals. As Thompson has remarked: "We may well say of them what Hobbes said of words: 'They are wise men's counters, they do but reckon by them, but they are the money of fools.'" (20) Constructs of this kind may be generated by science *ad libitum*, for since they are merely counters by which to reckon, their nature and validity is essentially functional. And their function is to construct and shape a body of doctrine which will explain natural phenomena. Though they do not correspond to anything encountered in experience, they serve to give systematic form to the data of experience. As Cassirer has observed: "thought only separates itself from intuition in order to turn to it with new instruments, thereby to enrich it in itself... They render insight into relations possible, and guarantee it, although they themselves can never be perceived after the fashion of isolated objects." (21) They differ from the data of experience by their essential interconnectedness. Because of this interconnectedness they can serve to erect a coherent organism which can substitute for the disconnected mass of experiential data and thus rationalize it. In other words, by mapping the elements of nature which by themselves appear as incoherent, contingent and unpredictable upon constructs, science is able to create a symbolic system which is more coherent, more necessary, more rational than nature. More or less arbitrary rules of combinations may be employed in relation to these constructs which gives great freedom for the mind to reason about them and which gives great pliancy to the constructional system.

The results of this rational transformation are ultimately mapped back upon nature in such a way as to predict phenomena. (22)

In this way science succeeds in building up a world of its own . . a world that is rationally organized, and intrinsically coherent, and all the elements of which mutually imply each other. The validity and significance of the individual constructs which go to make up this symbolic system cannot be established by themselves alone by appealing to experience. In so far as the notion of verification can be applied to them, it cannot mean the establishment of any direct referends in reality. Their validity and significance is derived from the role that they play as members of a theoretical complex.

It is evident that these pseudo-sensible constructs go far beyond the strictly physical models in their capacity to serve as symbols. But in so far as they resemble in some respects these physical models they share to some extent in the limitations attached to the latter. Both types of constructs provide the sensible support that physical speculation needs. But though they may for a while stand the weight of speculation placed upon them, they tend eventually, as Jeans has remarked, "to break in our hands." (23)

That is why physics must reach beyond the limitations of these constructs to a more perfect type of symbolism.(24)

This more perfect type of symbolism is found in mathematics.(25) As is well known, mathematics, especially in its modern dialectical form, is admirably suited to play the role of symbolism. Its abstraction from existence, from nature, and from all specific substances, and its empty forms make it an apt instrument to signify collections and the relations among manifolds without signifying the nature of the <u>relata</u>. Through mathematical symbolism alone can the diverse phenomena of nature be reduced to a high degree of interconnectedness. That is why physics is learning to express itself more and more fully in the abstract forms of mathematics. One has only to recall Heisenberg's, Dirac's and Schrodinger's recent developments in quantum physics to realize how far this tendency has gone. As we have already remarked, sensible and pseudo-sensible constructs will never be completely dispensed with, but as Jeans has put it, they will remain mere parables - - mere clothing which we drape over our mathematical symbols.(26)

3. A World of Shadows.

"The frank realisation that physical science is

concerned with a world of shadows," writes Eddington, "is one of the most significant of recent advances. I do not mean that physicists are to any extent preoccupied with the philosophical implications of this. From their point of view it is not so much a withdrawal of untenable claims as an assertion of freedom for autonomous development."(27) Nothing could be more striking than the paradoxical fact that by attempting to introduce the brilliance of cartesian clarity everywhere in the physical world, science has made of it a world of shadows. We must now try to see why the world of physics has necessarily become a world of shadows and what some of the philosophical implications of this fact are.

The shadowy character of the physical world derives principally from its symbolic nature. But even independently of the use of symbols there are a number of reasons why the world with which physics deals can be truthfully called a world of shadows. To begin with, all human knowledge is by its very nature shadowy. For the human intellect is the lowest intellect that could possibly exist; it is essentially united with matter, and dependent upon it (at least extrinsically) for its functioning. As a consequence its realm of knowledge is at best a mere shadowland.

That is why Aristotle tells us that it is like the eyes of an owl which can see well only in the deep twilight and in the dark. And the more it attempts to penetrate into the realm of the sensible, the more does its knowledge become shadowy. Sense knowledge is truly an obscure knowledge. For it is at the utmost extreme of knowledge, where immateriality peters out into materiality, where the light of the intentional world is mingled with the darkness of the purely physical world. It is a very late twilight when darkness has almost entirely taken over, and when only obscure shadows can be seen. Now physics deals with everything in terms of sensible matter. Not only that, but it is the part of natural doctrine that is the farthest advanced in the direction of concretion, that is the most profoundly immersed in the obscurity of matter. That is why its object is essentially a shadowland.

The dialectical character of physics gives us another reason why it necessarily deals with shadows. For since it is a *scientia quia* and not a *scientia propter quid*, it can get at phenomena alone; it is restricted to mere appearances. The nature behind the appearances remains in the dark. In attempting to get at this nature, physics throws up a scaffolding against reality - - a scaffolding

which is like a shadow of reality, roughly, and sometimes grotesquely reflecting its outline. Though there is always some relation between the proportions of a shadow and the reality, this relation is not definite, particularly with regard to specific details. The relation between the world constructed by the physicist and the world of reality, is of this kind.

By the fact of its being subalternated to mathematics, the world of physics takes on an even stronger resemblance to a shadowland. For a shadow is something that reduces the heterogeneity of the object it represents to pure homogeneous exteriority. The qualitative is swallowed up in the quantitative. To be more specific, the mathematical line is a shadow of the physical line, and when the physicist studies the physical line in terms of the mathematical line, he is getting at reality only by means of its shadow.

But it is principally because of its symbolic character that the world of physics is a world of shadows. And the reason for this should be fairly evident. We have seen that symbols differ from names in that they do not stand for natures in the strict sense of the term. That is why when they are used as signs, the precise nature of the things signified remains blurred and hidden in the background. And no manipulations of symbols can make them emerge from this background.

As science perfects its symbolic forms, the physical world takes on more and more the character of a self-authenticating formal system in which the interrelatedness of nature's manifold is seized upon and reflected. The principal criterion for the use of these symbolic forms is not that they should individually have a direct correspondence with something intuitively given, but that they be able to fit coherently into the self-authenticating system. From one point of view the increasing perfection of the symbolic reflection of nature's interrelatedness throws greater light upon the <u>relata</u>, but from another point of view it makes them more like shadows.

Sir Arthur Eddington has laid great emphasis upon this point. In the introduction to <u>The Nature of the Physical World</u> he writes:

> Science aims at constructing a world which shall be symbolic of the world of commonplace experience. It is not at all necessary that every individual symbol that is used should represent something in common experience or even something explicable in terms of common experience. The man in the street is always making this demand for concrete explanation of the thing referred to in science, but of necessity he must be disappointed. It is like our experience in learning to read. That which is written in a book is symbolic of a story in real life. The whole intention of the book is that ultimately a reader will identify some symbol, say BREAD, with one of the conceptions of familiar life. But it is mischievous to attempt such identifications prematurely, before the letters are strung into words and the words into sentences. The symbol A is not the counterpart of anything in familiar life. To the child the letter A would seem horribly abstract; so we give him a familiar conception along with it.
> "A was an Archer who shot at a frog." This tides over his immediate difficulty; but he cannot make serious progress with word-building so long as Archers, Butchers, Captains, dance round the letters. The letters are abstract and sooner or later he has to realize it. In physics we have outgrown archer and apple-pie definitions of the fundamental symbols. To a request to explain what an electron really is supposed to be we can only answer. "It is a part of the A B C of physics. The external world of physics has thus become a world of shadows...
> It is difficult to school ourselves to treat the physical world as purely symbolic. We are always relapsing and mixing with the symbols incongruous conceptions taken from the world of consciousness. Untaught by long experience we stretch a hand to grasp the shadow, instead of accepting its shadowy nature. Indeed, unless we confine ourselves altogether to mathematical symbolism it is hard to avoid dressing our symbols in deceitful clothing. When I think of an electron there rises to my mind a hard, red, tiny ball; the proton similarly is neutral grey. Of course the colour is absurd - - perhaps no more absurd than the rest of the conception - - but I am incorrigible. I can well understand that the younger minds are finding these pictures too concrete and are striving to construct the world out of hamiltonian functions and symbols so far removed from human preconception that they

do not even obey the laws of orthodox arithmetic. For myself I find some difficulty in rising to that plane of thought; but Iam convinced that it has got to come.

Later in the same work he brings out this point more specifically in connection with his explanation of the cyclic method employed in physics. (28) All of the constructs out of which the structure of physics is formed, such as point-events, potentials, matter, etc. are definable and translatable only in terms of each other, not in terms of anything else, and in particular not in terms of any underlying reality that is independent of the mind of the scientist or the physical objects of the perceptual world. These constructs form a closed circle. By beginning at any point on this circle we may define any one of the members which form it in terms of the others, and from it deduce the others. But as we travel around the circle at no point do we make fresh contact with reality. At a certain point, e.g. "matter" we may think that we are talking about something which has a direct embodiment in the world of reality, but in point of fact, the "matter" that is dealt with in physics has no direct counterpart in nature. It is by working around this circle that we derive the physical laws.

In this way physics remains within its own domain; it constitutes a closed world of its own, and this world is but a shadowland reflecting the underlying reality which can never be made to emerge from its obscurity:

> And you can see how by the ingenious device of the cycle physics secures for itself a self-contained domain for study with no loose ends projecting into the unknown. All other physical definitions have the same kind of interlocking. Electric force is defined as something which causes motion of an electric charge; an electric charge is something that exerts something that produces motion of something that exerts something that produces . . <u>ad infinitum</u>. The supposed approach through the <u>physical world</u> leads only into the cycle of physics, where we run round and round like a kitten chasing its tail and never reach the world-stuff at all . . . However much the ramifications of the cycles may be extended by further scientific discovery, they cannot from their nature trench on the background in which they have their being - - their actuality. (29)

It is particularly in its use of the theory of groups that the physical world takes on the character of a world of shadows. As we saw in the last Chapter, it is possible to give an exact mathematical description of patterns, while the nature of the entities involved in them remain in the dark. "It (mathematics) dismisses the individual elements by assigning to them symbols, leaving it to non-mathematical thought to express the knowledge, if any, that we may have of what the symbols stand for . . .

Every path to knowledge of what lies beneath the structure is then blocked by an impenetrable mathematical symbol." (30)

All this discussion about the shadow world of physics calls to mind the famous shadows of the Platonic cave. In fact, the well-known passage from the *Republic* is so relevant here that we cannot refrain from quoting it:

> And now, I said, let me show in a figure how far our nature is enlightened or unenlightened:— Behold! human beings living in an underground cave, which has a mouth open towards the light and reaching all along the cave; here they have been from their childhood, and have their legs and necks chained so that they cannot move, and can only see before them, being prevented by the chains from turning around their heads. Above and behind them a fire is blazing at a distance, and between the fire and the prisoners there is a raised way; and you will see, if you look, a low wall built along the way, like the screen which marionette players have in front of them, over which they show the puppets.
> I see
> And do you see, I said, men passing along the wall carrying all sorts of vessels, and statues and figures of animals made of wood and stone and various materials, which appear over the wall? . . .
> You have shown me a strange image, and they are strange prisoners.
> Like ourselves, I replied; and they see only their own shadows, or the other shadows which the fire throws on the opposite wall of the cave.
> True, he said; how could they see anything but the shadows if they were never allowed to move their heads?
> And of the objects which are being carried in like manner they would only see the shadows?
> Yes, he said.
> So then, he said, the truth would be literally nothing but the shadows of images. (31)

All that has been said in the course of this study about the nature of experimental science makes it evident how much the scientist is like a prisoner in a dark cave. The very method to which he is committed are the chains which bind him and prevent him from turning his head and seeing reality in its objectivity. As Plato's observer saw both other shadows and his own thrown against the wall of the cave, so in the shadow world of physics the scientist sees both the shadows of objective reality and his own, but in this case the two are inextricably blended together.

The following parable brings out still further the similarity between the physicist and the cavedweller of Plato:

> An aged college Bursar once dwelt secluded in his rooms devoting himself entirely to accounts. He realised the intellectual and other activities of the college only as they presented themselves in the bills. He vaguely conjectured an objective reality at the back of it all - - some sort of parallel to the real college - - though he could only picture it in terms of the pounds, shillings and pence which made up what he would call "the common sense college of everyday experience." The method of account-keeping had become inveterate habit handed down from generations of hermit-like bursars; he accepted the form of accounts as being part of the nature of things. But he was of a scientific turn and he wanted to learn more about the college. One day in looking over his books he discovered a remarkable law. For every item on the credit side an equal item appeared somewhere else on

> the debit side. "Ha" said the Bursar, "I have discovered one of the great laws controlling the college. It is a perfect and exact law of the real world. Credit must be called plus and debit minus; and so we have the law of conservation of L s. d. This is the true way to find out things, and there is no limit to what may ultimately be discovered by this scientific method. I will pay no more heed to the superstitions held by some of the Fellows as to a beneficient spirit called the King or evil spirits called the university Commissioners. I have only to go on in this way and I shall succeed in understanding why prices are always going up."
> I have no quarrel with the Bursar for believing that scientific investigation of the accounts is a road to exact (though necessarily partial) knowledge of the reality behind them. Things may be discovered by this method which go deeper than the mere truism revealed by his first effort. In any case his life is especially concerned with accounts and it is proper that he should discover the laws of accounts whatever their nature. But I would point out to him that discovery of the overlapping of the different aspects in which the realities of the college present themselves in the world of accou ts, is not a discovery of the laws controlling the college; that he has not even begun to find the controlling laws. The college may totter but the Bursar's accounts still balance.(32)

However much symbols and shadows may cut off the scientific observer from reality their essential purpose is to unite him to it. For the nature of symbols is to signify something and the nature of shadow is to be a reflection of reality.(33) That is why, after having seen the nature of the physico-mathematical world, we must now

try to analyze its relation to the objective world. The nature of this relation has been more or less implicit in much that has been said thus far, and has, we feel, already begun to take on fairly definite outline. But it is of supreme importance for a right understanding of the validity of scientific knowledge to endeavor to make it as explicit as possible. That is not an easy thing to do, for it should be evident from all that has been said up to now that this relation is far from being the simple thing that the classical physicists and the majority of modern Scholastics have imagined it to be. We can only hope to treat the problem in its general aspects without descending to details.

CHAPTER ELEVEN

THE ABSOLUTE WORLD CONDITION

1. Isomorphism.

By the absolute condition of the universe is meant the objective world as it is itself — the world as it is contemplated by supramundane intelligences which do not have to depend upon the manifold subjective and relative conditions that necessarily accompany all knowledge derived through the senses, which are free of the barriers that result from the limitations of the human intellect, which do not have to probe the world with appliances that are within it, and a part of it, and subject to its laws, and which do not have to reconstruct the world, and thus remodel and change it, in order to know it.(1)

That this absolute world condition is not identified with the physico-mathematical world is only too evident. We must beware of the ambiguity of the term "physical world."(2) Originally it was employed to designate the objective cosmos. Physical science was born of a desire

to lay hold of this cosmos in its objectivity. But as science grew, it gradually evolved, for reasons already set forth, a world quite distinct from the objective cosmos — a world of its own making. It is to this latter world that the term "physical world" now usually refers when it is employed by physicists.

