oportet quod ista abstractio seu immaterialitas prius reluceat in mediis, seu principiis: et ita ex diversitate mediorum, prout diversimode illuminant, sumitur diversa ratio formalis scientiae, ut D. Thomas docet (II-II q. 1, a.1; et q. 9, a.2 ad 3)." (21)

When we do not possess sufficient understanding of the subject in question in order to grasp it by its proper principles, there remains one way of discovering something about it, even though what we discover will not be certain. We can proceed by the extraneous logical principles that attach to second intentions, to topical argument. It is this usefulness for inquisition that merits for dialectics the name of <u>logica inventiva</u>.

CHAPTER SIX

SITUATION

In considering any created substance it is possible to distinguish what is essential, what makes it to be in act a substance, from those accidental perfections which complete it. In material substances, the first of these perfections is that of quantity, that is the order of the parts in the whole. Because a given substance is by its essense, say, a stone, it has virtually and indistinctly parts. What orders these parts and causes them to be distinct is the accident of quantity. This is why we speak of quantity as being parts outside of parts, of order in extension.

> "In sententia S. Thomae propria et formalis ratio quantitatis est extensio partium in ordine ad totum quod est reddere partes formaliter integrantes. Unde remota quantitate substantia non habet partes integrales formaliter in ratione partis ordinatas et distinctas." (1)

To prove that this is really St. Thomas's opinion, John of St. Thomas quotes several texts, notably one from the <u>Summa Totius Logicae</u>:

> "Quod positio uno modo dicitur ordo partium in loco, et sic est unum de praedicamentis, quod dicitur situs; alio modo positio est ordo partium in toto, et sic positio est differentia quantitatis". (2)

It may be objected that the <u>Summa Totius Logicae</u> is not a work of St. Thomas. John of St. Thomas refers to several authentic texts, however, that state the same doctrine. It will be sufficient to give one from the commentary on the <u>Physics</u>:

> "Nam situs, secundum quod ponitur praedicamentum, importat ordinem partium in loco: licet secundum quod ponitur dif-

ferentia quantitatis, non importat nisi ordinem partium in toto." (3)

It is necessary to point out, as John of St. Thomas has done in explaining a similar passage in De Trinitate, that here situs is not taken formally sed pro radice, and he gives as his reason that otherwise it would not be necessary to point out the difference between the predicament of situs and the formal difference of quantity. (4)

In Metaphysics V, XIII, 1020 a 7-8, Aristotle gives a definition of quantity based on one of its properties, its divisability into homogeneous parts:

"Quantity means that which is divisible into constituent parts, each or every one of which is by nature some one individual thing." (5)

The explanation of the significance of this passage is given by John of St. Thomas:

"Et tota explicatio reducitur ad hoc, quod definit Aristoteles quantum per divisibile, non in partes physicas, id est materiam et formam, nec in partes potentiales, sicut anima dividitur in intellectivum et sensitivum, nec in subiectivas, sicut universale dividitur in inferiora, sed in partes integrales et quantitativas, quae ita sunt compositae, ut facta divisione maneat unaquaeque aliquid unum, sicut patet cum aqua dividitur in varias partes. In has partes integrales divisibilis est res quanta." (6)

In material things quantity is the first of the accidents, the one that immediately follows substance and constitutes it in the perfection that existence requires. So close is the connection between substance and the first of the accidents that many have mistaken quantity for material substance itself.

"Negaverunt hoc aliqui Nominales, qui cum ex una parte sentirent sine quantitate nullas dari partes nec divisibilitatem, ex alia vero parte cuiuscumque rei entitatem nihil aliud quam partes suas existimarent, dixerunt quantitatem non distingui realiter a re habente partes, sive substantia sit sive accidens." (7)

This confusion springs, no doubt, from the fact that the two are never separated in any existing thing. (We speak here of the natural order, and not of such mysteries as the Holy Eucharist). To conclude from the fact of their inseparability to their identity is an example of the fallacy of accident. They are not separable, but they are distinguishable a parte rei. A material substance is constituted in act a substance by its form, that which is expressible in its definition. Because it is thus constituted, it has virtually and confusedly parts. What makes these parts exist distinctly and in their due order is the accident of quantity.

Because of its close connection with substance, quantity alone of all the accidents can found a science. St. Thomas has pointed out this peculiarity of quantity:

> "Sciendum autem est, quod quantitas inter alia accidentia propinquior est substantiae. Unde quidam quantitates esse substantias putant, scilicet lineam et numerum et superficiem et corpus. Nam sola quantibas habet divisionem in partes proprias post substantiam. Albedo enim non potest dividi, et per consequens nec intelligitur individuari nisi per subjectum. Et inde est, quod in solo quantitatis genere aliqua significantur ut subjecta, alia ut passiones." (8)

Quantity, being the first of the accidents, and the most fundamental one, is treated as if it were a substance and the science of mathematics demonstrates its properties. Even in mathematics it is ultimately the substance as modified by the accident of quantity that we study. We divide quantity first into its two main species; the discrete and the continuous. After this we further divide number into its various species and the continuous into line, surface, and solid. Finally we demonstrate the properties of each of these kinds of continuous and discrete quantities in the sciences of arithmetic and geometry.

It is not only the mathematician who treats of quantity, though. To be able to distinguish well the mode of consideration of the mathematician, it is well to note the manner of procedure of the metaphysician and the natural philosopher.

To define quantity and its main species belongs to the metaphysician. It is he alone who considers it according to its absolute essence as an accident of material substance. This consideration is an absolute one and must be reducible to the intelligible nature of things alone without reference to the sensible mode of existence as such. (9)

The natural philosopher considers the quantity of things when he notes that one thing is larger than another and seeks the cause for it. He speaks of the quantity of something as it is physically united to the other accidents and as determined within definite limits by form and figure. It is possible to ask why the neck of the giraffe is so disproportionately long. Here we have to deal with a neck of a certain (perhaps statistically) fixed length and with a quantity not conceived separately from the sensible qualities of the giraffe. It is clear that such a problem and any tentative solutions that may be proposed must be tested by whether or not they conform to our sense experience of the giraffe. This is what is meant by saying that in natural philosophy the reduction is to the senses.(1

The mathematician considers quantity neither absolutely, like the metaphysician, nor according to the sensible conditions of its existence, like the natural philosopher. His mode of consideration is an altogether special way of regarding quantity. He considers it in its pure homogeneity as extension, either discrete or continuous, and without any sensible qualities whatever. In fact any quantity is qualitatively determined by form and figure so that it has a precise physical limitation. The quantity of the mathematician is interminate quantity. By this is meant that it is thought of as having an underdetermined size. The triangle of the geometer is any triangle that the proposition requires, and not a triangle with sides six inches long.