Progress in science has resulted, from one point of view at least, in an ever widening gap between these two worlds. The scientific universe has become more and more independent of the objective universe, more and more closed in upon itself, more self-sufficient. This has come first of all from the steadily increasing use of hypothetical elements logically interwoven into a coherent structure, but it has been carried to great lengths by the subalternation of physics to mathematics, which, as we have seen, is independent of existence and of any necessary order to existence, and which constitutes a closed and autonomous universe determined only by its own intrinsic logic. In this way, physical science has tended to become more and more a formal, self-authenticating system, even the raw materials of which are no longer taken directly from the objective world, but are subjectively created constructs.

From this point of view, then, the scientific world is a self-contained world, distinct from and independent of the absolute world condition. Science has become like a platonic demiurge, fabricating a universe out of its own subjective constructs and rationalizing it by means of mathematics. And in this perspective there is a great deal of truth in Maritain's remark: "ce n'est pas la réalité qui demandera à la science d'être vraie, c'est la science qui demandera à la réalité d'être 'scientifique', et de lui présenter ses papiers." (3) In order to know that there is a vast difference between the scientific world and the absolute world condition it is not necessary, as some might be tempted to suppose, that we have direct knowledge of the world in itself and thus be able to compare the two. For in the first place we know that there is a negative distance between the two universes by our experience with the kind of knowledge we have, which must go from the more general to the more concrete without ever being able to exhaust the concrete. The history of science brings out this point and underscores our great ignorance. In a positive way we know that there is a vast difference between the two universes because we know that in order to carry on scientific endeavor we must construct and must inject mathematics. (4)

But this independence of the scientific world

from the absolute world condition is only one side of the picture, and to exaggerate it to the extent of obscuring the other side would be to vitiate the whole meaning of scientific knowledge. (5) The objective world is not merely a malleable matter which allows the scientist to make any constructions he may wish. In erecting his scientific world he enjoys a great measure of freedom, but he is not completely free. (6) Though mathematical physics is formally mathematical and from this point of view independent of the real world, it is <u>terminative naturalis</u>; its whole purpose is to get to know objective nature. The scientific world remains bound down to the objective world at both ends; that is to say, the scientist must both begin and terminate his work in contact with nature. (7) While it is true to say that in one sense, the theory of Relativity, for example, as it pursues its constructive elaborations never returns to the world of experience but seems to draw farther and farther away from it, in another sense it does return. Einstein knew before he started that all of his mathematical calculations and constructive elaborations had, in the end, to lead back into the black - bands of the Michelson interferometer. The scientist must solve problems that are initially <u>given</u> in the objective world; his solutions must explain facts <u>as found</u> in experience. While the experimental operation measures the world condition, there is a sense in which it is true to say that the absolute world condition measures the experimental operation. As Eddington has observed, "The study of physical quantities, although they are the results of our own operations (actual or potential), gives us some kind of knowledge of the world-conditions, since the same operations will give different results in different world-conditions."(8) Moreover, there is a sense in which it is true to say that the scientist deals with familiar objects of the objective world. A sign of this is found in the fact that commercial companies concerned with these objects always have recourse to the help of scientists.(9)

All this helps us to understand the problem that the meaning of real existence presents to the mind of the modern scientist.(10) If the question is raised: "Does the scientific world really exist?" or "Does an electron really exist?" it is impossible to answer either yes or no, for we are dealing with constructs composed of both reality and mind. Taken from the point of view of the subjective elements they contain, they do not really exist. But taken from the point of view in which they are a reflection of reality, they do really exist. In fact, in the latter perspective we may

say that they exist in a more real sense than the sense world or the world of philosophy of nature, for science, in coming closer to concrete objectivity becomes more like the knowledge that God and the separated substances have of the absolute world condition than any other type of knowledge we have.(11) That is why Eddington, writing of Rossetti's Blessed Damosel who contemplates the world from heaven, can say: "If the Blessed Damosel sees the earth in the Einsteinian way she will be seeing truly — I can feel little doubt as to that — but she will be missing the point. It is as though we took her to an art gallery, and she (with that painful truthfulness which cannot recognize anything that is not really there) saw ten square yards of yellow paint, five of crimson, and so on."(12) The scientific world is made up of yards of paint taken from the objective world; but these yards of paint have been caught up into a composition that is not found in nature.

In the light of these remarks a number of passages in the writings of modern scientists which at first sight might appear baffling are rendered perfectly intelligible. A good example is the following passage of Eddington:

> However, so far as I can judge the meaning of the question, the answer appears to be in the affirmative — the external world described in physics [E.G. O. L.] really exists.

> One thing can perhaps usefully be added. I do not think that with any legitimate usage of the word it can be said that the external world of physics is the only world that really exists.(13)

There are in fact an infinite number of "physical universes". There was for example the original universe of Einstein which was full of matter and static. That has now been abandoned. There was likewise the universe of De Sitter which was empty. There is now the universe of Abbé Lemaître, which contains matter in constant expansion. These "physical universes" may be multiplied endlessly. All of them can be said to really exist in the sense just determined, but none of them can be considered the only one that really exists.

Perhaps the central problem with which we are concerned in this Chapter can be made clearer by casting it in the following form: is the scientific world true? Is it the truth about objective nature? What exact sense can be attached to Eddington's statement that if the Blessed Damosel sees the objective world in the Einsteinian manner she sees it truly? (14) As is well known, truth may be defined either in terms of intrinsic coherence or in terms of extrinsic conformity. Every science in so far as it constitutes a body of doctrine and takes on systematic form must possess truth in the former sense. There are some sciences in which this kind of truth is of pri-

mary concern. These are particularly the mathematical sciences which deal with abstracta ut abstracta, and which prescind from any actual order to existence. But in those disciplines which deal with reality and which are sciences in the strict meaning of the term it is truth in the sense of extrinsic conformity that is of primary concern.

Now from what was said above about the scientific world constituting a closed and intrinsically coordinated system and about the criteria for the choice and elaboration of constructs being not correspondence with objective entities but their capacity to serve as principles of internal coherence, it would seem to follow that it is truth in the first sense of the term that is characteristic of experimental science. This would seem to derive both from the vast use of hypothesis and especially from the introduction of mathematics. It is true that there is some connection between scientific constructs and objective reality, but it would seem that this connection must be viewed not so much in terms of truth as in terms of goodness, since the validity of these constructs is judged by their functional role, by their explanatory efficacy. The whole question comes down to this, then: can the conformity definition of truth be applied to the relation between the scientific world and the absolute world condition?

It should be immediately evident that if the conformity definition be taken in its full and absolute meaning, the answer must be no. Truth in this sense has the implication of uniqueness and to apply it to the ever changing scientific world would make of it an extremely protean thing. On the other hand, it is equally evident that there is some correspondence and some kind of conformity between the scientific world and the absolute condition of the universe, that some relation similar to truth obtains between them, if for no other reason than that verisimilitude is, as we saw in Chapter V, of the very nature of experimental science. This conformity is found even with regard to the most theoretical parts of science, for since theory is the source from which the phenomena of nature logically flow, and the objective essences of things are the source from which they really flow, it is obvious that there must be some kind of correspondence between the two, even though theory may not give an explanation of reality that is true in the strict sense of the word. And as theory is perfected this correspondence becomes more and more exact.

Moreover, the scientific world is made up of reality as well as of mind, and it must not be forgotten that even the subjective elements derive their whole meaning from their orientation towards the real world.

In other words, scientific symbols like all symbols are a mixture of truth and fiction, as Urban has observed:

> It is, as we have seen, of the very nature of the symbol that it contains both truth and fiction, both the real and the unreal. This principle follows, in a sense, from the two preceding. We have already seen that a symbol must stand for something, otherwise it would not be a symbol. We have also seen that it cannot stand for anything in a wholly unambiguous way. If it did it would not be a symbol. A fictional element in every symbol is made necessary by the principle of dual reference. It is of the nature of the symbol that if either reference is taken exclusively it becomes unreal or else a mere substitutional sign.
> A relation of two domains is involved in every symbolic function. If the symbol is taken literally, as we say, if, in other words, the reference to the primary domain is taken exclusively the symbol is a fiction and misrepresents. If it is taken wholly as a sign without any reference to the intuitive domain out of which it springs, it is again a fiction, in this case a merely conventional sign. The symbolic function, as distinguished from literal representation or description and from the merely conventional, is not only this dual reference but the combination of truth and fiction which arises out of it. This is as true in the region of scientific symbolism as in any other. It is, in fact, one of the main issues in modern scientific concepts is truth and how much fiction.(15)

It is clear, then, that in spite of its self-authenticating character, mathematical physics has a definite relation of correspondence with the real world. By the very fact that it is terminative naturalis, it must in some way realize the conformity definition of truth that is characteristic of all sciences which deal with reality.(16) And if mathematical physics appears as something that is from one point of view essentially dominated by the coherence definition of truth and at the same time from another point of view principally dominated by the conformity definition, it is chiefly because it is a *scientia media.* Let us try to fix upon the nature of the correspondence between the two worlds.

This obviously depends upon one's theory as to the nature of scientific knowledge. For those who press operationalism to the limit of maintaining that science reveals *nothing but* a set of operations carried on by the scientific worker, this correspondence is extremely tenuous when it exists at all. At least this is true if the notion of correspondence be considered from the point of view of speculative truth, as it is being considered in this context. For many operationalists, scientific symbols do not *represent* the objective universe at all; they merely reveal how one has operated upon nature and how one must operate upon nature in order to control it.(17) These authors fail to realize that the art that is involved in experimental science is purely functional and that its whole purpose is to serve science by

helping to disclose the objective logos. In other words, scientific symbols are like poetic symbols in this that they turn aside from a direct expression of reality only that in some sense they may express it more profoundly.(18)

The majority of modern scientists and philosophers of science hold that the physico-mathematical world must be considered at least a partial representation of reality. This opinion is held by Einstein and Planck, among others, and according to Cassirer it constitutes the essential modern scientific standpoint. It adopts a mediate position between the copy theory of the classical physicists and extreme operationalism. For most of those who hold this view, the scientific representation of reality consists in a reflection of nature's order, structure and interrelatedness, rather than in a direct representation of intuitively given natural phenomena.

We believe that this opinion is essentially correct. But it is necessary to try to give greater philosophical determination to the correspondence between the scientific world and the absolute world. Some have sought to solve this problem by saying that the scientific universe is analogically true. Hoenen has been particularly favorable to this solution.(19) He holds that physical theories express an analogical relation to reality and that if all the superfluous elements in them are eliminated by means of experiment and reasoning, it is possible for the relation to become univocal. We shall not linger over the latter part of this opinion, for all that has been said in preceding Chapters makes it abundantly evident how utterly untenable such a view is. In so far as analogy is concerned, we believe that this opinion is extremely ambiguous. It is clear that if the term analogy be taken in a broad and loose sense it may be applied to the knowledge that science gives of the objective world, in that the scientific world is partly like and partly different from the absolute world condition. But it is extremely important to keep this use of the term distinct from the proper use that is found in metaphysics. In true analogy we find a totum acutale that is the analogum in which the parts are known. In the case in hand, on the contrary, the parts are not known well enough. The objective and subjective elements in the scientific world are so intimately interpenetrated and fused, that it is impossible to distinguish between them; it is impossible to say what is in conformity with objective reality and what is not; it is impossible to determine which particular part is due to nature and which is due to mind.

We believe that the correspondence between the scientific world and the absolute world condition can best be explained in the following terms. In the first place, the scientific universe is a sign of the objective universe. Every sign represents an object distinct from itself to a cognitive power. But because there are two essentially different ways in which this representation can be effected, there are, as is well known, two essentially different kinds of signs: formal and instrumental. Since every sign is a means by which a cognitive power gets to know an object, even a formal sign is a kind of instrument. But it differs from an instrumental sign in this that it delivers the object it represents so directly and immediately to the mind that in this deliverance it does not itself constitute an object of knowledge. Thus the concept which the mind forms of an objective entity is a formal sign of that entity because it does not interpose itself as an object between the mind and the entity. An instrumental sign, on the other hand, is one that is first known in itself as an object in its own right, and only by being known in this way does it represent another object distinct from, but virtually implied in itself.(20) In other words, as Cajetan has remarked,(21) there are two kinds of beings; some are primarily designed to be and only secondarily do they represent; others are primarily designed to represent other things. The former are instrumental signs and the latter are formal signs. In Thomistic terminology, a formal sign is *id in quo aliquid cognoscitur*, an instrumental sign is *id per quod aliquid cognoscitur*, the first is a *forma intra potentiam informans*, the second is an *obiectum extra potentiam movens*.

Now the great error of many of the classical physicists and of the majority of modern scholastics is that they have looked upon the scientific world as a kind of formal sign directly and immediately revealing the absolute world condition. To view the scientific world in this light means to fall a prey to a great illusion. It means to destroy the scientific world's character as a sign, for it wipes out the true revelation it gives of the objective universe.

The physico-mathematical world is not a formal sign, but an instrumental sign of the absolute world condition. It constitutes an object in its own right, and must be known as such before it can reveal the objective universe. Like all instrumental signs, it hides the object it represents at the same time that it reveals it. And it is only by viewing the scientific world in this light that through it we can in some fashion come to know the objective world as it is in itself.

No other notion brings out so accurately the true character of the relation between the two worlds than this notion into which enters both instrumentality and signification. It explains how the physico-mathematical world can be at the same time completely closed in upon itself and completely opened to the objective world. It reveals why the criteria of the validity of the scientific structure can be both goodness and truth, with the goodness entirely subservient to the truth, why the scientific universe is at once practical and speculative, with the practical completely orientated towards the speculative, at once art and science, with the art entirely ordered to the science, (both in the sense in which fine art reveals an original, and in the sense in which useful art serves a purpose — the practical purpose in this case being found in the speculative order). Neither pure instrumentality alone, nor pure formal signification can bring out all of these paradoxical elements and serve to establish them in their proper relations.

The physico-mathematical world is in many ways a particularly perfect type of instrumental sign and it tends towards the perfection of a formal sign. Even those elements in it which are not taken directly from the objective universe and which consequently from one point of view serve to hide it, are introduced into it only to reveal the absolute world condition all the more. In this it is similar to a work of art into which the artist's own logos has been injected only for the purpose of revealing the original with greater clarity.

But the scientific world is an even more perfect sign than a work of art in that the fabrication found in it, while interposing an object between the mind and the real world, can never constitute an end in itself. The scientific world is art, but not simpliciter. It is essentially speculative knowledge, and as such its whole raison d'etre consists in its orientation towards the real world. In this it is similar to a formal sign.

John of St. Thomas assigns five conditions which must be present if one thing is to be the sign of another.[22] First, the sign must be something distinct from both the object signified and the cognitive potency. This condition is fairly obvious and needs no comment. Secondly, it must have the nature of a representation. This establishes a transcendental relation between the sign and the thing signified. Thirdly, the sign must be more knowable than the

thing signified. By reconstructing the objective universe, by injecting his own logos into it, by introducing the rationality of mathematics, the scientist succeeds in rendering it more intelligible. Fourthly, the sign must be less perfect than the thing signified and inferior. This recalls to mind what we said in an earlier chapter about the physico-mathematical world being worse than the real world precisely because it is better. It ever remains a mere substitute for the real world. Its role is purely functional. This means that over and above the transcendental relation mentioned a moment ago, there is a predicamental relation between the scientific world as sign and the absolute world condition as thing signified. This relation belongs to the species of relation that exists between a measure and a thing measured, (in the sense explained in Chapter VIII in connection with the various types of relation). The absolute world condition is the measure of the physico-mathematical world. It is this predicamental relation and not the transcendental relation that constitutes the latter as the sign of the former.[23] The fifth condition laid down by John of St. Thomas is that the sign and the thing signified must be dissimilar. The vast difference between the scientific universe of discourse and the objective universe has already been sufficiently stressed.

The foregoing makes it clear that in experimental science the mind does not assimilate the objective world directly, but rather reflects it by constructing a schema of its own that is founded upon reality. But it is important to try to determine the nature of this schema and thus bring out as accurately as possible the exact character of the instrumental sign. We believe that this can be done by having recourse to the notion of isomorphism. Isomorphism, as the word implies, signifies identity of structure or form, and it is commonly defined in the following terms: Given two classes: S, composed of elements a, b, c, ..., and S', composed of elements a', b', c', ...; if the elements of S can be placed in one-one correspondence with those of S', in such a way that a corresponds to a', b corresponds to b', etc.; and if for every relation R between the elements of S (e.g. a R b) there exists a relation R' between the corresponding elements of S' (e.g. a' R b'), the two classes are said to be isomorphic. A familiar example of isomorphism is found in an ordinary map. There is identity of structure between the relations between the points on the map and the corresponding points on the countryside to which the map is related. It is important to insist upon the fact that isomorphism is not founded upon

a material correspondence between the elements involved, but the identity of structural form. It prescinds from the proper nature of the matter to which the forms are applied. But this prescinding is not a negation. In fact if the heterogeneity of the matter of the different systems were destroyed, the isomorphism would also be destroyed.

Now this notion of isomorphism brings out the nature of the relation between the physico-mathematical world and the absolute world condition. For mathematical physics is a search for system and order. As we have seen, it constructs its own organized system, but in so doing it is determined in its every move, either directly or indirectly, by measurements made upon the real world. In spite of the arbitrary elements in measurement, the absolute world condition remains the measure of the measuring process, in such a way that although different codes of measurement employed in relation to the same world condition will render different results, as long as the same code is employed in relation to the same world condition, the results will be identical.[24] That is why, after the physicist has constructed his schema he is able to map it back upon nature and predict natural phenomena.[25]

As Duhem has observed, the relation between the scientific world and the objective world may be compared to the relation between the form of a suit of armour and the form of the body of the knight who wears it.(26) There is always a similarity of structure in this relation no matter how imperfect the suit of armour may be. This similarity grows as the suit becomes more perfect, as the number of pieces of metal which compose it increases, and as its structure becomes more complex. At the limit the form of the suit would be identified with the natural form of the body. This limit can never be actually reached, obviously; but it can be indefinitely approached. And as the artificial form of the suit gets closer to the natural form of the body, it is at the same time drawing farther away from it in the sense that it is constantly becoming more artificial.

2. Logical Identity.

The gap that exists between the ideal to world condition and the structures manufactured by the scientist is something that the mind must seek to bridge. It must seek to go beyond the relation of isomorphism of which we have been speaking and arrive at some kind of identity. In order to see how this may be accomplished, how what is at

once both reality and artifice can be erected into a unified object, it is necessary to have recourse to the notion of predication of identity.

Aristotle and St. Thomas speak of this notion in several places, notably in the fifth book of the _Metaphysics_(27) and the fourth book of the _Physics_.(28) In the latter text we read:

> Genus potest cum additione unitatis vel identitatis praedicari de pluribus individuis existentibus in una specie, et similiter genus remotum de pluribus speciebus existentibus sub uno genere proximo; neque tamen species de individuis, neque genus propinquum de speciebus diversis potest praedicari cum additione unitatis vel identitatis... Et huius assignat (Aristoteles) rationem: quia cum idem et diversum seu differens opponantur, ibi possumus identitatem dicere, ubi differentia non invenitur, sed non possumus dicere identitatem ubi invenitur differentia.

In order to make a predication of identity of things that are different it is necessary to ascend to a genus that is not divided by their proper differences. Aristotle and St. Thomas explain this by having recourse to examples taken from mathematics. Thus it is possible to say that a scalene triangle and an equilateral triangle are the same figure. But it would be incorrect to say that they are the same triangle. The reason is fairly obvious. For the one condition for identity is absence of difference. Now the scalene triangle and the equilateral triangle divide the genus triangle by a difference that is proper to the triangle, since they are different species of triangle. That is why we cannot say that they are the same triangle without falling into a contradiction. But they do not differ by a difference of figure, since they both fall under the same difference which divides the genus figure, namely triangle. And that is why we can say that they are the same figure.

Aristotle and St. Thomas give another example taken from the realm of number. Even though it is impossible to say that ten cows and ten dogs are the same ten, it is possible to say that they are the same number. In other words, there are two different species of ten, but the same number. The <u>same</u> number is neither the ten cows (for then either the dogs would not be ten or they would be identified with the ten cows), nor the ten dogs (for similar reasons). It is neither the one nor the other determinately, but different from both. It is not different, however, in the sense of being non-ten, as three or twelve. It is ten, but indifferent to the particular species of ten.

From this example it is clear that the relation

of identity is something created by the mind. For to be the same number does not mean to be identical, otherwise the ten cows and the ten dogs would be the same. Hence the identity in question in this whole context is constituted by a relation of reason added to that which is predicable as genus of individuals or as remote genus of species.

What has been said of figures and numbers may be applied to the <u>ratio entis</u>. Both real being and logical being may be said to be the same being, provided that the <u>ratio entis</u> in question be not identified with either the one or the other. In other words, the <u>ratio entis</u> can be said to be the "same" only on condition that it be "other", that is to say, it can be the "same" only if it is not identified with any of the terms in relation to which it is said to be the same. It must be like the <u>ratio "ad"</u> of relation, which is indifferent to "inesse", or "non-inesse", or like mathematical quantity which is indifferent to real or logical being.

It is to be noted that this predication of identity is not tautological. When we say that a scalene triangle and an equilateral triangle are both the same figure, we do not merely wish to say that figure is predicable of both of them in so far as both of them are figures. For the same could be said of triangle since both of them are triangles. Predication of identity does not merely have to do with what is the "same" in the species, namely the genus, or with what is the "same" in the individuals, namely the species. It has to do with the differences in their very difference - - not in an absolute way, of course, for that would make them absolutely identical, but in their relation to the genus that is predicated of them by identity. Thus, this predication is not made after the terms in question have been stripped of their difference, for any genus may be predicated of its inferiors in this way. On the contrary it presupposes the differences. It is this, in fact, that gives it its special significance.

That in relation to which the differences are said to be the "same" is something purely logical, namely the logical genus in so far as it takes on a potentiality that derives from our mode of conception. The indetermination in question is not found either in the terms themselves to which identity is attributed or in that which is attributed to them, for both a scalene and an equilateral triangle on the one hand, and figure on the other, are in themselves

definitely determined things. The indetermination is found in the figure in so far as it is considered as a predicable genus. In other words, predication of identity can exist only because it involves logical intentions.

It is clear, then, that by withdrawing into the potentiality of the logical order where differences can be blended it is possible to predicate the "same" of things that are essentially diverse, to unite into one things that are divided secundum rem. And this is of extreme importance for the question of the relation between scientific constructions and the absolute world.

In order to see why this is so, let us take a simple example. When after an ordinary process of measurement we declare that the proper length of a certain body is two meters, this statement can be taken in two ways. It may, in the first place, mean simply that a meter measure has been placed twice end to end along the body; in other words that the length of the body is equal to the length of two meters. As a matter of fact, however, when we say that a certain body has a length of two meters, we are not speaking formally of the relation of equality between the body and the meter placed twice end to end along its surface. We are not speaking formally either of the absolute length of the body, nor the absolute length of the meter placed twice along its surface, nor of the relation of equality between the two, though all this is presupposed. In order to be able to say that a certain body has a length of two meters it is necessary to go beyond a mere relation of equality and arrive at identity. If the length of the body is equal to the length of two meters they are the same length, but they are not the same length of two meters, just as ten cows and ten dogs are the same number but not the same number ten.

In other words, we have seen that operational definitions do not allow absolute attributions, since the practical operation involved separates us from the terminus to which it is ordered. Now when we say that a body has a length of two meters we have in a certain sense surmounted the gap created by this separation, for merely to describe the measuring operation and to say that the body has a length of two meters are not the same thing. This has been done by ascending to a logical genus to which we have added the relation of identity. In this way it has become possible to predicate the 'same length' of the body in question. But, we repeat, the same length is not the same length of two meters. In other words, we have attributed to the body a

logical genus which cannot be identified with it. We are not in the real order, but merely turned towards it. If in this predication we actually reached the real order there would be contradiction, for the length that is said to be the same for the body of two meters and for the meter placed twice along its surface would be identified with both of them and one would be two.

It is clear that this identity adds something to the *unum secundum quid* constituted by the operational experiment and the absolute condition of the world. By arriving at identity even though it be merely logical, we have in some way surmounted the diversity involved in the *unum secundum quid*, and have achieved a kind of counterfeit *unum per se*.

What has just been said about the simple process of measurement can be applied in a general way to all of the constructions manufactured by the scientist. Mathematical physics deals neither with the world of its own constructions as such, nor with the absolute world as such; it deals formally with a world that is a logical identity of the two.

But this logical identity is not an end in itself; it is only a means. And its purpose is to draw the scientist closer to the absolute state of the universe. In so far as it keeps the scientist in the logical order, and in so far as the goal sought for is the world *in se*, experimental science must ever strive to escape from this purely logical identity and to draw ever closer to the real world. In other words, logical identity is not sufficient. Science must seek to surpass it by tending towards real identity. We have seen that mathematical physics is dialectics and that "omnis dialectica est tentativa". From the construct which is the physico-mathematical world it must ever strive to reach the real world. To this dialectical movement we must now turn our attention.

3. Movement towards Real Identity.

That the scientific world is constantly in movement is a fact of history. But there are two things to be noted about this movement. First, it is something that is essential to the scientific world. Without it science would lose its meaning. In this experimental science differs radically from all the sciences in the strict sense, which, though caught in the flux of history and in some measure subject to it, are intrinsically independent of all movement. The reason for this character-

istic property of experimental science has already been emphasized: the scientific universe is essentially a dialectical construct which must ever seek to go beyond itself; it is a vehicle of progress and not a mansion of residence.

The second thing to be noted about this movement is that it has a very definite direction. "It is plain, writes Planck, "that when regarded as a whole, all the changes in the different views of the world of Physics do not constitute a rhythmical swing of the pendulum. On the contrary, we find a clear course of evolution making more or less steady progress in a definite direction."[29] From this point of view it is interesting and instructive to constrast the history of experimental science with the history of philosophy. Though philosophy in its inner essence is independent of movement, as we pointed out a moment ago, it appears to be much more a prey of the irrational flux of history. When viewed in its entirety, the history of philosophy presents no definite direction; it is constantly repeating and refuting itself.

As Poincaré has observed, to those who are unacquainted with the true meaning of experimental science the ephemeral character of scientific views and the constant succession of new theories may seem to have the same aimlessness. As a matter of fact, however, these views and theories are continually tracing out a definite pattern. "Sans doute, au premier abord, les théories nous semblent fragiles, et l'histoire de la science nous prouve qu'elles sont éphémères; elles ne meurent pas tout entières pourtant, et de chacune d'elles il reste quelque chose." (30) The following comparison of Duhem brings out with great exactness the existence of a definite direction in the movement of science underneath a superficial appearance of aimlessness:

> Celui qui jette un regard de courte durée sur les flots qui assaillent une grève ne voit pas la marée monter; il voit une lame se dresser, courir, déferler, couvrir une étroite bande de sable, puis se retirer en laissant à sec le terrain qui avait paru conquis; une nouvelle lame la suit, qui parfois va un peu plus loin que la précédente, parfois aussi n'atteint même pas le caillou que celle-ci avait mouillé. Mais sous ce mouvement superficial de va-et-vient, un autre mouvement se produit, plus profond, plus lent, imperceptible à l'observateur d'un instant, mouvement progressif qui se poursuit toujours dans le même sens, et par lequel la mer monte sans cesse. Le va-et-vient des lames est l'image fidèle de ces tentatives d'explication qui ne s'élèvent que pour s'écrouler, qui ne s'avancent que pour reculer; au-dessous, se poursuit le progrès lent et constant de la classification naturelle dont le flux conquiert sans cesse de nouveaux territoires, et qui assure aux doctrines physiques la continuité d'une tradition." (31)

As science advances it often happens that the new theories which supplant each other appear, in their external form at least, to become increasingly divergent. This is in itself a significant fact, for it is a sign that the scientific world is becoming more and more a subjective construct. But no matter how divergent new theories may be, they are never born in a vacuum; there is always a continuity with the past. "It happens", says Weyl, "that broadened or more precise experiences and new discoveries do not overthrow old theories but simply correct them. One looks for the least possible change in the historically developed theory that will account for the new facts." (32) The Bohr atom did not destroy the Rutherford atom, but merely corrected and developed it. And the same is true of other changes through which physical science has passed. This does not refer merely to the gradual changes that take place in physics. Even in the so-called revolutions there is always continuity with the past. The formulation of the Quantum Theory, as Planck himself admits, was prepared by Lummer's, Pringsheim's, Ruben's and Kurlbaum's measurements of the spectral distribution of energy, by Lenard's experiments on the photoelectric effect, and by Franck and Hertz's experiments on the impact of electrons. In the same way, the theory of

Relativity was prepared by Michelson's experiments on optical interference. But more than that, it is a mistake to believe, as many do, that the theory of Relativity and the theory of Quanta mean a complete destruction of classical physics. For it is necessary to assume the classical theory in order to define the experimental conditions in which the theory of Relativity obtains to a higher approximation. And that is why Einstein begins his first paper on the special theory of Relativity with the statement: "Let us have given a system of coordinates, in which the equations of Newtonian mechanics hold to the first approximation."