> "Pro cujus intelligentia advertendum est ex Cajetano (in praesenti quaest. 5, a.3), quod quantitas potest dupliciter abstrahi. Uno modo secundum abstractionem generis vel speciei ab individuis, remanente tota natura et quidditate quantitatis, sicut omnes aliae naturae quando in universali concipiuntur: et haec abstractio fit ab intellectu universalizante naturam; et hoc modo quantitas in abstracto consideratur a metaphysico, et sic non amittit rationem perfectionis neque boni. Alio modo fit abstractio quantitatis denudando illam a sensibilitate, et fit per imaginationem: sicut imaginamur distantiam quantitatis in vacuo, lineas aut superficies in eo imaginantes; et talis abstractio non est universalis a particulari, sed solum quantitatis interminatae, seu imaginatae, a sensibili; sicut si in relatione aliquis consideraret puram rationem _ad_, et non rationem _in_, solum consideraret id quod est commune relationi reali et rationis: non autem id quod perfectionis et realitatis est in relatione."
> "Constat autem ad demonstrationes mathematicas perinde se habere lineas et figuras imaginarias, et reales; nam etiamsi in vacuo imaginemur lineam, aeque bene potest ibi fieri demonstratio mathematica. Si vero fiat linea in aliqua materia reali, non considerat illam quantum ad suam quidditatem realem (id enim pertinet ad metaphysicum), sed solum quantum ad proportionem mathematicam. Quare ex vi talis abstractionis omittit rationem boni, quia non attenditur ejus perfectio vel convenientia, aut conducentia ad aliquid: sed solu est extensio imaginata prout continuitatem, commensutationem vel proportionem habet, ut docet D. Thomas." (11)

John of St. Thomas says the quantity of the mathematician is midway between the real quantity existent in physical things according to a real determination and the imaginary quantity of a pure being of reason like, say, a line in a vacuum.

"Ita mathematica considerat quantitatem, quantum ad id praecise quod habet de extensione interminata, et secundum id quod habet a materia: non secundum terminationem et modum quem habet a forma, ratione cujus redditur sensibilis. Quare quantitas mathematica habet conceptum positivum quantitatis interminatae, eo modo quo quantitas potest inveniri, sive imaginarie, sive sensibiliter in ratione entis veri. Unde permissive se habet ad rationem entis realis et veri: neque positive includendo et considerando adaequate, neque positive excludendo per repugnantiam, realitatem ipsius quantitatis. Et in hoc differt a quantitate pure imaginaria, quae est ens rationis: haec enim repugnanter se habet ad quantitatem realem, quia ens rationis est. At vero quantitas mathematica non repugnanter se habet, sed indifferenter: quia aeque bene potest facere suas demonstrationes in lineis realibus, vel imaginariis; sicut si relatio consideretur secundum rationem ad praecise, nondum consideratur ut ens rationis: nec tamen ut determinate ens reale: sed indifferenter ad illud; quia non consideratur adaequata ratio ejus ex omni parte quae requiritur ad realitatem, ad quam etiam requiritur rationi: sed ex ea parte qua indifferens est ad realitatem, et solum explicat rationem ad. Sic quantitas consideratur a mathematico inadaequate, et sub ea ratione praecise extensionis interminatae; quae indifferenter se habet ad imaginariam et realem, et sic non excludit rationem entis, sed permittit: neque repugnanter se habet ad illud, sed indifferenter. Und nec ens rationis est determinate, nec ens reale determinate: sed indifferenter et permissive se habet ad utrumque." (12)

It is perhaps for this reason that Aristotle confines the use of the term "abstraction" to the mode of conceiving of the mathematician. (13)

Because he envisages quantity in this manner, the mathematician is in a peculiar way confined to working with abstractions. He not only considers things apart from their particularity, as does the metaphysician and the natural philosopher, but according to a mode incompatible with existence.

Since all of the external senses must convey their object according to the mode in which they have received it, and quantity exists completely separately from any qualitative determination, it is impossible for the mathematician to reduce the object of his consideration to the external senses. The imagination can make this separation and convey the pure homogenity of extended parts. It is for this reason the St. Thomas, following Aristotle, says that mathematics is reducible to the imagination.

This is particularly clear in the passages where intelligible matter, the matter of mathematical objects, is treated.

In Met. VII, lect.X, St. Thomas is contrasting the objects of natural philosophy with those of mathematics:

> "Nec differt utrum singularia sint sensibilia vel intelligibilia. Singularia quidem sensibilia sunt sicut circuli aerei et lignei. Intelligibilia singularia sunt sicut circuli mathematici. Quod autem in mathematicis considerentur aliqua singularia, ex hoc patet, quia considerantur ibi plura unius speciei, sicut plures lineae aequales, et plures figurae similes. Dicuntur autem intelligibilia, hujusmodi singularia, secundumquod absque sensu comprehenduntur per solam phantasiam, quae quandoque intellectus vocatur secundum illud in tertio de Anima: "Intellectus passivus corruptibilis est."
> "Ratio autem hujus est, quia materia, quae principium est individuationis, est secundum se ignota, et non cognoscitur nisi per formam, a qua sumitur ratio universalis. Et ideo singularia non cognoscuntur in sua absentia nisi per universalia. Materia autem non solum est principium individuationis in singularibus sensibilibus, sed etiam in mathematicis. Materia enim alia est sensibilis, alia intelligibilis. Sensibilis, quidem ut aes et lignum, vel etiam quaelibet materia mobilis, ut ignis et aqua, et hujusmodi omnia; et a tali materia individuantur singularia sensibilia. Intelligibilis vero materia est, quae est in sensibilibus, non inquantum sunt sensibilia, sicut mathematica sunt. Sicut enim forma hominis est in tali materia, quae est corpus organicum, ita forma circuli vel trianguli est in hac materia quae est continuum vel superficies vel corpus." (14)

To this we should add a passage from the commentary on Met. VIII

> "Deinde cum dicit "est autem". Solvit praedictam dubitationem in mathematicis: et dicit quod duplex est materia: scilicet sensibilis et intelligibilis. Sensibilis quidem est, quae concernit qualitates sensibiles, calidum et frigidum, rarum et densum, et alia hujusmodi, cum qua quidem materia concreta sunt naturalia, sed ab ea abstrahunt mathematica. Intelligibilis autem materia dicitur, quae accipitur sine sensibilibus qualitatibus vel differentiis, sicut ipsum continuum. Et ab hac materia non abstrahunt mathematica." (1

Lastly there is a passage in the Summa that brings the whole matter into the sharpest light. What this contributes beyond the other passages is the relating of intelligible matter to substance:

> "Dicendum quod quidam putaverunt quod species rei naturalis sit forma solum, et quod materia non sit pars speciei. Sed secundum hoc in definitionibus rerum naturalium non poneretur materia. Et ideo aliter dicendum est quod materia est duplex, scilicet communis, et signata vel individualis; communis quidem, ut caro et os; individualis autem, ut hae carnes et haec ossa. Intellectus igitur abstrahit speciem rei naturalis a materia sensibili individuali, non autem a materia sensibili communi. Sicut speciem hominis abstrahit ab his carnibus et his ossibus, quae non sunt de ratione speciei, sed partes individui, ut dicitur in VII Metaph.; et ideo sine eis considerari potest. Sed species hominis non potest abstrahi per intellectum a carnibus et ossibus.
>
> Species autem mathematicae possunt abstrahi per intellectum a materia sensibili non solum individuali, sed etiam communi; non tamen a materia intelligibili communi, sed solum individuali. Materia enim sensibilis dicitur materia corporalis secundum quod subiacet qualitatibus sensibilibus, scilicet calido et frigido, duro et molli, et huiusmodi. Materia vero intelligibilis dicitur substantia secundum quod subiacet quantitati. Manifestum est autem quod quantitas prius inest substantiae quam qualitates sensibiles. Unde quantitates, ut numeri et dimensiones et figurae, quae sunt terminationes quantitatum, possunt considerari absque qualitatibus sensibilibus, quod est eas abstrahi a materia sensibili; non tamen possunt considerari sine intellectu substantiae quantitati subiectae, quod esset eas abstrahi a materia intelligibili communi. Possunt tamen considerari sine hac vel illa substantia; quod est eas abstrahi a materia intelligibili individuali.
>
> Quaedam vero sunt quae possunt abstrahi etiam a materia intelligibili communi, sicut ens, unum, potentia et actus, et alia huiusmodi, quae etiam esse possunt absque omni materia, ut patet in substantiis immaterialibus." (16)

It is important to realize that there are two irreducible species of quantity: the continuous and the discrete. (17)

Continuous quantity is the order of undivided parts in a whole; it is undivided extension. Extension, in turn, is of three kinds: length, breadth, and thickness. It is of these that geometry treats.(18)

Discrete quantity is the order of parts, distinct and separate, though homogeneous. Here, the word "whole" in the definition of quantity (as the order of parts in a whole) has a distinct meaning. The "whole" of continuous quantity is something physically one, as, for instance, this stone. That of discrete quantity is a unity of order in separate things, as, for instance, seven oranges on my desk. There is something about each orange in the group that allows it to be regarded as as joining with the others to make up semen oranges. They do not lose their identity and become one large orange, but, each remaining separate, they make up seven together. What formally constitutes them as seven is the last unity, the seventh. This is not to say that one is eternally predesignated as the last one I will count. It does not make any difference which one I count last; for this will in no way affect the total. We say that there are seven because there are one more than six present. It is the additional unit that changes the number from one species to another, and hence we speak of the last unit as the constitutive one.