> Like the system of Euclid, or Ptolemy, of Newton, which have served their turn, so the systems of Einstein and Heisenberg may give way to some fuller realization of the world. But in each evolution of scientific thought new words are set to old music, and that which has gone before is not destroyed but refocussed. Amid all our faulty attempts at expression the kernel of scientific truth steadily grows; and of this truth it may be said — The more it changes, the more it remains the same thing. (33)

It is clear, then, that the development of the scientific world does not take place in a haphazard fashion but follows a very definite direction. At the end of chapter V we noted that there is a similarity between experimental science and the type of knowledge described by Russell in Mysticism and Logic in which deductions are drawn from

freely chosen hypotheses. Now it is necessary to see that there is also a vast difference between them. For in the type of knowledge considered by Russell, there is no direction; we may, as he says, take any hypothesis which seems amusing. Experimental science, on the contrary, is knowledge that is essentially ordered towards a definite goal.

Now the relation between the scientific world and the absolute world condition cannot be properly grasped unless it be viewed in terms of a movement that is essential to the former and essentially orientated towards the latter. And we know of no way of bringing out accurately this dynamic relation except by having recourse to a notion which plays its most familiar role in mathematics and especially in the calculus, but which can be fruitfully applied to other fields as well. We have in mind the notion of a variable ordered towards a limit. A brief analysis of this notion will throw great light upon the orientation of the scientific universe towards the absolute world condition. (34)

This notion, in its most simple and generic form, is usually expressed in terms similar to the following: A variable quantity x is said to tend towards a determined limit if the successive values of x approach a certain fixed number a in such a way that the difference $x-a$ becomes less than any given number e, no matter how small it may be. Thus, for example, the number 2 may de defined as the limit towards which the following series tends:

$$1,\ 1+\tfrac{1}{2},\ 1+\tfrac{1}{2}+\tfrac{1}{4},\ \ldots$$

In the same way, a circle may be defined as the limit towards which tends a regular inscribed polygon whose sides increase indefinitely. Applying this now to the question in hand, we hold that the scientific world may be considered as a variable quantity which by passing through the successive stages of its evolution approaches the absolute world condition as its limit.

An analysis of this notion reveals that it involves both a heterogeneity and a homogeneity, both an otherness and a likeness. The heterogeneity, the otherness, consists in the fact that there are necessarily two terms which belong to different orders or to different species: e.g. discontinuous-continuous; point-line; line-surface; polygon-circle; curved-straight, etc. Heterogeneity is essential to the notion of limit, even under the aspect in which the limit is considered as a value of the variable term: it is precisely in its heterogeneity that it is the limit value of the variable. It is not a polygon (no matter of how many sides) that is the limit of the polygon

whose sides increase indefinitely, but a circle. On the other hand, even though the polygon becomes more and more like a circle, it is not changing in its species (in which it remains essentially a polygon), but merely in its values. Now this heterogeneity is found in the relation between the scientific universe and the absolute universe. A great deal of emphasis has already been laid upon their essential otherness. It is not an advanced stage in its own development that the scientific world is attempting to reach in the movement that is essential to it, but something beyond itself and essentially other than itself, namely the absolute state of the universe. On the other hand, even though the scientific world in its development comes ever closer to this absolute state, it does not in any degree lose the otherness which derives from the fact that it is essentially a construct. On the contrary, this otherness increases, just as the polygon becomes, in a sense, more of a polygon, i.e. a many-sided figure, the more its sides are increased.

But along with this heterogeneity there is an essential homogeneity involved. This is evident by the very fact that one term is said to be the limit of the other. When we say that x has a as its limit (lim x a), the fixed term a is considered as the limit value of x, in such a way that lim ($\underline{x}$ - $\underline{a}$) = 0. From this point of view the heterogeneous terms are considered as belonging to the same order, that is to say, one is considered as a value or a case of the other. A polygon with a hundred sides is considered as a case of the polygon; a circle is considered (in a hypothetical way) as another case — the limit case: if the limit could be reached the case of the polygon which is the circle would differ from all the other polygons in that it would have the greatest number of sides possible. From this point of view there is an order of continuity between the variable and the limit. And it must be noted that the "more" or "less" of the formal order of the variable quantity is not merely quantitative. That is to say, a certain given value of the variable is not merely greater than any preceding value; it is at the same time more like the limit. In other words, by running through its values the variable is related to the formal structure of the limit. The increasing structural similarity tends towards structural identity.

A homogeneity of this kind is found in the relation between the scientific world and the absolute world. The former tends to issue into the latter. If the limit of scientific development could be reached there would be

identity of structure between the two. And as the limit is approached the likeness of structure which we explained above by the notion of isomorphism, increases. While at any given stage of the development there is a certain likeness of structure between the two worlds, it is inadequate and often extremely misleading to consider this static relation independently of the dynamic relation that the movement which is essential to the scientific world involves. This is suggested in the following passage of Sir Arthur Eddington:

> Scientific discovery is like the fitting together of the pieces of a great jig-saw puzzle; a revolution of science does not mean that the pieces already arranged and interlocked have to be dispersed; it means that in fitting on fresh pieces we have had to revise our impression of what the puzzle-picture is going to be like. One day you ask the scientist how he is getting on; he replies, "Finely. I have very nearly finished this piece of blue sky." Another day you ask how the sky is progressing and are told, "I have added a lot more, but it was sea, not sky; there's a boat floating on the top of it." Perhaps next time it will have turned out to be a parasol upside down; but our friend is still enthusiastically delighted with the progress he is making. The scientist has his guesses as to how the finished picture will work out; he depends largely on these in his search for other pieces to fit, but his guesses are modified from time to time by unexpected developments as the fitting proceeds. These revolutions of thought as to the final picture do not cause the scientist to lose faith in his handiwork, for he is aware that the completed portion is growing steadily. Those who look over his shoulder and use the present partially developed picture for the purposes outside science, do so at their own risk.(35)

There is, then, in the notion of limit the paradox of heterogeneity and homogeneity. And the key to this paradox, as has just been intimated, is found in movement. For one term is ordered towards another as its limit, not in its proper specific character, but only in so far as it is considered as a variable whose successive values approach the term which is the limit. These successive values must be indefinite; between any given value and the limit there must always be an infinity of other possible values in potency. But this potential infinity is not sufficient. It is merely the foundation of something more, namely a progression, a movement, a becoming. Because of this movement the difference between the two terms decreases indefinitely. In this way the variable tends to enclose the limit as its own final value. Heterogeneity tends towards homogeneity. The variable tends to bound itself by going beyond itself, that is to say by going beyond any value actually given within itself; it tends to break through its own form and thus destroy itself by taking on the form of the limit.[56] In other words, both the variable and the limit have a double state: an absolute state which consists in their irreducible otherness, and a state of becoming by which they tend to reduce this otherness to sameness. The variable is always essentially other than

the limit, but at the same time it is always becoming the limit. In the same way, the limit has an absolute state by which it is essentially different from the variable, but at the same time it has a state of becoming - - a state of "coming from" the variable. "The limit must be coming from the otherness that is the variable, as if it were precontained in that otherness. The variable - - whose proper values are being more and more actualized, so that the variable itself is becoming more and more the self that it ever more can be - - must at the same time be moving away from itself and becoming identical with what is otherness to it, viz. the limit."[57] In so far as the limit is considered as coming from the variable it may be said to be generated by the progression of the variable. Thus this progression triumphs over the givenness of the limit and in this sense rationalizes the irrationality of this mere givenness.

This movement of which we have been speaking is *sui generis*, for by its very nature it is a movement that can never arrive. Whereas the terminus of every other movement, such as the becoming of a house, is defined by the possibility of its actually being reached (whether it actually will be reached or not) the limit of this movement is defined by the impossibility of its being reached. Whereas the terminus of movement in the ordinary sense can still be considered the terminus even though the movement towards it has actually ceased, the limit of this movement ceases to be a limit once the getting closer ceases. In other words, the notion of limit supposes an actual and indefinitely prolonged movement. Just as all relations consist in an "esse ad", so all movements are towards something other. But just as some relations are of such a nature that they cannot "be in" that "toward" which they are, so this movement cannot actually reach the limit towards which it tends. From one point of view this movement seems to be an end in itself, since it can never arrive at anything beyond itself. But from another point of view, it is not an end in itself, since it must ever tend towards the limit which is beyond itself.

Now all this has an application to the relation between the scientific world and the objective world. Both of them have an absolute state by which they are essentially heterogeneous. But they also have a state of becoming which tends to reduce this irreducible heterogeneity to homogeneity. In so far as the scientific world is concerned this state of becoming consists in a continuous development by which it

draws ever closer to the objective world. In so far as the objective world is concerned this state of becoming does not, obviously, mean a real change; it merely means that as the scientific world draws closer to the absolute world, the latter may be considered as coming from the former. In this way, the absolute world condition may be viewed as being generated by the construction of the scientist; thus its pure givenness is triumphed over and the irrationality of this givenness rationalized. As we remarked in Chapter IV, if the scientist could reach his goal, man would be God. But there is one difference to be noted here between the movement of a variable towards a limit and the movement of the scientific world towards the objective world. In the former case the limit is already known before the movement towards it begins. In the latter case, this is not true: the absolute world is an unknown quantity that gradually reveals itself as the movement towards it progresses. In this way the state of becoming of the objective world has more of the nature of a generation.

It is clear that the objective world as a limit cannot be reached by the progress of science. The aim of science, writes Planck, "is an incessant struggle towards a goal which can never be reached. Because the goal is of its very nature unattainable. It is something that is essentially metaphysical and as such is always again and again beyond each achievement." (38) The very method to which experimental science and especially mathematical physics is committed makes it impossible for it to ever reach the objective universe as it is *in se*. And yet by a strange paradox, it is only by remaining faithful to this method that it can be carried closer and closer to this goal. (39)

All this brings us back to what was said earlier in this study: experimental science is essentially a vehicle of progress and can never become a mansion of residence. And to consider it as a mansion of residence is the most effective way of destroying its true relation to the absolute world condition. From this point of view, the movement of the scientific world may be considered as an end in itself, and in this sense we may accept the dictum of Gotthold Lessing to which frequent reference is found in the writings of modern scientists: "Not the possession of truth but the effort in struggling to attain it brings joy to the researcher." (40) But from another point of view it is obvious that the movement of the scientific world is not an end in itself. The end must ever remain the absolute world condition. The scientist who loses himself in the develop-

ment of his own subjective constructions is not true to his science. It must be noted, moreover, that while it is better to be able to move towards truth than not to be able to approach it at all, it is absolutely speaking far better to be in the full possession of truth than merely to be approaching it.

It is obvious that the reason why the variable cannot arrive at the limit is that this arrival would involve a contradiction. The limit of a polygon would be both a circle and a polygon, that is to say, both a circle and a non-circle, both a one-sided and a many-sided figure, both an unbroken and a broken line. This contradiction is an essential condition for terms to be related as variable and limit. When it is stated that a polygon and a circle meet at infinity, this merely means that they would meet if *per impossibile* "at infinity" could be. The variable tends towards its ultimate value and at the same time at something that is essentially other than any of its values. In other words, the tendency to realize itself is a tendency to destroy itself. But this does not mean that the dialectical movement towards the limit is in itself contradictory and meaningless. The contradiction that *would* be is only at the limit, which cannot be attained. The movement itself cannot be considered contradictory simply because it cannot attain a contradiction. The possibility of this movement does not depend upon the possibility of attaining the limit but upon the possibility of considering the term toward which the movement tends as the limit of this movement. The movement in itself is meaningful precisely because it never goes beyond the stage of "being towards".

Now the movement of the scientific world is a movement towards contradiction. This has already been alluded to on several occasions throughout our study. We have seen in a general way that the scientific universe in seeking to posit itself more fully tends to negate itself and to vanish into emptiness. Several particular forms of this tendency towards contradiction have already been indicated. But it is of extreme importance to examine this question more closely here, for nothing could bring out more clearly and fully the noetic structure of the scientific world. And this can best be done by showing that the most fundamental and most proper principles of experimental science are such that they could not be really true without contradiction, that is, they could not be true without being false. These fundamental principles are the methodological principles such as the principle of definition, of

identity, of unity, of order, of induction, of simplicity, etc. Let us consider a few examples in detail.

The first example to be examined is the principle of definition. We have seen that in mathematical physics all definitions are in terms of operations of measurements. Now both from the point of view of measurement and from the point of view of operation this principle of definition involves mathematical physics in a movement towards a limit, the attainment of which would imply a contradiction. In so far as measurement is concerned this is evident from all that was said in Chapter VIII about the search for a *minima mensura*. Progress in measurement must consist in a movement towards greater precision and certitude. The limit of this movement would be an absolutely minimum measure. But such a measure is a contradiction since it implies a quantity that is at once continuous and non-continuous.

A similar movement towards contradiction is discovered when the nature of operational definitions is analyzed. We saw in Chapter IV that these definitions express a mixture of nature and art, of a *quod* and a *quo*, of subject and object. The thing defined is neither a pure operation, nor a pure objective quantity, but an inextricable mixture of the two. In other words, the *definitum* is only an *unum per accidens* and not an *unum per se*. The unity is conferred upon it by the mind. If it were a *per se* unity, the world would be at the same time nature and a human work of art. This is the position of the Marxists.

It is clear, then, that while operational definitions are destined to help us to know the real *in se* (for operations are not carried on for their own sake, and physics does not consist in mere descriptions of what physicists do), a reality which could be known *in se* by means of operational definitions is an impossibility. By means of operational definitions we tend towards a limit which cannot be attained by means of operational definitions. The practical operation involved separates us from the terminus towards which it leads us. Arrival at the limit would involve a complete arrival at the limit and a complete separation from it at the same time.

Another good example is found in the principle of induction. Poincaré's statement that all generalization is an hypothesis is true of the type of induction that is characteristic of experimental science — induction by enumeration. When a general proposition "Every A is B" is founded merely upon the enumeration: "A is B", "A is B", "A is B", etc., it cannot be true. For if "Every A is B" is true, "Some A is non-B" is false. But in so far as

"Every A is B" is founded merely upon a collection of particular cases, it cannot be said that "Some A is non-B" is false. Hence, "Every A is B" is a logical proposition that tends towards reality without being able to attain it. It is, so to speak, a relation "ad" without "inesse". If "Every A is B" were true in so far as founded upon a collection, all A's would not only be alike; they would be identified — they would be the same A. For if induction by enumeration could give a universal in the strict sense, this universal would be the particular cases, and the particular cases would be the same particular case. Hence there would be contradiction.

The principle of causality as employed in physics offers a third example for our analysis. Events are knowable by us only in so far as they are determined. Hence the future can be adequately known by us only to the extent in which it is already determined in the present. The future is, of course, of great importance in the fluid universe that constitutes the object of physics. The future is a part of our world, for without before and after there could be no time. Now it is evident that there must be a certain amount of determination in the relation between present and future, since the universe is not run by pure

If the identities which we posit in science were real identities, the logical and the real order would be identified.