In all this we suppose a homogeneity among the things that are numbered. It is evident that we cannot add horses, bottles, and angels, and get a predicamental number, since these things are not differentiated mainly by number as are oranges, but rather by their forms. The most fundamental note of predicamental quantity is homogeneity. This permits us to ignore qualitative differences and consider nothing but pure quantity. (17)

Number, the class to which all discrete quantities belong, is defined as a multitude measured by one. (18)

"Sic igitur unum, secundum quod simpliciter dicitur ens indivisibile, convertitur cum ente. Secundum autem quod accipit rationem mensurae, sic determinatur ad aliquod genus quantitatis, in quo proprie invenitur ratio mensurae.

Et similiter pluralitas vel multitudo, secundum quod gnificat entia divisa, non determinatur ad aliquod genus. Secundum autem quod significat aliquid mensuratum, determinatur ad genus quantitatis, cujus species est numerus. Et ideo dicit quod numerus est pluralitas mensurata uno, et quod pluralitas est quasi genus numeri. Et non dicit quod sit simpliciter genus; quia sicut ens genus non est, proprie loquendo, ita nec unum quod convertitur cum ente, nec pluralitas ei opposita. Sed est quasi genus, quia habet aliquid de ratione generis, inquantum est communis.

Sic igitur accipiendo unum quod est principium numeri et habet rationem mensurae, et numerum qui est species quantitatis et est multitudo mensurata uno, opponuntur unum et multa, non ut contraria, ut supra dictum est de uno quod convertitur cum ente, et de pluralitate sibi opposita; sed opponuntur sicut aliqua eorum quae sunt ad aliquid, quorum scilicet unum dicitur relative, quia alterum refertur ad ipsum. Sic igitur opponitur unum et numerus, inquantum unum est mensura et numerus, est mensurabilis.

Et quia talis est natura horum relativorum quod unum potest esse sine altero, sed non e converso, ideo hoc invenitur in uno et numero, quia si est numerus, oportet quod ubicumque est unum, quod sit numerus." (20)

It is important to understand the nature of the one that is the principle of number. There is a transcendental predicate of being whereby it is undivided. This predicate belongs to all that is inasmuch as it has being. One, the principle of number adds the notion of measure to this formal indivision of each thing. Predicamental measure is that whereby the quantity of a thing is known. Its essential characteristics are homogeneity with what is measured and greater simplicity and knowability than it.

"Deinde cum dicit "semper autem". Ponit secundum quod considerandum est circa mensuram; dicens, "quod metrum", idest mensura, semper debet esse cognatum, scilicet ejusdem naturae vel mensurae cum mensurato sicut mensura magnitudinum debet esse magnitudo: et non sufficit quod conveniat in natura communi, sicut omnes magnitudines conveniunt: sed oportet esse convenientiam mensurae ad mensuratum in natura speciali secundum unumquodque, sic quod longitudinis sit longitudo mensura, latitudinis latitudo, vox vocis, et gravitas gravitatis, et unitatum unitas.

Sic enim oportet accipere ut absque calumnia loquamur; sed non quod numerorum mensura sit numerus. Numerus autem non habet rationem mensurae primae, sed unitas. Et si unitas mensura est, ad significandum convenientiam inter mensuram et mensuratum, oportet dicere, quod unitas sit mensura unitatum, et non numerorum. Et tamen si rei veritas attendatur, oportebit hoc etiam concedere, quod numerus esset mensura numerorum, aut etiam unitas numerorum similiter acciperetur. Sed non similiter dignum videtur dicere unitatem esse mensuram unitatum, et numerum numeri, vel unitatem numeri; propter differentiam, quae videtur esse inter unitatem et numerum. Sed istam differentiam observare, idem est, ac si quis dignum diceret quod unitates essent mensurae unitatum, sed non unitas; quia unitas differt ab unitatibus ut singulariter prolatum ab his quae pluraliter proferuntur. Et similis ratio est de numero ad unitatem; quia numerus nihil aliud est quam pluralitas unitatum. Unde nihil aliud est dicere unitatem esse mensuram numeri, quam unitatem esse mensuram unitatum." (21)

Thus one must be identical with each of the things to be counted, and further, it must simply by repeating itself render an adequate account of the number of the group of homogeneous objects. (22)

One, the principle of number, is not a number, since it is not a multitude. It must be indivisible or it is not one. When we divide an orange, we have not two oranges, but rather no orange at all. Likewise with anything else that is said to be one; it cannot be further divided into homogeneous parts or it is not one.

"Assignat autem rationem, quare mensuram oportet esse aliquid indivisibile; quia scilicet hoc est certa mensura, a qua non potest aliquid auferri vel addi. Et ideo unum est mensura certissima; quia unum quod quod est principium numeri, est omnino indivisibile, nullamque additionem aut subtractionem suscipiens manet unum. Sed mensurae aliorum generum quantitatis imitantur hoc unum, quod est indivisibile, accipiens aliquid minimum pro mensura secundum quod possibile est. Quia si acciperetur aliquid magnum, utpote stadium in longitudinibus, et talentum in ponderibus, lateret, si aliquod modicum subtraheretur vel adderetur; et semper in majori mensura hoc magis lateret quam in minori." (23)

It is only because modern mathematicians conceive of number dialectically as containing the property of infinite divisability that they have accepted fractions as numbers.

St. Thomas says that number arises from the division of the continuum. (24) The fundamental notion of quantity is the order of parts in the whole. When we add to this the notion of actual division between homogeneous parts, we have the idea of number. It is only the division of the continuum that is capable of giving us the notion of number, because the number system is potentially infinite and any actual number of real things is finite.(25) The number system is potentially infinite, since no matter how large a number one conceives, it is always possible to think of a larger by adding one to it. The continuum is actually not divided at all, but it is capable of being divided indefinitely. Thus it is the ideal basis for generating the number system.

The proper notion of number includes nothing then but the order of absolutely homogeneous parts. Because of its abstraction from all qualitative determinations it can be applied to all homogeneous objects. The proper notion of seven applies to seven oranges or seven blocks of wood. It is not with these applications of number that arithmetic deals, but rather with the numbers themselves conceived as species of discrete quantity. Othe wise, arithmetic would not be founded on the mode of defining proper to mathematics.

The traditional designation for predicamental number is <u>numerus numeratus</u>. (26) This applies strictly to homogeneous parts ordered under the last unity in them. In contrast to this proper sense of number, there is also the dialectical notion of number which is called <u>numerus numerans</u>. It is by a conception of this kind that even non-homogeneous things can be counted. We can say that an angel, a bottle, and a grain of sand are three things. This will be true, but the three which is said of these things will not be a predicamental number, since it does not measure really homogeneous things. A quasi-homogeneity is given them by the mind when it unites them in the logical genus of thing, but this is not a real homogeneity such as is required to constitute the number which is the subject of arithmetic.

This second sense in which number is used, namely, numerus numerans, is formed by the mind through a further prescinding grom the properly quantitative notion of number. Just as numerus numeratus is attained by leaving aside all qualitative differences between different things and retaining only the pure notion of parts of a homogeneous continuum, so numerus numerans leaves aside even homogeneity and considers only the order that can be conceived of as existing between non-homogeneous objects when they are treated by the mind as if they shared in some genus. (27)

Classical arithmetic, that of Euclid in Books VII through IX of the Elements, for example, treats only of numerus numeratus. Modern mathematics is founded rather on numerus numerans. We will here consider the mode of procedure of these two disciplines in order to see the profound differences that there are between them; how, for instance, one is perfectly scientific in its procedures and the other is only dialectical.

The first proposition of the seventh book of Euclid's Elements is: "Two unequal numbers being set out and the less being continuously subtracted in turn from the greater, if the number which is left, does not measure the other, the original numbers will be prime to one another". For instance, if I choose two numbers 29 and 7, and then set them forth thus: "29-7 equals 22; 22-7 equals 15; 15-7 equals 8; 8-7 equals 1. Of the original numbers, 29 and 7, the proposition states that I can say that they are prime to one another. By this is meant that they have no other common measure than one. The proof is as follows:

> "For, the less of two unequal numbers AB, CD being continually subtracted from the greater, let the number which is left never measure the one before it until an unit is left;
> I say that AB, CD are prime to one another, that is, that an unit alone measures AB, CD.
> For, if AB,CD are not prime to one another, some number will measure them.
> Let a number measure them, and let it be E; let CD, measuring FH, leave an unit HA.
> Since, then, E measures CD, and CD measures BF, therefore E also measures BF.
> But it also measures the whole BA; therefore it will also measure the remainder AF.
> But AF measures DG; therefore E also measures DG.
> But it also measures the whole DC, therefore it will also measure the remainder CG.
> But CG measures FG; therefore E also measures FH.
> But it also measures the whole FA; therefore it will also measure the remainder, the unit AH, though it is a number: which is impossible.
> Therefore no number will measure the numbers AB, CD; therefore AB, CD are prime to one another."(28)

Euclid's method of proof is to suppose that they have another common measure and then to show that as result we must assume that a greater number can measure a lesser one. Everything in the enunciation and the proof depends on the simple notion of numbers and the operations that are possible following the nature of each number. It is possible to subtract one number from another because the second is greater than the first, and it is impossible that a greater number should measure, that is divide evenly, a smaller one. This is all that is necessary to state the proposition and to prove it. In other words, in Euclidean arithmetic problems and proof can be reduced to

1. certain self-evident principles, as, for instance, those about greater and less quantities
2. to the simple notions of unity and the numbers.

All operations performed in it are justified by these same perfectly certain starting points.

Let us contrast this scientific mode of procedure with modern algebraic handling of a problem. For instance, it can be stated that <u>a</u> - x equals y. If this is to have meaning in the sense of classic arithmetic <u>a</u> must be greater than x, but modern algebra is not content with this limitation. It strives to be perfectly general, that is to make a statement that will be true for all values of its symbols. If <u>a</u> equals 8 and x equals 10, algebra invents -2 in order to permit the interpretation of the equation. Likewise in order that all numbers can be divided, fractions are invented. Furthermore, other symbols are used to allow the extraction of the square roots of all numbers including such numbers as 2 which do not have square roots. Algebra thus reverses the method of arithmetic by inventing what passes for numbers to justify its operations instead of making the range of possible operations depend on the nature of the numbers involved. Even the rules for operation become extremely mysterious sometimes. It is impossible in the range of elementary algebra to understand why a minus number times a minus number gives a positive one, though this must be adopted as a rule of procedure from the very first.

"The use however, of the same terms in these two sciences will by no means imply that they possess the same meaning in all their applications. In Arithmetic and Arithmetical algebra, addition and subtraction are defined or understood in their ordinary sense, and the rules of operation are deduced from the definitions: in Symbolical Algebra, we adopt the rules of operation which are thence derived, extending their application to all values of the symbols and adopting also as the subject matter of our operations or of our reasonings, whatever quantities or forms of symbolical expression may result from this extension: but, inasmuch as in many cases, the operations required to be performed are impossible, and their results inexplicable, in their ordinary sense, it follows that the meaning of the operations performed, as well as of the results obtained under such circumstances, must be derived from the assumed rules, and not from their definitions or assumed meanings, as in arithmetical algebra."

....

"In as much as the result of symbolical addition and subtraction are obtained from an assumed rule of operation, and not from the definition of the operation itself, it will follow that their meaning, when capable of being interpreted, must be dependent upon the conditions which they are required to satisfy."

....

"It appears, therefore, that in the case of negative symbols, the operation of addition is no longer associated with the fundamental idea of increase, nor that of subtraction with that of decrease: and thus a change of sign from plus to minus, in the symbol operated upon, is equivalent to a change of operation from addition to subtraction and conversely."

....

"The signs plus and minus, when prefixed to symbols denoting quantities of the same kind, cannot denote modifications of magnitude, but only such affections or qualities of the magnitudes represented, as are convertible by the operations of addition and subtraction: it is on this account that -<u>a</u> can admit of no interpretation, as compared with <u>a</u> or plus <u>a</u>, when <u>a</u> denotes an abstract number, to which no qualities are attributed."

....

"Quantities and their symbols are said to be real or possible, when they can be shewn to correspond to real or possible existences: in all other cases, they are said to be unreal, impossible or imaginary. It will follow, therefore, that when positive symbols represent real quantities, the same symbol with a negative sign will be said to be impossible or imaginary, whenever they are not capable of an interpretation, which is consistent with the conditions they are requred to satisfy. It remains to shew that there exist large classes of magnitudes which possess qualities which can be correctly symbolized by the signs plus and minus, and that consequently the terms negative and impossible are not coextensive in their application."

(29)

Algebra is justified as a means of calculating the measurement of continuous quantities. Its great operational facility makes it an apt instrument in the physical sciences where it is necessary to have a formula that measures one sort of length or area in terms of another. Here, what is sought is not the properties of a subject in terms of its own intelligibility, but an approximation of one definite (and in this sense even variable quantities are definite) quantity in terms of another.

What would be unjustified intellectually would be to think that algebra and arithmetic are mathematics in the same sense. One starts with simple self evident statements immediately graspible by the mind without experience; the other, with a dialectical meaning of number justified only in terms of operation. (30) One is intended as a means of understanding the nature of the objects it studies; the other is used as a means of calculation merely. One is absolutely certain and scientific. The other is a dialectical since it effectuates its operations through beings of reason and is only significant when its results are capable of interpretation in terms of the physical objects to which it ultimately relates.

While it is justifiable to teach algebra to even young students, even necessary in view of the needs of the experimental sciences, it is sophistry to pretend that its "numbers" are numbers in the same sense as those of arithmetic, or that its operations are the same. This is the effect of the teaching of algebra to those and especially by those not perfectly capable of distingUishin dialectical reasoning from what is scientific. Those who have not this ability may with perfect safety undertake the teaching or even the prolonged study of algebra. Otherwise the mind is warped by being exercised in a discipline where it is not possible to know the definitions of what is being studied and where one reasons according to rules that one does not understand. If such procedures are confused with science, then the natural tendency is to believe that all reasoning is postulational, and that it is impertinent to ask what one is talking about. Perhaps the widespread atrophying of the philosophic temper of mind sometimes observed in modern students can be traced to such sophistical formation.