In spite of the contradiction at the limit, science tends to emerge from mere logical identity to real identity. We find this tendency on every level of the scientific structure. In the definitions we tend to pass from logical identity to the absolute world condition, even though the arrival would be contradictory. The same is true of scientific laws: generalization tends towards a universal nature, even though if such a nature were achieved it would be contradictory. The case of hypotheses is very much the same: they are destined to make the truth known, but they cannot provide truth <u>ex propriis</u>. The truth which they help to reveal does not depend in any way on them. If hypotheses could be identified with their terminus (which is known by experience) they would destroy themselves as hypotheses. Finally, scientific deduction is orientated towards a true conclusion. But it cannot provide this true conclusion, that is to say, the conclusion cannot be true <u>qua</u> conclusion. Between the conclusion taken as such and the truth that it permits us to discover there is only an accidental connection, since an other deduction could serve

to reveal the same truth.

Since therefore, the initial definitions cannot give us the real as it exists in itself; since physical laws are only generalizations which are never really founded in any absolute sense; and since deductions cannot be true as such, it is evident that the physical world cannot be identified with the absolute world condition. It is, consequently, merely a construction of the mind - - a construction which imitates more or less the absolute world. It is turned towards the absolute world, and can approach it indefinitely without ever being able to reach it.

The Marxists have sought for a proof of their dialectical materialism in this characteristic nature of science. Their line of argument may be reduced to this: The methodological principles are true. But if they are true, the world is contradictory; it is at the same time affirmation and negation of itself, at the same time true and false; there is no absolute truth. Consequently, since this state of things cannot satisfy speculative thought, man is not made for thought, but for action.(41) The error of this argument consists in an exploitation of the ambiguity of the term "true" in the proposition "the methodological principles are true". The foregoing analysis has

It is clear, then, that the principle of causality in physics has a meaning, and can be said to be true, only in so far as it is not really true. It is merely a methodological principle: it tells us how to proceed and not what things are objectively. In so far as it tells us how to proceed it is true. In so far as it attempts to tell us how things are in themselves, it involves a contradiction.

These examples suffice to show that the movement of the scientific world tends towards a contradiction. The meaning of this tendency will be made clearer if we return to the notion of predication of identity discussed earlier in this Chapter. We saw that in this predication we consider the terms which are either specifically or individually different not merely in what they have in common absolutely, but in their very formal differences. It is this,[1] in fact, which characterizes predication of identity. Polygon and circle, for example, have an identity in their very differences (considered of course in relation to their remote genus). Now in our discussion of the notion of limit we saw that it supposes two terms which are at once the same and different. That is to say, the limit must be comprised in the variable; since it is the limit of the variable it must be considered as comprised in the order of the variable. Now this identity

which the limit supposes is accomplished by passing to the genus that is predicable by identity. Consequently, the notion of limit is founded on a predication of identity of the differences.

Now the dialectical movement consists precisely in the tendency of one difference towards another difference within their abstract identity. This identity in the difference is a principle of dialectical movement, but it is not the terminus. The tendency of one difference towards another difference within one abstract identity, is a tendency towards an identity of another order, namely real identity. It is the realization of this real identity that is impossible.

All this makes it evident once again how much truth there is in Meyerson's central theme that if science could arrive at the goal which it is constantly seeking the result would be a vast tautology, and how correct DeBroglie is in quoting in connection with his description of Meyerson's doctrine the remark of Valiary to the effect that what science seeks to achieve is an absurdity. It is clear, however, that this absurdity is not merely that of a vast tautology, but that of an intrinsic contradiction.

to claim that philosophical and theological knowledge are essentially subjective and that only experimental science is capable of giving true objective knowledge. We must try to see why just the opposite is the case.

CHAPTER TWELVE

OBJECTIVE SUBJECTIVITY

1. Subjectivity and Objectivity.

As we explained earlier in this essay, all knowledge is by its very nature objective, since to know is to become another thing in its very otherness. But not all knowledge is equally objective, for there is a direct proportion between objectivity and the perfection of the knower. In God alone is perfect objectivity found.

Now the word subject can be taken in two ways. In the first place, it can be understood to mean simply a knower. In this sense, all knowledge, in so far as it implies that a known thing is in a knower (cognitum est in cognoscente) involves both a subject and an object. In its proper meaning, however, the term subject implies a state of subjection. This involves passivity, and consemade it clear that they are true only as logical principles and not in the sense in which they would signify the truth of the world *in se*. In other words, there is a confusion here between the logical and the real order. Logical possibilities have greater freedom than real possibilities. In the logical order it is reasonable to build structures with elements that are not capable of realization. Nor does the lack of this capability prevent the possibility of drawing closer and closer to the real. It is possible for logical constructions to comprehend being and non-being at the same time. "Non homo", for example, is an indetermination which comprises at the same time both being and non-being.

Because the scientific world is a logical construction, because it is dialectics, there is deep within it an essential conflict from which it ever seeks to deliver itself. In the first place, there is the conflict between being and non-being. Experimental science tends towards being by means of the impossible. It tends towards the real by means of the purely logical. There is, moreover, a con flict between the one and the many: it tends towards the one by means of the many. There is a conflict between the speculative and the practical, between science and art.

Because of its operationalism, mathematical physics tends in its experimentation towards the *res* in its physical, entitative status; at the same time it tends towards pure science in the intellect. For this reason it tends to issue into two contrary directions: on the one hand pure science, independent of physical operations of things in their entitative status; on the other hand, pure operation by which things are mastered through action. That is why there will always be two fundamental tendencies in mathematical physics: one towards a kind of Platonic mathematicism, and the other towards a kind of dialectical materialism whose ultimate aim is to master things through and for practical action.

Perhaps the general drift of this whole Chapter can be summed up by saying that the scientific world is a structure composed of both the subjective and the objective and that if the goal towards which it strives could be reached it would be at the same time completely subjective and completely objective. For this reason it is necessary before bringing this study to a close, to turn our attention to the question of the subjective and the objective in mathematical physics. It has been customary for scientists judgments, by fabricating formal discourses in its processes of reasoning. This active subjectivity is also an obstacle to pure objectivity. For all of these reasons it is necessary to agree with Eddington that "it is the inexorable law of our acquaintance with the external world that that which is presented for knowing becomes transformed in the process of knowing." (1)

But this subjectivity of the human intellect must not be exaggerated. For there is a sense in which it is true to say that the mind is capable of a kind of pure objectivity. In its ordinary processes and in the way in which it functions in the philosophical sciences it is able to disengage the quod quid est of things — their objective essences. There is always a certain amount of subjectivity involved, to be sure, but it is a kind of subjectivity that attaches not so much to that which is known as to the way in which it is known or the state in which it is known. To use Scholastic terminology, it is a subjectivity that affects rather the modus quo cognoscitur than id quod cognoscitur. There is, of course, a kind of subjective element entering into the object known, but it is more of a negative than a positive thing. That is to say, in comparison with the object in se the object as known is always

imperfect and inadequate. But this does not transform the object in the sense of making it a new object. Definitions of the mind can apply with perfect truth to things as they are in se. In other words, the mind does not project a new positive element into the essence it knows in such a way that this essence is reconstructed into something different. In this the intellect differs essentially from the senses which in knowing their object necessarily transform it into something different because of the physical interaction which takes place between object and organ.

Now, as we have seen, physics deals with sensible things under the aspect in which they are the most profoundly immersed in sensible matter. That is why the obscurity of sensible matter and the subjectivity and anthropomorphism attached to sensibility are of major concern for it. We have seen what means it has devised to triumph over these obstacles and how great has been their success. We have noted that Planck was correct in writing "that as the view of the physical world is perfected, it simultaneously recedes from the world of sense; and this process is tantamount to an approach to the world of reality." (2) But we have also insisted upon the fact that quently limitation and imperfection.

When the term is understood in the first way there is no opposition between it and objectivity. In this sense it may be applied even to God, in whom knowledge is so perfect and therefore so objective that there is no real distinction between the knower, the knowledge and the object known. In its proper meaning, however, there is an opposition between it and objectivity. In fact, a pure subject in this sense is an object which does not know at all.

Now in the knowledge of all creatures, the knower is in some measure a subject in the proper sense of the word. For all creatures receive their knowledge from without and their state of being recipients involves passivity and subjection. This is true even of the angels, for their intelligible species are impressed upon them by God. An object, in its full formality as object, is above every created intellect, for in so far as an intellect is a subject in the proper sense of the term it is measured by the object, and a measure, from the point of view in which it is a measure, is always more perfect than the thing measured. Creatures cannot be the measure of objects because

their being is not the source of these objects. Their cognitive powers cannot reach the very root of these objects because they are not the root.

This subjectivity (in the sense of the term in which it is opposed to objectivity), already found in the highest angel, increases as we descend the hierarchy of created beings. It is found in the fullest measure in which it can be found in sense knowledge, for here a material organ, which in itself is a pure subject and hence absolutely opposed to objectivity, enters into the very intrinsic structure of the cognitive power. But already in the human intellect (which is the lowest type of intellect that could possibly exist) a large measure of subjectivity is found. For the human intellect has this in common with the senses that it receives its species from things. This involves a greater measure of subjection and passivity than is found in angelic knowledge in which the species though coming from the outside, do not come from things (they are, in fact, prior to things) but from God. Now the obscurity arising from this passive subjectivity forces the human intellect to have recourse to a kind of active subjectivity. That is to say, it can know only by constructing logical beings, by composing and dividing in its irretrievably confused with the way by which it is known. Secondly, there is an intellectual intrusion consisting in *a priori* hypothetical construction. Mathematical physics has no other means of getting to know reality except by refashioning it in these two ways. It cannot assimilate reality directly; it can only reconstruct it. It is, as Einstein and Infeld have suggested,(7) in a position something like that of a man trying to understand the mechanism of a closed watch. Since he has no way of opening the case, he cannot know the inside of the watch as it is in itself. All he can do is construct something that will account for the moving of the hands and the ticking. As Meyerson has remarked, "nous voulons le réel conforme à la raison, mais nous comprenons en même temps que s'il était, la raison devrait pouvoir le recréer."(8)

Since, then, the scientific world is formally a subjective construction, it follows that its constitution is predetermined by the methodological principles employed in constructing it. "Operabilia sunt quorum principia sunt in nobis." It also follows that to the extent in which it is so predetermined it can be known *a priori* by a close analysis of these principles and their implications. This, it seems, is the gist of Eddington's *The Philosophy of Physical Science*, the substance of which he has expressed in the following passages:

> Let us suppose that an ichthyologist is exploring the life of the ocean. He casts a net into the water and brings up a fishy assortment. Surveying his catch, he proceeds in the usual manner of a scientist to systematise what it reveals. He arrives at two generalisations:
> (1) No sea-creature is less than two inches long.
> (2) All sea-creatures have gills.
> These are both true of his catch, and he assumes tentatively that they will remain true however often he repeats it.
>
> In applying this analogy, the catch stands for the body of knowledge which constitutes physical science, and the net for the sensory and intellectual equipment which we use in obtaining it. The casting of the net corresponds to observation; for knowledge which has not been or could not be obtained by observation is not admitted into physical science.
>
> An onlooker may object that the first generalisation is wrong. "There are plenty of sea-creatures under two inches long, only your net is not adapted to catch them." The ichthyologist dismisses this objection contemptuously. "Anything uncatchable by my net is *ipso facto* outside the scope of ichthyological knowledge, and is not part of the kingdom of fishes which has been defined as the theme of ichthyological knowledge. In short, what my net can't catch isn't fish." Or - - to translate this analogy - - "If you are not simply guessing, you are claiming a knowledge of the physical universe discovered in some other way than by the methods of physical science, and admittedly unverifiable by such methods. You are a metaphysician. Bah,'"
>
> The dispute arises, as many disputes do, because the protagonists are talking about different things. The onlooker has in mind an objective kingdom of fishes. The ichthyologist is not concerned as to whether the fishes he is talking about form an objective or subjective class; the property that matters is that they are catchable. His generalisation is perfectly true of the class

this movement away from the world of sense and towards the world of reality is at the same time a movement away from the world of reality towards a subjective world in such a way that if it be asked which of the two famous tables of Eddington,[3] (the familiar table and the scientific table) is the more objective and which the more subjective, it is necessary to make a very important distinction: the scientific table is at once more subjective because of the essential subjectivity of scientific method, and more objective, i.e. more like a table as it is known by a superior intellect.

The profound subjectivity of the physico-mathematical world is now generally admitted by all the better scientists.[4] But it is important to try to determine the nature of this subjectivity. By a strange paradox, the movement of science away from the sense world towards the world of reality is at the same time a movement away from the world of reality to a world that is, from one point of view, subjective in essentially the same way as the sense world. What we mean here is that, just as the sense world is subjective in a way that puts a positive subjective element into the object and reconstructs it to the extent of transforming it into something different, so mathematical

physics projects a positive subjective element into its object and reconstructs it into something essentially different. There is therefore, a sharp distinction to be drawn between the type of subjectivity that is characteristic of experimental science and the type mentioned a few moments ago that accompanies other kinds of intellectual knowledge. In the latter case, art merely surrounds the object, whereas in the case of experimental science art enters intrinsically into the object and constructs it. And just as in the case of sense knowledge the objective and the subjective are so interpenetrated that it is impossible for the knower to draw a line between them and thus set forth the object in its pure objectivity, so in mathematical physics the subjective and the objective are so fused that it is impossible for the scientist to disentangle them.[5] In order to do this he would have to have direct intellectual intuition of the real world.

In the course of this study we have endeavored to indicate the most important ways in which subjectivity enters into scientific knowledge.[6] All of them, as has already been suggested, may be traced back to two sources. First there is a physical intrusion of the subject in the experimental operation in which the object known becomes

Lord Rutherford showed us the atomic nucleus did he *find* it or did he *make* it? It will not affect our admiration of his achievement either way - - only we should rather like to know which he did. The question is one that scarcely admits of a definite answer. It turns on a matter of expression, like the question whether the spectroscope finds or whether it makes the green colour which it shows us. But since most people are probably under the impression that Rutherford found the atomic nucleus, I will make myself advocate of the view that he made it. The tendency of writers on quantum theory has been perhaps to go farther than I do in emphasising the *physical* interference of our experiments with the objects which we study. It is said that the experiment puts the atoms or the radiation into the state whose characteristics we measure. I shall call this Procrustean treatment. Procrustes, you will remember, stretched or chopped down his guests to fit the bed he had constructed. But perhaps you have not heard the rest of the story. He measured them up before they left next morning, and wrote a learned paper "On the Uniformity of Stature of Travellers" for the Anthropological Society of Attica . . .

Suppose an artist puts forward the fantastic theory that the form of a human head exists in a rough-shaped block of marble. All our rational instinct is roused against such an anthropomorphic speculation. It is inconceivable that Nature should have placed such a form inside the block. But the artist proceeds to verify his theory experimentally-- with quite rudimentary apparatus too, merely using a chisel to separate the form for our inspection, he triumphantly proves his theory. Was it in this way that Rutherford rendered concrete the nucleus which is scientific imagination had created? . . .