CHAPTER VII

GEOMETRY

Quantity is, as we have seen, the order of parts in a whole. When these parts are discrete we have numbers; if they are not, we have continuous quantity. Aristotle has pointed out several ways in which a quantity can be continuous. (1) If, for instance, the parts are united by a ligature or by contact or in any other such artificial way we have a kind of continuity, but this is not the kind of union of parts in which continuity properly consists. It is possible to characterize the properly continuous as that wherein the end of one part is the beginning of the other. (2) It is this infinite density that is proper to the continuum, and it is from this characteristic that all its other properties, (its infinite divisability, for instance,) follow.

As we pointed out in the previous chapter, mathematics does not study quantity as quantity. More particularly, geometry does not study the continuum as such, but rather it studies the interminate continuous quantity according to the special manner it is grasped in mathematical abstraction. It is this special manner of consideration that distinguishes the geometer and the arithmetician from the metaphysician and the natural philosopher. These remarks are prefaced only to avoid a possible confusion between a study like this one for instance, and one that would be properly mathematical.

Extension, which is synonomous with continuous quantity, can be of three kinds: length, breadth, and thickness. (3) These are the primary species of all that belongs per se to continuous quantity.

Point, which is defined by Euclid as that which has no parts (4) is the principle of a line, just as a line is the term of a surface and a surface is the term of a solid. A line must be terminated by points, otherwise it would be infinite and this is impossible.

> "Similiter, se esset longitudo infinita, non esset linea. Linea enim est longitudo mensurabilis." (5)

These points are actual, that is to say, they are the real terminations of the line. Besides these terminal points, every line has an infinity of points in potency. None of these are actual, however, or we would have not one line but many.

What is said here about the termination of every line does not conflict with the interminateness of mathematical lines, since mathematics does not envisage lines as infinite, but merely as not having any definite length.

Point differs from one, principle of number, in that it is not actually present in the line, and hence it cannot measure it. It is for this reason that there is no minimum measure in continuous quantity. Hence it is, also, that any measure in the genus of continuous quantity is imperfect. Ideally a measure must be first and absolutely simple in order to make known the quantity of what is being measured. If it is not perfectly simple it requires something beyond itself to make its own quantity known.

Even position belongs to point only per accidens. John of St. Thomas gives a satisfying explanation of this.

> "SECUNDO COLLIGITUR punctum separatum non esse per se in loco; caret enim quantitate, et illud dicitur per se esse in loco, quod potest per se loco moveri. Puntum autem non potest per se moveri loco, (ut dicemus infra q. 20. art. ult.,) quia correspondet tantum indivisibili loci. Si autem moveretur per se, immediate post corresponderet alteri indivisibili; non dantur autem duo indivisibilia immediate. Solum ergo punctum est per accidens in loco ratione partium, quae ponuntur in loco."(6)

This can also be seen from the fact that points interpenetrate, and that no two things can be in the same place at the same time.

> "Itaque plura indivisibilia non requirunt distinctum locus, sed tanguntur se totis, quia idem est in illis tangi et tangi se totis, cum careant partibus; faciunt tamen, ne partes penetrentur, quia faciunt ne partes se totis tangantur, sed solum pense extremitates...... quod punctum continuativum inhaeret parti inadaequate, scilicet toti parti, sed non totaliter, nec eam comprehendo et undique penetrando, cum sit illi improportionatum, utpote indivisibile cum divisibili, sed inhaeret parti tamquam subiecto ut principium eius vel finis."(7)

Line, surface, and solid are the ultimate subjects of geometry. They, like all that belongs per se to the predicament of quantity, have a quasi-substantial character, as St. Thomas remarks, and thus we can demonstrate their proper passions of them. These ultimate subjects are not constructed, but they are abstracted from material being. We are not studying something of our own creation when we study geometry, but something objective, an aspect of material being.

It is important to insist on this objective character of geometry, since Knat's view (8) that its object is something purely ideal has obtained a wide currency. This is no place to enter into an explanation of his position. For our purpose it is sufficient to oppose the Aristotelian doctrine of mathematical abstraction to it, and to insist on the consequent objectivity of all mathematics in the proper sense, namely all that which deals with species of what is per se quantitative.

No less false than the Kantian view of the object of geometry, is the opinion that geometry is the study of space, that is of the actual dimensions and shape of the material universe. This view is expressed in some mathematical writings from at least the time of Descartes. (9) In a way Kant did not deny this opinion, but merely transformed it by making space a category of perception rather than something physically existent in the universe as his predecessors had done.

It is sometimes alleged in favor of modern dialectical geometries like those of Riemann and Lobachevski that they are or may be truer than the Euclidean because they describe more accurately the actual physical universe than Euclid's geometry does. Such a statement is based on a false understanding of the nature of geometry. Continuous quantity as abstracted from material beings contains nothing but the notion of pure extension in its three dimensions. Attached to these fundamental concepts are properties like straight and curved for line, the species of polygon for figure and the various kinds of solids. What constitutes the science of geometry is the demonstration of the properties that follow upon the demonstration of its ultimate subjects. It is completely per accidens that a given theorem manifest or does not manifest the properties of the whole physical universe or one of its parts. It belongs to a scientia media like astronomy to apply the findings of geometry in the physical world.

Thus the true concept of the object of geometry is that it is something real, something not fabricated by the mind, but rather abstracted in such a way that all sensible qualities that belong de facto to the physical universe are not included.

It is important to see that it is sensible qualities that are excluded from mathematics and not necessarily all qualities. St. Thomas gives the example of hot and cold and others like them to illustrate what mathematics leaves out.

> "Sensiblis quidem est, quae concernit qualitates sensibiles, calidum et frigidum, rarum et densum, et alia hujusmodi, cum qua quidem materia concreta sunt naturalia, sed ab ea abstrahunt mathematica. Intelligibilis autem materia dicitur, quae accipitur sine sensibilibus qualitatibus vel differentiis, sicut ipsum continuum. Et ab hac materia non abstrahunt mathematica." (10)

This is clear up to a point, but it must not be forgotten that "triangle" and all such determinations are qualitative.

> "Mathematica enim sunt numeri, et magnitudines; et in utrisque utimur nomine qualis. Dicimus enim superficies esse quales, inquantum sunt quadratae vel tringulares. Et similiter numeri dicuntur quales, inquantum sunt compositi. Dicuntur autem numeri compositi, qui communicant in aliquo numero mensurante eos; sicut senarius numerus et novenarius mensurantur ternario, et non solum ad unitatem comparationem habent, sicut ad mensuram communem." (11)

The explanation of this seeming contradiction between a form of abstraction that excludes sensible qualities and yet seems to include qualitative determinations that are sensible like hot and cold would seem to be that "triangle" and all similar notions are qualitative determinations of quantity as such, and not merely sensible qualities that inhere in the subject as determined by quantity but do not determine further what is specifically quantitative. It is rather to this last kind of qualities that heat and cold, etc. belong.

If geometry does not construct, but rather receives its object, it does use construction a great deal. St. Thomas has pointed out that there are two kinds of subject in geometry.