It is difficult to see where, if at all, a line can be drawn. The question does not merely concern light waves, since in modern physics form, particularly wave form, is at the root of everything. If no line can be drawn, we have the alarming thought that the physical analyst is

an artist in disguise, weaving his imagination into everything - - and unfortunately not wholly devoid of the technical skill to realise his imagination in concrete form . . .

The question is raised whether the experimenter really provides such an effective control on the imagination of the theorist as is usually supposed. Certainly he is an incorruptible watch-dog who will not allow anything to pass which is not observationally true. But there are two ways of doing that - - as Procrustes realised. One is to expose the falsity of an assertion. The other is to alter things a bit so as to make the assertion true. And it is admitted that our experiments *do* alter things. (11)

All this undoubtedly conjures up the dreadful spectre of idealism in the minds of many and particularly the neo-Scholastics for whom the stigmatizing phrase "ducit ad subiectivismum" is sufficient to demolish every doctrine which does not propose the univocal type of realism which they consider inseparable from all knowledge. As a matter of fact however, it is only by recognizing the essential subjectivity of scientific knowledge that one can be a true realist. It is for this reason that we have entitled this Chapter "Objective Subjectivity". Most of the critics who have belabored with the redoubtable club of accusation of idealism Eddington and other modern scientists who have tried to bring to light this subjectivity are far more idealists than their victims. For they project into the objective world something that is essentially the product

> of creatures he is talking about - - a selected class perhaps, but he would not be interested in making generalizations about any other class. Dropping analogy, if we take observation as the basis of physical science, and insist that its assertions must be verifiable by observation we impose a selective test on the knowledge which is admitted as physical. The selection is subjective, because it depends on the sensory and intellectual equipment which is our means of acquiring observational knowledge. It is to such subjectively-selected knowledge, and to the universe which it is formulated to describe, that the generalizations of physics - - the so-called laws of nature - - apply.
>
> It is only with the recent development of epistemological methods in physics that we have come to realise the far-reaching effects of this subjective selection of its subject matter. We may at first, like the onlooker, be inclined to think that physics has missed its way, and has not reached the purely objective world which, we take it for granted, it was trying to describe. Its generalizations, if they refer to an objective world, are or may be rendered fallacious through the selection. But that amounts to condemning observationally grounded science as a failure because a purely objective world is not to be reached by observation...
>
> Suppose that a more tactful onlooker makes a rather different suggestion: " I realize that you are right in refusing your friend's hypothesis of uncatchable fish, which cannot be verified by any tests you and I would consider valid. By keeping to your own method of study, you have reached a generalisation of the highest importance—to fishmongers, who would not be interested in generalizations about uncatchable fish. Since these generalisations are so important, I would like to help you. You arrived at your generalisation in the traditional way by examining the fish. May I point out that you could have arrived more easily at the same generalisation by examining the net and the method of using it?"
>
> The first onlooker is a metaphysician who despises physics on account of its limitations;

> the second onlooker is an epistemologist who can help physics because of its limitations. It just because of the limited - - some might say, perverted - - aim of physics that such help is possible . . .
>
> Generalisations that can be reached epistemologically have a security which is denied to those that can be reached empirically... some laws of nature may have an epistemological origin. These are compulsory; and when their epistemological origin is established, we have a right to our expectation that they will be obeyed invariably and universally. The process of observing, of which they are a consequence, is independent of time or place.(9)

It would take us too far afield to analyze and assess the validity of the development and applications which Eddington subsequently makes of the principles laid down in these passages. But after all that has been said about the subjective construction of the scientific world we do not see how the principles themselves can be called into question. Moreover, we feel the implications of these principles are so far reaching that all of the laws of (10)
physics without exception must be recognized as subjective. Later in the same work Eddington lays great stress upon a point that is vital for the question which forms the subject of this chapter: the scientific world is not simply discovered, it is manufactured by the scientist:

> The question I am going to raise is - - how much do we discover and how much do we manufacture by our experiments? When the late

objective world, the latter is reflected in the former and is grasped in some way through it. Whereas idealism seeks to arrive at a maximum of ideas with a minimum of experience, physical science tends towards a maximum of experience in order to arrive at a minimum of ideas. Meyerson has shrewdly pointed out that having started with sensible reality, it is the sensible rather than the reality that physical science tends to dissolve and that this dissolution of the sensible actually results in a reinforcement of the reality. Idealism does just the opposite - - the sensible remains but the reality becomes nothing apart from the ego.(18) It may readily be admitted that as physics advances in its theoretical elaborations it seems to take on more and more the character of idealism.(19) But the likeness is only superficial. For in idealism subjectivity is an end in itself. In physics, on the contrary, it is only a means; its character is purely functional. Because the whole purpose of the subjectivity of physics is to carry the mind to a greater measure of objectivity, it is essentially different from the subjectivity of idealism. There can be no doubt that Relativity physics for example is much more subjective than Classical physics was. But at the same time it is far more objective, for it has purged physics of innumerable subjective elements that were lurking unsuspected in the

Classical system. It delivered physics from the subjectivism of individual observers and made all systems of coordinates equivalent for the expression of the general laws of nature.

> There is another side to the theory of relativity. We have pointed out in the beginning how the development of science is in the direction to make it less subjective, to separate more and more in the observed facts that which belongs to the reality behind the phenomena, the absolute, from the subjective element, which is introduced by the observer, the relative. Einstein's theory is a great step in that direction. We can say that the theory of relativity is intended to remove entirely the relative and exhibit the pure absolute.(20)

2. Mathematical Physics and Kantianism.

Eddington sums up the substance of his *Philosophy of Physical Science* in the following terms:

> The subjective laws are a consequence of the conceptual frame of thought into which our observational knowledge is forced by our method of formulating it, and can be discovered *a priori* by scrutinising the frame of thought as well as *a posteriori* by examining the actual knowledge which has been forced into it.(21)

It is impossible to read these lines without finding them reminiscent of Kantianism. And as a matter of fact, as we noted in Chapter I, Eddington himself draws explicit of the mind. They are in many respects worse than the Platonists of whom St. Thomas writes: "Ex hoc in sua positione erravit (Plato) quia credidit quod modus rei intellectae in suo esse sit sicut modus intelligendi rem ipsam."(12) From what was said in the last Chapter about logical identity it is evident that they identify the logical and the real in reality, and that is essentially idealism. Nor can the subjectivity of scientific knowledge be considered a falsification of reality, as Professor De Koninck has pointed out:

> Ne disons pas que les concepts de la science reposent en définitive sur une distorsion du monde et que dès lors les documents du physicien sont par avance forgés et trahissent la réalité. Mais justement il ne faut pas se laisser abuser par cette distorsion. Les documents sont fidèles à leur façon et ne nous trompent que lorsque nous leur prêtons une signification à laquelle ils ne prétendent pas. Est-ce que la lumière est un malin génie qui se joue de nous lorsqu'un bâton plongé dans l'eau paraît brisé? Pas plus que mon poste de T.S.F. n'est responsable de ce que mes enfants croient qu'il y a un monsieur caché dans la boîte. (13)

It is futile to try to rule out the subjectivity of mathematical physics as some modern Scholastics have done by appealing to the Thomistic doctrine that ideas are not id quod sed id quo cognoscitur.(14) For while it is true that in non-reflexive knowledge an idea is a mere quo which

which carries the mind to a quod and not just to itself known as an idea, the quod to which the mind is thus carried may be either objective reality or a construction of the mind. We hold that in mathematical physics the quod to which the mind is carried is formally something that is manufactured by the mind - - though not without dependence upon objective reality.

If the subjectivity which we have attributed to the scientific world be rightly understood there is no reason to fear idealism. While insisting upon this subjectivity Eddington likewise insists upon the fact that it can never be more than partial - - that objectivity is also essential to physical science.(15) Meyerson has shown how great and how constant is the concern on the part of all the greatest scientists to remain in as close a contact as possible with an objective universe. This is true even of physicists like Einstein and Schrodinger whose theories seem to have the greatest likeness to idealism.(16) Whereas idealism begins with a denial of the objective universe, physical science begins by postulating its existence.(17) All through its development the contact with this objective universe remains unbroken. And even though science constructs its own subjective world as something distinct from the speculative knowledge is reducible to objects of experience alone.[24] At the same time, however, he insists upon the fact that experience alone is not sufficient to explain scientific knowledge, that the mind cannot simply be measured by external reality but must in some way become its measure; in other words, that true scientific knowledge must be a priori knowledge. His intimate acquaintance with the physics of his time made it evident to him that the universality and necessity of scientific concepts could not be derived from the singularity and contingency of experience and consequently had to be a contribution of the mind.

We have already intimated, particularly in Chapter IV, to what extent Kant was justified in arriving at this conclusion. We have seen that experimental science by its very nature demands that the mind by means of hypothetical constructions of its own making supplies for the universality and necessity which experience cannot provide, and even predetermines experience. We have seen that he was correct in maintaining that in experimental science the mind cannot know reality as it is in itself; it can only approach it provisionally. And in getting to know reality, it necessarily fashions and forms it according to

its own preconceived ideas. Kant's great mistake as we said a moment ago consisted in making experimental science the pattern and norm of all speculative knowledge. This mistake did not derive from the fact that he conceived all speculative science as necessarily composed of an a priori element as well as an element drawn from experience, for that is perfectly true, but rather in the fact that he failed to recognize that there are two essentially different kinds of a priori elements. For in so far as philosophy of nature, for example, is universal and necessary it contains an a priori element in the sense that this universality and necessity rises above, and hence is independent of singular contingent experience. This a priori element, however, does not consist in something posited by the subject, but in something revealed by the object, namely an analytical and hence necessary truth concretized in the singular contingent experience.

As we saw in Chapter IV, it is precisely because the mind is unable to discover truths of this kind in experimental science that it is forced to have recourse to another kind of a priori element which is conferred by the mind. And in so far as this type of knowledge is concerned, Kant was justified in making synthetic a priori judgments the attention to the remarkable affinity between Kantian epistemology and the modern developments of physics. Let us recall his words once again:

> If it were necessary to choose a leader from among the older philosophers, there can be no doubt that our choice would be Kant. We do not accept the Kantian label; but, as a matter of acknowledgement, it is right to say that Kant anticipated to a remarkable extent the ideas to which we are now being impelled by the modern developments of physics. (22)

Nor is Eddington the only one who has drawn attention to this affinity. From the start the Theory of Relativity has seemed to have profound philosophical implications and it has been a natural tendency to attempt to associate it with some philosophical system. And, as Meyerson has remarked,[23] the philosopher whose name has been mentioned the most frequently by the relativists themselves (Einstein seems to be an exception) has been Kant.

As is well-known, Kant was perfectly conversant with Newtonian physics, and had a vast admiration for it. This admiration led him into two serious errors. First, he considered Newtonian physics to be definitive. For him it was not merely dialectical; on the contrary it had the supreme certitude of science in the strict sense of the word. Secondly, not only was it a perfect science,

but it was the perfect science. In other words the properties of physics became for him the criteria for all speculative science. And that is why the *Critique of Pure Reason* is in the last analysis nothing but a critique of physical science, or more exactly, a critique of speculative knowledge in terms of physical science. These two fundamental errors necessarily compromised the validity of the whole epistemological structure of Kant, but they did not prevent him from seizing upon the proper nature of physical science - - at least in an obscure way. That is what we must now try to see. And our brief analysis will consider two points: first we shall try to see how Kant seized upon the general nature of physical science; secondly we shall consider the relevance of his doctrine for mathematical physics in particular, and especially with regard to its object. It is this second point that is of greater interest for us.

It is well-known that Kant erected his philosophical system as a reaction to the empiricism of Hume in which he recognized the utter destruction of all true science. But this reaction did not blind him to the essential role that experience plays in science. In his introduction to the *Critique of Pure Reason* he makes it clear that all methodological principles which constitute the very essence of the scientist's approach to reality, and that as a consequence a close examination of these principles makes it possible to know a priori the fundamental lines of this construction, just as the examination of the fisherman's net makes it possible to know a priori a great deal about the nature of his catch. Because these methodological principles do not change, because they are fixed forms which are essential to the very nature of experimental science, the laws which are known in this a priori way have a necessity that those deriving from experience do not have. And in all this there is certainly a striking affinity with the Kantian categories.

But of greater importance in this study of the relation between Kanticnism and mathematical physics is the consideration of the similarity between Thomistic doctrine with regard to the object of mathematical physics and Kant's doctrine of sensible intuition. (25) Let us recall the substance of what Kant has to say about sensible intuition. Early in his Critique of Pure Reason he explains what he means by intuition in general. (26) He defines it as the necessary means by which all knowledge is related to objects and which all thought uses in order to attain them.

Kant agreed with Aristotle that all our knowledge begins in the senses and he held that all intuition as found in man is necessarily sensible — it has to do with an object furnished by sensation. Nevertheless, he felt that sensible intuition could not consist merely in the reception of physical data coming from external reality. For his whole purpose, as is well known, was to save science from the devastation it had received at the hands of both the extreme rationalists who had followed in the wake of Descartes and of the extreme empiricists such as Hume. And he thought that this could be accomplished only by considering the whole structure of science as determined by a kind of noetic hylemorphism in which the matter would be a posteriori and furnished by physical reality and the form would be a priori and provided by the subject. That is why in setting out to disclose and analyze the a priori forms of cognition he felt that such forms should be found even in our sensible intuition of the external world, in such a way that even our direct experience with nature would consist in a fashioning of physical reality by the subject.

And in order to explain how this is possible he distinguished between two aspects of intuition: pure intuipivotal point of science. It should be recalled that for Kant synthetic a priori judgments were those in which there is added to a subject a predicate that is essentially extrinsic to it. As a result such judgments were a purely artificial synthesis consisting in an accidental composition whose unity derived from the mind. Their truth was not founded upon the principle of contradiction as was that of analytical judgments, but on the possibility of experimental verification.

Now all this is a fairly accurate description of the type of judgments that are characteristic of experimental science. We have seen that experimental science is based essentially upon induction by enumeration. If it were to limit itself to the individual cases of the enumeration ("This A is B") its judgments would be purely synthetic, and it would be completely deprived of the character of science. On the other hand, induction by enumeration can never give true universal natures and hence analytical judgments with the a priori knowledge that is characteristic of such judgments. That is why experimental science must necessarily have recourse to synthetic a priori judgments in which the a priori element is something conferred by the mind. When, therefore, experimental science declares: "Every A is B",

this judgment is at once synthetic, because based on purely synthetic judgments ("This A is B", "That A is B", etc.) and a priori, because the form of universality is conferred by the mind without adequate foundation in nature. However, because of the regularity found in the multiplicity of cases, it must be noted that such a judgment is neither purely synthetic nor purely a priori.

Because judgments of this kind are not founded upon the principle of contradiction but upon the possibility of experimental verification they can never be anything more than hypothetical. Because of his belief in the definitive character of Newtonian physics Kant failed to recognize their hypothetical nature and attributed to them perfect necessity that derived from absolutely fixed forms of thought which were his categories. The dissolution of the Classical system has shown how unwarranted his assumptions were in this regard. Nevertheless it must be noted that, in spite of the essentially transitory character of the hypothetical constructions of experimental science, Kant was not wholly wrong in attributing a fixed and necessary character to the a priori element found in it. For earlier in this Chapter we saw that the construction of the scientific world is predetermined and shaped by the

> lui-même sensation, il suit que, si la matière de tout phénomène ne nous est donnée qu' a posteriori, la forme en doit être a priori dans l'esprit, toute prête à s'appliquer à tous, et que par conséquent, on doit pouvoir la considérer indépendamment de toute sensation.(87)

It should be fairly evident that pure intuition and the form of the phenomenon are merely two aspects of the same thing. Pure intuition is the a priori form in so far as it is considered as a determination of pure sensibility. The form of the phenomenon is the same a priori form considered in relation to the manifold of sensations to which it is applied and to which it gives order and unity.

It is to be noted that in the passage just cited, Kant, in speaking of the union of the a priori form with the matter of sensation, uses the word "application". This is significant. For it brings out the fact that in this union the form is essentially extrinsic to the matter. If the very being of the phenomenon arises from the extrinsic application of one of its composing elements to the other, it follows that it can be nothing but an artificial composite whose unity is purely accidental.

Now the close affinity between this object of sensible intuition and the object of mathematical physics

as analyzed in this study should be immediately apparent. This affinity is found both in the fact that the two objects are accidental composites, and in the very nature of the elements which enter into the composition. In so far as the composition itself is concerned, it is clear that in both cases there is a union of two elements one of which plays the part of matter and the other that of form. In both cases the form is something essentially extrinsic to the matter, and as a result the union consists merely in an application of one to the other effected by the knowing subject. Consequently, the union is in both cases something purely accidental, something due to the mind rather than to nature, and hence the resulting composite is an artefactum.

A similar affinity is found in the very elements which go to make up the composite. For in both cases the material element is a sensible datum, something deriving from physical nature, and the formal element is something drawn from mathematics. In both cases the mathematical form orders and rationalizes the physical datum and gives it scientific significance.

It is easy to see why for Kant the application of mathematics to nature is not only possible but even tion and empirical intuition. The former is sensible intuition considered from the point of view of pure sensibility, that is to say, from the point of view of the capacity of the knower to receive objects coming from the sensible world, prescinding from actual sensation and from any particular objects that such sensation might furnish. The latter is sensible intuition considered from the point of view of actual sensation of physical objects. In pure sensibility he discovered certain forms or determinations which were *a priori* in the sense that they were prior to all actual sensation and hence completely independent of it. These *a priori* forms of sensibility which constituted pure intuition were space and time.

Now it is extremely significant that for Kant space and time were the object of mathematics. He defined mathematics as the science which considered these two *a priori* forms of sensibility in abstraction from all concrete sensible data. Space constituted the object of geometry which deals with lines and figures; time constituted the object of arithmetic because it deals with numbers which are a succession of units.

It is evident from what has just been said that

for Kant sensible intuition involves something more than just sensibility in the ordinary sense of the word. It is in fact not merely sensible knowledge, but intellectual knowledge. It is called sensible because of its dependence upon sensation which provides it with the matter to which the *a priori* forms are applied.

Now the two *a priori* forms of space and time which when taken by themselves in abstraction constitute the object of mathematics, when applied to actual sensation caused by physical reality constitute something that Kant calls a *phenomenon*. This phenomenon is a composite made up of two elements: a material element which is *a posteriori* and derived from nature through actual sensation, and a formal element which is *a priori* and consists in the forms of pure sensibility. Only by the application of the latter to the former can the raw materials of knowledge coming from nature be unified, ordered, rationalized, made significant, and rendered capable of entering into the structure of science.

> Ce qui, dans le phénomène, correspond à la sensation, je l'appelle matière de ce phénomène; mais ce qui fait que le divers qu'il y a en lui est ordonné suivant certains rapports, je le nomme la forme du phénomène. Comme ce en quoi seul les sensations peuvent s'ordonner, ou ce qui seul leur permet de les ~~[illegible]~~ à une certaine forme, ne saurait être ramener ...

extreme there is the position of those who remove the mathematical world so far from the physical world that in mathematical physics the former remains a pure instrument, a pure logical or linguistic tool, in relation to the latter. In this position the object of mathematical physics is also simply and perfectly one; that is to say, it is a pure physical object to which mathematics remains completely extrinsic.

There is something highly significant in the wide divergence of these two opinions. For it brings out the fact that the mathematical world is at once extremely close to and extremely distant from the physical world. When this is grasped, it becomes easy to understand why modern authors such as Einstein have divided geometry into two branches of which one is very distant from the physical world, and the other identified with it. The first branch consists in purely formal knowledge based on free creations of the mind and schematic concepts devoid of all content, and the second in a natural science known as practical geometry. As we noted in Chapter VI this is actually a denial of the true nature of geometry, since the first branch seems to be nothing but dialectics, and the second nothing but a physical science. The distance between

the physical world and the mathematical world and the closeness of them was also a problem for Plato, as we saw in Chapter I. On the one hand he drew them into a union that was extremely intimate in the sense that he made the physical world indefinitely amenable to mathematization and conceived of this mathematization as a revelation of a logos that is proper to nature. On the other hand, he created an immeasurably wide gulf between them by conferring upon the mathematical world an ontological existence that was independent of the physical world. There is this to be noted immediately about the distance created by Einstein between the two worlds and that created by Plato: in the first case the gulf can be bridged in the sense that the dialectics can be successfully and fruitfully applied to the physical universe as an instrument, even though it must ever remain essentially extrinsic to the object of physics, whereas in the case of Plato, as we intimated in Chapter I, in the measure in which the mathematical world is conceived to have an ontological existence of its own, not only must it remain extrinsic to the object of physics, but it cannot even be used as an instrument in relation to the physical world.

We believe that it is possible to hit the very heart of the problem of mathematical physics by saying that both Plato and the moderns have erred by making the mathematical world at once both too close to the physical world, and too distant from it. In the Thomistic solution of the problem they are brought together without identification and separated without the creation of a gulf between them. And once this has been understood it becomes possible to see how mathematics can enter intrinsically into the object of physics and at the same time remain extrinsic to it and serve as an instrument. It also becomes possible to see that the object of mathematical physics is not something simply and perfectly one, but rather something that is under one aspect one, and under another dual. Because it is one, Aristotle and St. Thomas could conceive of mathematical physics as a science. But because it is at the same time dual, they found it necessary to conceive of it as a <u>scientia media</u>. Let us try to analyze these points and see how they fit together.

In the first place, Aristotle and St. Thomas make a definite and clear-cut distinction between the physical world and the mathematical world by means of their doctrine of the different degrees of formal abstraction.

The physical world must be studied in the light of the first degree of formal abstraction. It is a world of mobility and everything in it must be defined in terms of sensible matter. The mathematical world is the result of the second degree of formal abstraction. It is a world of immobility and everything in it must be defined without sensible matter. Once we have made this initial distinction and turn to examine the nature of the abstraction by which the mathematical world is set off from the physical world something very significant immediately strikes us. For there is a peculiar quality about mathematical abstraction that is not found in either physical or metaphysical abstraction. In both of these latter cases there is a correspondence between the way the object concerned exists outside the mind and the way it exists inside the mind. The object of physics depends upon sensible matter both for its being and for its "being known". The object of metaphysics is independent of sensible matter both for its being and for its "being known". But the object of mathematics is on the one hand dependent on sensible matter for its being, that is to say, for any existence it can be said to have outside the mind, and on the other independent of sensible matter for its "being known". In this dichotomy between the way mathematical objects are conceived and the way they exist lies the secret of the distance between the mathematical world and the physical world and their closeness. But before attempting to see why this is so, it is significant to note that both Plato and the moderns conceive of the distance between the two worlds in a way that puts mathematics in a state which can in some sense be said to correspond to the third degree of abstraction. We explained in Chapter II that both metaphysics and logic fall within the general category of those sciences whose object is free of all matter. Metaphysics arrives at this state by means of positive abstraction, logic by means of negative abstraction. Now in so far as Plato attributes an ontological existence to abstract mathematical forms he conceives of them as though they were separated substances. And that is why, as we noted in Chapter I, his metaphysics is a kind of mathematical metaphysics. On the other hand, in so far as the moderns identify mathematics with dialectics they make of it a kind of logic. To put mathematics into the third degree of abstraction is to separate it too far from the physical world and at the same time not far enough. It is only by analyzing the proper nature of the second degree that we can understand the true nature of its separation. But before insisting upon this separation, let us try to see why mathematics ever remains in close contact with physical reality.

The mathematical world is intrinsically and essentially linked to the physical world. As we remarked in Chapter VI, if the material world were impossible, the mathematical world would also be impossible. Since prime matter is the principle of homogeneity, and since homogeneity is the fundamental postulate of all mathematics, there is no possibility of mathematics without an intrinsic reference to prime matter. In other words, it is only in a world of composed essences, in which formal oppositions are incomplete because of the common matrix of prime matter that the mathematical world can originate. All mathematical notions are drawn from the physical universe, and even after the separation of abstraction has taken place, they still retain a necessary connection with the world of matter. For unlike the case of metaphysical abstraction, the separation effected by the mind in simple apprehension cannot in the case of mathematics be transposed to the second operation of the mind. The essence of the judgment is the copula, and this expresses existence, and if mathematical entities are to exist at all they must exist in the physical world. In the universe of matter there are lines and circles and triangles which may be considered the physical counterparts of mathematical lines and circles and triangles, even though the realization of the latter in the former is not perfect since they lose what is proper to them as abstract entities through their realization in the material universe.

The fact of this loss suggests how far the mathematical world is from the physical world in spite of the nearness upon which we have just been insisting. In a sense the mathematical world is farther removed from the physical world than is the world of metaphysics. For while mathematical being has a necessary relation with the real physical world, it never retains the ontological essence of the thing with which it is connected. Metaphysical abstraction does. And that is why the communia entis can be said to be realized directly in the physical world as well as in the world of separated substances. Mathematical entities are not realized directly in the physical world. In other words, by the very fact that metaphysics deals with sensible beings in so far as they are beings, its notions can be predicated of the physical universe. Mathematical entities on the other hand can be predicated directly of nothing existing in physical reality, precisely because they are defined in a way in which they cannot exist, that is, as separated from sensible matter.

While all sciences deal with the abstract, the mathematical sciences are the only sciences which deal with the abstract precisely as abstract. Their world is an

autonomous world, set apart from reality, and governed by its own intrinsic laws. In it the mind is eminently free. It deals with notions originally drawn from the physical world, but notions which have been transformed into a condition that is especially congenial to its own nature. Though dealing with things originally connected with sense matter, it is not bound down to the necessity of having its processes terminate in the external senses. Though its notions always retain some kind of physical reference, they acquire a pliancy and a capacity for manipulation that are utterly foreign to the physical world.

All this is at the basis of the doctrine of John of St. Thomas that the mathematical world prescinds not only from the actual exercise of existence, but also from any intrinsic order to existence, and that as a consequence mathematical being is indifferent to either real or logical being, just as the essence of relation consisting in the esse ad is indifferent to either real existence or purely logical existence. And this explains why it has been possible for modern mathematicians to build elaborate dialectical superstructures upon mathematical foundations — dialectical superstructures which, while essentially distinct from mathematical structures, are nevertheless based upon them and in some way patterned after them. These dialectical superstructures have immeasurably increased the pliancy and instrumentality of mathematics.

The foregoing makes it clear that the mathematical world is an intermediary world between the purely material and the purely immaterial worlds. And this explains why mathematics can at the same time enter intrinsically into the object of mathematical physics and at the same time remain extrinsic and serve as an instrument. And while being a medium between the material and the immaterial, mathematics is at the same time a medium between the objective and the subjective, as is evident from the last paragraph. This immeasurably increases its effectiveness as a scientific instrument, because it gives freedom to the mind to elaborate its own rational schemas, and at the same time provides the possibility of these schemas being applied to cosmic reality.

Having in this way solved the problem of the distance and the closeness between the mathematical world and the physical world and explained in a general way how it is possible for mathematics in mathematical physics to enter intrinsically into the object and at the same time remain extrinsic as an instrument, it remained for Aristotle, St. Thomas, Cajetan and John of St. Thomas to work

out this possibility in fuller and more specific detail. This they did in their doctrine of subalternation and <u>scientia media</u>.

In mathematical physics, physics is subalternated to mathematics in the fullest sense of the word; that is to say, there is subalternation by reason of the object. This means that object of the subalternated science contracts the object of the subalternating science by adding something to it. The addition, however, can be only an accidental difference, for otherwise there would be no formal distinction of sciences. This is an important point because it means that the matter of the subalternated science remains extrinsic to that of the subalternating science even though the two enter into composition.

As soon as we examine the nature of the elements entering into mathematical physics another reason for this extrinsic character presents itself. For mathematical entities are united with physical elements in the state of idealization that is proper to mathematical abstraction. This union is, therefore, not a direct concretion of mathematical entities in sensible matter. It does not consist in something that would be merely the reverse of mathematical abstraction - - the mere putting back of mathematical entities into the sensible matter from which they were drawn. This means that the composition of the two can never be anything more than the application of the former to the latter. In other words, it is a composition that is not discovered, but created by the mind; it is a logical composition. It is something remarkably similar to the Kantian "phenomenon", and from this point of view as well as from the point of view of the innumerable predetermining a priori elements that the mind contributes to reality in all experimental science, many concessions must be made to Kantianism by a realistic philosophy of mathematical physics.

Now the union between the two worlds is effected by the mind principally through a process of measurement which lays hold of the quantitative determinations in nature directly, and indirectly of the other determinations in so far as the former can serve as surrogates of the latter. But our processes of measurement can never be anything more than approximative, and herein we find a third reason why the mathematical world remains essentially extrinsic to the physical world. If it were merely a question of the first two reasons, mathematical physics could still be a science in the strict sense of the word. The third reason, however, prevents it from being a true science and makes it dialectical.

In fact, at this level it has already become doubly dialectical. For by the very fact that it is experimental science, physics is without a true propter quid and has to have recourse to mere probable reasoning; and the attempt to find a propter quid in mathematics only results in an approach to nature which is so extrinsic that it provides nothing better than a substitutional and approximative propter quid.