> "In illis autem scientiis, quae sunt de aliquibus accidentibus, nihil prohibet id, quod accipitur ut subiectum respectu alicuius passionis, accipi etiam ut passionem respectu anterioris subiecti. Hoc tamen non in infinitum procedit. Est enim devenire ad aliquod primum in scientia illa, quod ita accipitur ut subiectum, quod nullo modo ut passio; sicut patet in mathematicis scientiis, quae sunt de quantitate continua vel discreta. Supponuntur enim in his scientiis ea quae sunt prima in genere quantitatis; sicut unitas, et linea et superficies et alia huiusmodi. Quibus suppositis, per demonstrationem quaeruntur quaedam alia, sicut triangulus aequilaterus, quadratum in geometricis et alia huiusmodi. Quae quidem demonstrationes quasi operativae dicuntur, ut est illud. Super rectam lineam datam triangulum aequilaterum constituere. Quo adinvento, rursus de eo aliquae passiones probantur, sicut quod eius anguli sunt aequales aut aliquid huiusmodi. Patet igitur quod triangulus in primo modo demonstrationis se habet ut passio in secundo se habet ut subiectum." (13)

The first and most important are the ultimate subjects like line and figure, but one of the first tasks of the geometer is to prove that it is possible to take a simpler subject like line and from it to construct a more complex one like triangle. Hereby he arrives at a secondary kind of subject about which he makes further demonstrations. They remain secondary because they are reducible to the elementary notions that are not constructed but discovered. Indeed, the very possibility of the existence of the secondary subjects remains a property of the primary.

Also, geometry uses construction very extensively as a means of manifesting the properties of its subjects. Aristotle refers to the construction of a line through the vertex of a triangle parallel to one of the sides as a means of manifesting that the sum of the angles of a triangle are equal to two right angles. (13) That he was referring to one of the most indispensable methods of the geometer can be seen by merely looking at any geometrical work.

The geometer bases his science on definitions, axioms, and postulates. The definitions are either nominal definitions of what we have called the ultimate subjects of geometry or they refer to notions readily traceable to these ultimate notions.

The axioms are special geometrical adaptations of more general principles which are self evident in themselves. Evidently it does not belong to the geometer either to examine or defend these principles. Both of these tasks belong to the metaphysician.

Geometrical postulates are statements of the possibility of effecting certain constructions or of the results of such constructions. What is needed to show the legitimacy of these postulates is to show the nature of the possibility or the inevitability involved. Cajetan distinguishes two main kinds of possibility: physical possibility and what he calls equivocal possibility. The second of these includes mathematical possibility.

> "Aristoteles dividit potentiam in potentias, quae eadem ratione potentiae dicuntur, et in potentias, quae non ea ratione qua praedictae potentiae nomen habet, sed alia. Et has appellat aequivoce potentias. Sub primo membro comprehenduntur omnes potentiae activae, et passivae, et rationales, et irrationales. Quaecunque enim posse dicuntur per potentiam activam vel passivam quam habeant, eadem ratione potentiae sunt, quia scilicet est in eis vis principiata alicuius activae vel passivae. Sub

> secundo autem membro comprehenduntur potentiae mathematicales et logicales. Mathematica potentia est, qua lineam posse dicimus in quadratum, et eo quod in semetipsam ducta quadratum constituit. Logica potentia est, qua duo termini coniungi absque contradictione in enunciatione possunt." (14)

Cajetan does not expressly say of mathematical possibility that it really means non-contradiction, but this appears to be one of its meanings. As is consistent with a mathematical statement the test of possibility, the manifestation of non-contradiction, must be visualized in the imagination.

For instance, when it is said that it is possible to draw a line from any point to any other one, all that is expressed is the non-contradictoriness of such a construction. There is nothing in the nature of a line that prevents its terminating in any two given points. The verification of this is made by referring the suggested construction to the image of line in the imagination. Thus is certified to us that there is nothing contradictory in the supposition that such a line is or can be constructed.

The most celebrated of all the postulates is Euclid's famous fifth postulate. It is usually enunciated thus: "If a straight line falling on two other straight lines make the interior angles on the same side less than two right angles, the two straight lines if produced indefinitely meet on the side on which are the angles less than two right angles". (15) This postulate has given rise to endless controversy and it has been interpreted in such a way as to cast doubt not only on it but upon the whole of Euclidean geometry. Perhaps it is not too much to say that the widespread disbelief in the possibility of certain knowledge has as one of its main supports the theory that the fifth postulate is arbitrary and hence that geometry itself is uncertain. Geometry has always been regarded as one of the most certain achievements of the human mind. It is only reasonable, then, that any doubt cast upon it would also reflect on the power of the mind to know anything with certitude.

The way in which the fifth postulate is undermined is to declare that there is no reason to suppose that from the same point several lines cannot be drawn parallel to a given line. Hence, it is supposed that there are several such lines and all of them are called parallel. It will be admitted that with all except one of these lines the transversal cutting each of them and the line to which they are supposed parallel will form less than two right angles on the same side of the transversal. In other words, it is supposed that all the "parallels" except one will be inclined toward one another and still never meet. We must ask then what is the character of the inclined lines. Are they, for instance, straight or curved. If they are straight they must be homogeneous in every part or they will not conform to the famous descriptive definition that has straight lines extending evenly in every part. By this definition if the beginning of the line is inclined ever so slightly, even a billionth of a degree, all the other parts must continue to incline accordingly. As long as we suppose that there will continue to be homogeneous inclination there is a contradiction involved in supposing that lines which are separated by a finite distance will not meet.

Riemann's introduction of a dialectical consideration of the line based on the nature of the horosphere does not in any way contradict what we have said. It is true that, if we suppose a circle of infinite radius, any part of the circumference would be indistinguishable from a straight line. This means that the same line can be regarded as straight and curved. What is important to remember is that we have here nothing but a hypothesis useful in dialectical consideration but in the last instance expressive of a contradiction. No line, mathematical or real, can be at one straight and not-straight any more than it can be a line and not a line. It would be foolish to abandon perfectly clear and certain definitions like those of straight line for a purely dialectical consideration, however useful.

If we rule out this contradictory notion of the line, the only way in which from a given point more than one line can be drawn and of course prolonged in both directions so as never to meed a given line is for all save one to be asymptotic. As we have seen, they can be asymptotic only if they are not straight. Hence it is an equivocation to call the one straight line drawn from a given point so as not to meet a given line parallel in the same sense as a number of curved lines.

It is not astounding that non-Euclidean geometries should be useful in describing the space that seems to be included in our physical universe. All its main types are concerned with curves, whether hyperbolas or ellipses. What wonder is it that they apply very well to a space that experiment seems to show is a curve with a tremendously long diameter? Classic geometry has long since undertaken the strictly scientific study of the properties of curves. What has been added by the non-euclideans is the use of the algebraic method of analytic geometry. It is in no way a contribution to have added the equivocal use of the term parallel.

It is only this last aspect of their work that is undesirable. When they suppose that Euclid's work rests on an arbitrary assumption they make it dialectical. Alongside of it they pretend to set up equally arbitrary systems. Thus Euclid becomes one species of a wide variety of geometries and some have said that schools should teach geometry accordingly from the very first. What is lost here is the formation of the scientific <u>habitus</u> with the consequent scepticism we have spoken about. All this shows how important the regulatory role of the metaphysician is in examining and justifying the principles of the other sciences.

An excellent example of the contrast between the ancient scientific mathematics and their modern dialectical developement is afforded by comparing the work of Apollonius on Conic Sections with the treatment of them in analytic geometry. Take for instance the eleventh proposition in the first book of Apollonius.