In so far as the mathematical element which enters into composition with the physical element always remains extrinsic to it, the object of mathematical physics is dual. But from another point of view it is one. For in the first place, even though the composition in question is logical, it is not completely logical. The elements involved are brought together by the mind -- but for an objective reason. Even though the mathematical entities applied to nature retain their abstract and idealized state, the fact remains that they do have physical counterparts in nature. And the union between the two elements is so intimate that mathematical physics employs a unique type of abstraction, an intermediary abstraction which participates in the nature of both mathematical and physical abstraction at the same time.

But the most important point in connection with the unity of the object of mathematical physics is that a scientia media does not have as its object simply and directly the composite of the two elements considered as an accidental being. In mathematical physics only the physical element is considered directly. The mathematical element is considered obliquely, in so far as it is connoted by the physical element and in so far as it informs and modifies it and thus makes it scientifically fruitful by providing a source of new properties. In this way, even though there is no res media, there can be a scientia media.

In this notion of connotation we touch the very heart of the Thomistic philosophy of mathematical physics. For it explains how the object of the science can be at the same time one and dual, how mathematics can be brought into intimate contact with physics and yet retain its distance, its autonomy and freedom, and how it can enter intrinsically into the object which specifies mathematical physics and at the same time remain an instrument. The very fact that it is the physical element that is considered directly and per se, whereas the mathematical element is brought into the consideration obliquely and connotatively makes the role of the latter essentially functional. Moreover, while this gives wide

scope to the exercise of the functional role by leaving mathematics the autonomy that is native to it and by thus making it possible for it to exploit all of the conceptual richness and virtuosity that is intrinsic to its nature, it keeps the mathematical elaborations completely subordinated to, and always essentially orientated towards, the physical element.

One gets an idea of how wide is the scope granted to mathematics in Thomistic philosophy of mathematical physics when one recalls that in the structure of a mixed science an accidental element taken from the lower science is added to the object of the higher science. This means that from the point of view we have in mind here the physical element is merely an accidental addition to the mathematical element. Moreover, the latter plays the role of form in relation to the former. This means that in mathematical physics the illumination and conceptual determination comes from mathematics. As a result, even the things that are most proper to the study of nature lose their purely physical status and are mathematicized: motion is transformed from a becoming into a state; the flow of time becomes a dimension; the four causes are reduced to the formal cause; etc.

In taking advantage of the freedom that all this gives to mathematics, the mathematical physicist is not obliged to have a direct and immediate physical counterpart for every mathematical element he incorporates into his conceptual structure. The notion of connotation keeps the mathematical elaborations essentially orientated towards physical reality, but this orientation must not be understood in too narrow a sense. It is possible to maintain the essential contact that connotation implies even though mathematical elements which have no direct physical counterparts are introduced in order to enhance the theoretical power of mathematics in so far as it is employed as an instrument. In elaborate physico-mathematical theories the essential connotation is maintained by means of the text or dictionary.

The mathematical physicist, therefore, is free to push the pliancy and instrumentality of mathematics to the limit. In doing so, he may, if he wishes, go out beyond the limits of mathematics in the strict sense of the word and construct dialectical superstructures which will give greater scope to this theoretical explanation of physical reality. Even though the application of these dialectical constructions to physical reality does not

constitute mathematical physics in the strict Thomistic sense of the word, it is governed by the same general principles and follows the same general pattern as the latter. Through the use of these dialectical constructions mathematical physics, which is already doubly dialectical, becomes triply dialectical.

The objectum formale quod of mathematical physics is the physical considered as connoting the mathematical, and hence from this point of view it is more physical than mathematical ("magis naturalis quam mathematica"): its whole aim is to get to know the physical world and not the mathematical world. Its objectum formale quo is the special type of abstraction that is proper to it, which, while it participates in the nature of both mathematical and physical abstraction, is more mathematical than physical, since mathematics gives the propter quid and plays the part of form; hence from this point of view, mathematical physics is more mathematical than physical ("magis affinis mathematicis"). Though formally mathematical, it is not specifically mathematical. For in it mathematics is applied to a physical object in order to constitute a new subject and new principles proper to a science concerned with physical reality. Consequently it is specifically distinct from both pure physics and pure mathematics. Since it is not a science in the strict sense of the word, but dialectics, it has no habitus that is proper to it. The habitus that rectifies the intellect in it is the habitus of logic. However, mathematical physics is not pure dialectics. It proceeds per modum scientiae.

2. The Existence of Mathematical Physics.

Having seen how in relation to the problem of the essence of mathematical physics Thomism steers a middle course between the two extreme positions indicated at the beginning of this Chapter it will be helpful in order to round out this summary to explain how it likewise steers a middle course in relation to a problem which in a general way can be called the problem of the existence of mathematical physics. We have intimated that for some scholastics the grounding of physics upon mathematics is an error which should never have been committed or at best a mere historical accident. At the other extreme is the opinion of those who hold that this grounding of physics upon mathematics is so necessary that no other valid way of studying reality is [illegible]. True Tho[illegible] is [illegible] of these opinions.

Against the first opinion it holds that the subalternation of physics to mathematics is not only legitimate, but necessary and inevitable. In the course of our analyses we have indicated a number of reasons why this is so. Perhaps it would be well to recall the more important reasons. The very definition of science itself cognitio certa per causas, gives us the central reason. For experimental science is neither certain knowledge, nor is it knowledge of things in their proper causes. Hence physics has a double reason for reaching out to a scientia propter quid, i.e. mathematics, in order to obtain for itself at least a substitute certitude and a substitute propter quid. Doxa naturally aspires to the status of episteme; the "infirmus modus demonstrandi" that is characteristic of study of material nature, particularly in its concretion, seeks support in the more sure type of demonstration that is found in mathematics.

Moreover, physics is inevitably lead to abandon the attempt to treat nature in terms of the proper sensibles and to substitute the common sensibles for them. For sense cognition is to some extent necessarily subjective, and at the same time extremely limited, and as a consequence knowledge of nature in its concretion that [illegible] upon the proper sensibles is necessarily anthropomorphic. Hence it lacks the objectivity and intersubjectivibility that all science seeks to attain. Moreover, the proper sensibles are in many respects irrational: they cannot be defined; they are incapable of analysis; they are deficient in communicability; they can neither be demonstrated nor be the principles of demonstration; they are isolated. For all these reasons physics is lead to treat nature in terms of the common sensibles. And since these are all reducible to quantity, this inevitably results in the subalternation of physics to mathematics. For only the consideration of quantity in the light of mathematical abstraction has sufficient rationality to carry physics forward towards its goal.

Physics becomes subalternated to mathematics because through this subalternation the mind is able to realize its natural desire to triumph over the heterogeneity of reality through homogeneity. The mathematization of the cosmos provides a homogeneity which while it breaks down the barriers isolating the specific properties of nature and thus triumphs over their pure givenness, at the same time makes it possible to maintain contact with these specific properties through their quantitative surrogates.

In other words it affords at the same time both a unity to provide for what is lost by the emergence of physics from generalities, and a distinctness to enable the mind to follow its natural movement towards concreteness. The mathematization of nature makes it possible for the intellect to realize its instinctive desire to know reality in terms of what is most knowable for it (and thus make up for what is lost by drawing away from generalities) and at the same time in terms of what is most knowable <u>in se</u> (and thus make up for the deficiencies of purely generic knowledge.) In pure physics there is always an opposition between what is most knowable for the mind and what is most knowable <u>in se</u>. Hence the inevitable tendency to ground physics upon the one science in which what is most knowable for the mind is at the same time most knowable <u>in se.</u> And this grounding enables the intellect to realize its natural desire for deduction. Since the universals found in pure natural doctrine are merely universals <u>in praedicando</u>, natural science if left to itself cannot become a purely deductive system. Hence the inevitable turning to mathematics which is the deductive science par excellence because its universals are similar to universals <u>in causando.</u>

As natural doctrine moves towards concretion it is getting farther and farther away from the knowledge of nature that is most in conformity with the human intellect. In this can be found another reason for its turning to that science which is of all the sciences the most in conformity with the human mind. The least rational of the speculative sciences reaches out to the most rational to supply for its deficiencies. In this way the mind is able to study its most natural object (the essence of material things) through the science that has the greatest connaturality for it. The mathematization of the cosmos enables the mind to fulfill its natural tendency to dominate its object, to impose its laws upon it, to become prior to it, to triumph over its givenness, to construct it, and to get at its most profound aspect: the order of the whole.

A final reason for the subalternation of physics to mathematics must be added here. We have seen that by its very nature experimental science is led to express itself through symbols rather than through names. Mathematics provides the most perfect symbolic system for this expression.

Because of these reasons and many others that might be added, it is manifestly erroneous to consider the grounding of physics on mathematics an accident or a mistake. On the other hand it is equally erroneous to make this grounding so necessary that no other valid approach to reality remains possible. Thomism avoids this opposite extreme by situating mathematical physics accurately in the whole epistemological scheme. When this is done it becomes evident that not only is mathematical physics not the only approach to reality in general, since metaphysics is a valid science and the most important of all the purely human sciences, but it is not even the only approach to physical reality, since it is only the part of natural doctrine that is advanced towards concretion that requires subalternation to mathematics. Philosophy of nature remains a valid approach to the cosmos, and one which in many respects is of greater importance than the approach of mathematical physics, since it deals with the most fundamental problems of the universe and since it provides knowledge of the most noble natural form - - the human soul.

Thomism recognizes the worth and importance of mathematical physics. It believes that the most profound knowledge one can have of reality is knowledge of it in its *proper* causes, and from one point of view at least mathematical physics comes closer to this type of knowledge than philosophy of nature. Thomism even goes so far as to hold that in mathematical physics the mind possesses a knowledge of the cosmos which in many respects is like the knowledge that God has of nature, since it carries the mind far along the road towards knowing reality in its specific concretion. At the same time Thomism insists upon the many profound limitations that are inherent to the type of knowledge that mathematical physics provides. In the first place, it is not science in the strict sense of the word, but merely dialectics. It is not a mansion of residence, but a vehicle of progress - - a vehicle of progress that must travel over a road that has no end. Thomism believes that even though it is better to make progress than to stand still, *per se* a mansion of residence is more perfect than a vehicle of progress. By the very fact that it is experimental science mathematical physics can never arrive at universal and necessary propositions, and must remain in probable reasoning. Its definitions are operational and cannot give the *quod quid est* of things. It can get at the objective *logos* only by projecting a subjective *logos* into nature, in such a way that the two become inextricably intermingled. Because it is subalternated to mathematics, the only type of knowledge

it can give of nature is that provided by measurement. The data out of which its whole structure is built is, in the last analysis nothing but pointer readings. Now metric knowledge is at best an extremely meager kind of knowledge. For it comes to grips only with the quantitative determinations of nature; it is utterly blind to all the determinant properties of things in their specific essences, to the very inner nature of things, to all that is of greatest significance for philosophy, for art, and for human life itself. But it cannot even get at the quantitative determinations of reality in the sense of being able to tell us what these determinations are. By the very fact that it is "quantitative" knowledge it is not "quidditative" knowledge. It cannot answer the question "what", but only the question "how much?" And it cannot answer this question in any absolute way, since a *minima mensura* in continuous quantity is a contradiction in terms. It can give us only knowledge of ratios determined by arbitrary standards. Nor is it possible to progress indefinitely in the direction of a *minima mensura*. And besides all this, other innumerable limitations of metric knowledge result from the maze of hypotheses in which all measuring processes are involved, from the physical interactions between the measuring instrument and reality, from all the cosmic influences that enter into every measurement, etc.

For all these reasons the physico-mathematical world can be nothing more than a shadow world, in spite of (or rather precisely because of) all the Cartesian clarity with which it becomes suffused in the light of mathematical intelligibility. The true natures of things remain in the background. As a matter of fact, mathematical physics does not get to know the objective world in its absolute state directly; it knows it indirectly by constructing an imitation of it - - an imitation which is better than the objective world because more rational, but at the same time worse, because its whole purpose is to lead to the objective world in its absolute condition. The physico-mathematical world is not a formal sign, but an instrumental sign of the absolute world condition. Between the two there is a relation of isomorphism. The mind must ever try to bridge the gap between the two worlds by bringing the scientific world ever closer to the absolute world. But in coming continually closer, the two continually get farther apart. The reason is that the scientific world is at once essentially subjective and essentially objective, and the more objective it gets, the more subjective does it become.

This subjectivism of the scientific world does not favor idealism, since its whole purpose is to orientate the mind toward the absolute world condition. As a matter of fact it is only by admitting this subjectivity that it is possible to escape idealism, for otherwise one inevitably mistakes one's own mental constructions for objective reality.

While rejecting the exaggerations of scientism which have tended to make physico-mathematical method the only valid approach to reality, Thomism recognizes the truths which scientism has exploited for its own ends, and the source from which has come the spell that mathematical physics exercises over the mind. In mathematical physics the intellect is allowed to indulge in unlimited speculation in the realm that is most connatural with it - - that of mathematics, and this speculation is inseparable from construction in which the intellect posits its own object. At the same time this speculation brings it closer to the object that is most proper to it - - the essence of material things. And this intimate knowledge of material things reveals the plasticity and malleability that is native to them and thus gives the mind the power to re-fashion nature to its own image and likeness. Because man is composed of matter and spirit there are two fundamental tendencies in him: to draw everything from matter, and to draw everything from spirit. The quantitative homogenization of the cosmos and the study of it in the light of the abstract rationality of mathematics makes it possible for him to realize both of these tendencies simultaneously. Or to put the thing in a slightly different way: the combination of the first and second degrees of formal abstraction enables a man to be at once an idealist and a realist. The induction of experimental physics satisfies his desire to know cosmic reality; the deduction of mathematics satisfies his desire for perfect rationality. The first without the second leads him into impenetrable obscurity; the second without the first cuts him off from reality. The combination of the two provides a way out of obscurity and a way back to reality. More than that, it provides man with a kind of wisdom - - not the divine wisdom of metaphysics which is so far above him, which is only loaned to him in a very inadequate way and never really given to him, and in which he must make his way with continual strain and effort, but a human wisdom - - one to which his mind is particularly attuned, and in which he can move with comparative ease and security. It is a

wisdom whose ideal is to see the whole of cosmic reality in the light of a few fundamental mathematical formulae. Already the Einsteinian system has brought us far along towards this ideal. And if, as it is only natural to hope, Relativity and Quantum physics can eventually be integrated into a unified system, man will have come near to realizing his ideal. This is the wisdom to which Descartes dedicated himself - - a wisdom that is not restricted to an elite, but one in which all men can share on equal footing, a wisdom so connatural to man that as he tells us in his Regulae, if a student only follows the right rules "there is nothing, generally speaking, that any other man is able to know that he himself will not be capable of knowing." And this wisdom not only satisfies the mind's desire to dominate its object in the speculative order, it also satisfies its desire to dominate it in the practical order, for, as is well known, technological fruitfulness has inevitably followed in the wake of every advance in theoretical physics. Small wonder then that this type of knowledge has been transformed into a philosophy of life, that it has become the light of the world.

The great error of scientism has been to believe that the knowledge most connatural to man is also the knowledge most essential for him.

APPENDIX