> "Let a cone be cut by a plane through the axis, and let it be also cut by another plane cutting the base of the cone in a straight line perpendicular to the base of the axial triangle, and further let the diameter of the section

> be parallel to one side of the axial triangle; then if any straight line be drawn from the section of the cone parallel to the common section of the cutting plane and the base of the cone as far as the diameter of the section, its square will be equal to the rectangle bounded by the intercept made by it on the diameter in the direction of the vertex of the section and a certain other straight line; this straight line will bear the same ratio to the intercept between the angle of the cone and the vertex of the segment as the square on the base remaining two sides of the triangle; and let such a section be called a parabola." (16)

After the construction of the parabola has been thus described and one of its properties set forth, he proceed to prove the proposition. His methods are properly geometrical and every statement he makes can be justified in terms of the elementary notion of figure, line, proportion and the axioms and postulates.

Analytic geometry has as its initial postulate that for every point on a line there exists a number and for every number there corresponds a point on a line. Obviously, this supposes a dialectical conception of number that includes fractions and the roots of numbers that are not perfect squares. This means that the number system attempts to reproduce the density that belongs to the continuum. Also, the line must be thought of as being points in act, and so to be really discrete and not continuous.

Once this initial step has been taken it is possible to construct a graph dividing a page into four parts. The division is made by two lines at right angles to one another. These lines are divided according to a scale to represent numbers. Once this is done it becomes possible to represent any point on the paper in terms of its reference to the dividing lines. Formulae can be set up to describe the locus of any line during its whole length. This is done by substituting numerical values for general symbols like a, b, and x. One of the ways of representing the parabola is by the formula y^2 equals 4ax. This formula permits us to dispense with geometrical considerations and to work only with symbols. What is sought is ease of operability and nothing is easier than to work with precisely these symbols. They must be interpreted in terms of their correspondence on the graph, since some values of the symbols may involve quantities that have no sense in terms of the graph. An example would be a value that would make one of the symbols mean the square root of a negative number. Furthermore absolute accuracy cannot be had because there will always be a margin between the smallness of any fraction and the infinite divisability of the continuum. For most operational needs and for applications in engineering this discrepancy is of course negligible.

What is lost, though, is the manifestation of the properties of a subject through a proper concept of its nature. It is this that Appolonius supplies and that analytic geometry cannot. It is well that Descartes' discovery of Analytics permits us greater facility in operation. It is not well that this greater operability should have caused us to neglect a more scientific mode of procedure in the early steps of education.

Two things make a science perfect: (18) the certainty of its principles and the nobility of its object. Classical mathematics is preeminent amongst human sciences in the first respect. Its principles are not only most certain *per se* but also *quoad nos*. For this reason the young should study it so that they may begin to possess science to the degree and according to the mode possible for them at that age.

The object of classical mathematics is an accident of material substances considered inadequately, according to a mode that makes it incapable of existing. It is for this reason that mathematical objects are not good, since the good is object of desire and we can only desire what is actual or capable of becoming so. Desire carries us outside ourselves toward objects as they are in themselves. If they are incapable of being in themselves they cannot be desired.

Considered, then, from the point of view of their object the mathematical sciences are not the noblest of human disciplines. Natural doctrine gives us a far richer knowledge of material substances since it considers them adequately according to all the causes. It seeks to understand their principles, causes, and elements as Aristotle says in the first book of the *Physics*. (19) In other words we seek to penetrate not only into what is generically true but we try to understand the inner constitution of each thing as well as of its properties and inseparable accidents.

Metaphysics is our human mode of knowing those things that are higher than we are: the separated substances and the Cause of all being. Hence, it has a higher object than either natural doctrine or mathematics.

Inasmuch as the modern dialectical development of mathematics is used mainly in an attempt to manifest imperfectly and extrinsically the properties of material substances as these are reflected quantitatively, it tends to have a nobility that classical mathematics has less perfectly. This can be said because the less perfect operability of ancient mathematics makes them less adaptible to being used in natural doctrine.

To despise the dialectical developments of mathematics is to despise modern empirical investigations into nature and that is to underestimate the main achievement of the last four hundred years. Nothing like this is suggested when the dialectical character of these disciplines is pointed out. It is important, however, to be able to distinguish what is scientific from what is dialectical. Teaching which obscures or denies this distinction is sophistical. St. Thomas has said that logic (20) must be learned before mathematics, not because logic is easier, but because it is more necessary. It is needed precisely to protect one from being malformed by a discipline which (in its specifically modern form) proceeds from conceptions that only the most learned understand according to rules that are extremely mysterious. It is impertinent for a beginner in such a discipline to ask what is being talked about. If this is his only or his main training he will be spoiled for the intellectual life where the first questions are "an sit" and "quid sit".

NOTES ON CHAPTER ONE

1. *In Met.* lib. 1, lect. 1, n 23.
2. *In Met.* lib. 1, lect. 1n, 32.
3. *In Met.* 1, lect. 1, nn. 32, 33.
4. *In Met.* 1, lect. n.34.
5. *In Ethics* VI, n. 1152.
6. In Ethics VI, n 1145.
7. *Summa* I-II, q. 57, art. 3, ad 3.
8. *Summa* II-IIae, q. 47, art. 2, ad 3.
9. *In De Trin.* q.V, art. 1 ad 3.
10. The special problem posed by music will be discussed in the chapter on music.
11. *In Ethics* VI, n. 1211.
12. *De Trin.* q. V, art. 1, obj. 3 and answer to objection 3.
13. *In Post. Anal.* lect. 1, n. 1.
14. Idem, n.2.
15. Idem, n. 4.
16. Idem, n. 5.
17. Idem, n. 6.
18. Idem, n. 8.

NOTES ON CHAPTER TWO

1. Dositheus, *Ars Grammatica* in Keil, Vol. VII, p. 376.
2. Diomedes, *Ars Grammatica*, Lib. II, in Keil Vol. I, p. 426.
3. Quintillian, *Instit. Orat.*, Lib., Cap. 4, 10.
4. I lib. *Peri Hermeneias*, cap. I,.
5. St. Thomas, in I Lib. *Peri Hermeneias*, prologue.
6. In *I Post. Anal.*, lect. 1, n. 2.
7. Cf. *De Doctrina Christiana*, Lib. I, 4.

8. Cf. Arts libéraux, in <u>Dictionaire d'Histoire et de Géographie</u>, Tome 4, p. 827, col B et seq.

9. Amongst others that of Thomas of Erfurt, published in the complete works of Duns Scotus, vol. 1. Cf. also Gilson in the 2nd edition of his <u>Philosophie Médievale</u>.

Notes on Chapter Three

1. Cf. <u>De Poetica</u>, cap. 1-3.

2. I, q.35 ad 1 et 2; q. 93, 1c et 9c.

3. Idem et I, p. 93, a. 3 ad 4.

4. The fine arts, all of them arts of imitation, can be classified according to the species of quality they imitate. Painting, sculpture and architecture imitate form and figure, the fourth kind of quality in the list Aristotle gives in his categories. Music and the dance imitates passion, the third species of quality. Poetry (and for our purposes this includes the drama) imitates human actions, as we have said. Indirectly it also imitates passions, since many actions are influenced by passion and rear the sign of this influence in the way they are accomplished.

5. Ia IIae, q. 22, art. 3 sed contra.

6. IIa IIae, q. 141, art. 3 corpus.

7. II IIae, q. 142, art. 1 corpus

8. IIa IIae, q. 114, art. 1 et 2.

9. IIa IIae, q. 123, art. 2; q. 137, art. 1. et 2

10. II IIae, q. 153, art. 5 corpus

11. IIa IIae, q. 35, art. 4 corpus

12. Ia IIae, q. 63, art. 2 corpus.

13. Cf. <u>In De Anima</u> II, lect. XIII, ed. Pirotta, n. 387.

14. <u>Poetics</u> c. 9, 1451 b 5-7.

15. J. of St. Th. C. Phil., Logic II, q.XVIII, art. 3, p.538 b 44.

1. Aristotle, Rhetoric I, 2, 1355 b. 26.

2. Idem, 1355b. 28 b. et et seq., also 1356a 33.

3. Rhetoric, 1, chap. 3, 1358b. 7 et seq.

4. Id. 1357a 26 et seq.

5. In Post. Anal., Lib. 1, lect. 1, n.12. Cf. In Eth. I, lect. 3, n. 36.

6. In Post. Anal., lect. 1, n. 6.

6a. J. of St. Th., C. T. III, Disp. 27, art. 2, p. 341 n.3.

7. In III Sentent.

8. Rhet. 1, chap. 2, 1356b 1-4.

9. Prior Analytics II, cap. 23, 70a 11.

10. Id. 70a 15-24. Cf. Rhetoric I, 2, 1357b 4 et seq.

11. Prior Anal. I, 27a 1-5.

12. In Post. AnalI, lect. I, n. 12
Cf. Prior Anal., 70a 25. Here Aristotle says that when one premiss is supressed we have an enthymeme; otherwise there is a syllogism. This purely material indication (material here means exterior) of what an enthymeme is has led to the usual definition of it as a syllogism with one premiss omitted. (Cf. J. of St. Th., Cphil. Logic I) No attention whatever has been paid to the kind of premiss used in an enthymeme.

13. Rhetoric I,II, 1356b 10.

14. Cf. Prior Anal. II, chap. 33, 68b, 7-38.

15. In Post. Anal. I, lect. 6, n. 5.

16. St. Albert, In Prior. Anal., lib. 2, tract. VII, cap. 5.

17. Rhetoric I,II, 1355b39-1356a4.

18. Id. loc. cit.

19. Id. loc. cit.

20. Ia, IIae, Q. 71, art. 2 ad 3.

20a. Cf. In Eth. I, lec. 3.

21. In Peri., lib. 1, lect. 14, nn. 19, 20.

22. It is necessary to distinguish between the speculative truth possible in radically practical and virtually practical discourse from practical truth which belongs to prudence.

23. Cf. Father Kearney, "Cassirer's Conception of Art and History", in Laval Théologique and Philosophique, vol. I, n.2, pp. 131-153.

Notes on Chapter Five

1. St. Albert, In Lib. I Top. Tract. 1, cap. 2, p. 241.
2. J. of St. Th., Curs. Théol. T. I, Disp. 11, a. 1, nn.1&2.
3. St. Albert, In Lib. I Top. Tract. 1, cap. 2, p. 241,242.
4. St. Thomas, Met. IV, lect. 4, nn. 573-577.
5. Post. Anal., lect. 20, n.5.
6. In Boeth. De Trin., q. VI, a. 1 corpus.
7. Met. VII, lect. 3, n.
8. Met. VII, lect. 2, n. 1280.
9. III Phys., lect. 8, nn. 1-4.
10. De Coelo, Lib. I, lect. 15, nn. 1-3.
11. De Coelo, Lib. I, lect. 15, n.9.
12. I Poster., lect. 33, nn. 1-2.
13. I Poster., lect. 38, n. 6.
14. I Poster., lect. 27, n. 7.
15. De Anima I, lect. 2, nn. 24-25.
16. De Trin., q. 4, a. 2 corpus.
17. There are many other interesting passages, confirmatory and explanatory, which help clarify the notion of logical genus and dialectical definition. Let us mention De Trin., q. 7, art. 3; De Spirit. creaturis, art. 1 ad 24um, and Cajetan In De Ente et Essentia, edit. Laurent., 131.
18. I Poster. Anal. , lect.
19. IV Phys. Arist., lect. 1, n.1.

20. In IV Met., lect. 4, n. 572-577.

21. J. St. Th., C. Theol., T. I, Disp.2, art. 7, n. 16. Cf. also nn. 14, 15, 24, 31.

Notes on Chapter VI

1. Curs. Phil., Log. II, q. XVI, art. 1, p. 545b 25-28.

2. Id. loc. cit. p. 541 a 17-23.

3. In IV Phys. lect. 7, n. 4.

4. Idem loc. cit. p. 544 b 12-18.

5. Met. V, XIII, 1020 a 7-8.

6. Idem 1541 a 1-15.

7. Idem 541 a 34 et sequ. Cf. also Descartes, "Principes de la phil." n. 53.

8. In Met. V, lec. XV, n. 983.

9. In De Trin. q VI, art I, ad 3 partem in fine.

10. In De Trin. q VI, art. I, ad 1 partem in fine.

11. Curs. Theol. I, Disp. 6, art. 2, p. 533, n 17 & 18.

12. Curs. Theol. I, Disp. 6, art. 2, p. 534, n. 20.

13. In De Trin. q VI, art. 1, ad 2 partem.

14. In Met. VII, lect. X, 1494-1496.

15. In Met. VIII, lect. V, 1760. Cf. also, in VI Met. lect. 1, 1145, and in I Met. lect. V, 334. Also St. Albert.

16. I, q. 85 art. 1 ad 2.

17. Aristotle Categories Cap. VI, 4b 20.

18. Met. V, XIII, 1020a, 11-15.

19. Curs. Phil., Log. II, q. 16, art. 1.

20. Met. X, VI, 1057 a, 4. In Met. X, lect. 8, 2090-2094.

21. *In Met. X*, lect. 2, 1954 & 1955.

22. Cf. St. Th., Codl. X, q. 1, a.1 corpus and In III Phys. lect. 12, n. 5.

23. *In Met. X*, lect. 2, 1945.

24. *In Phys. III*, lect. 12, n. 5.

25.

26. *Phys. IV*. cap. 11, 219b 5. Cf. also J. of St. Th., *Curs. Phil.*, Log. II, q. 16, art.2.

27. *De Trinitate*, q. IV, art 2 corpus.

28. Euclid, *Elements*, Bk. VII, proposition 1. Heath, p. 296-297

29. Peacock, preface to Vol. I, pp. iii-ix & Vol. II, n. 545, 2 & 551, 554, 556, 557, 569, 600, 630, 631. Cf. also article "algebra" in *Encyclopedia Britannica*.

30. For instance, the use of fractions is contrary to the proper definition of number as a multitude measured by unity. Unity, by its nature implies indivision of a thing within itself.
Also, negative numbers, and square roots of numbers like two are justifiable only in terms of operation not in virtue of the nature of number.

Notes on Chapter VII

1. *In Met. V*, lect. 7, no. 850.

2. *In Met. XI*, lect. 13, n. 2413.

3. *In Met. V.*, cap. 13, 1020a 13.

4. Euclid, Book I, Def. 1, Heath, Vol. 1, p. 155.

5. *In Met. V*, 978.

6. J. S. Th., II, p. 347a 14-27.

7. Id. p. 429 a 27 and a 43.

8. *Critique of Pure Reason*, by Kant. Preface to the Second edition, Everyman edition, p. 10.

9. Cf. Pascal, *On Geometrical Reasoning*.

10. *In Met.* VIII, 1760.

11. *In Met.* V, lect. 16, n. 989.

13. *In Met.IX*, Cap. 9, 1051 a 22-30.
14. *In II PeriHermencias*, lect. 12, n.1.
15. Heath, vol. I, p. 155.
16. *Greek Math. Works*, vol. II, p. 305.
17. Cf. Brink, *Analytic Geometry*, chap. 1, n.1.
18. Cajetan, *Comment. In De An.*
19. Aristotle, *Physics I*, lect 1, n. 4.
20. *In De Trin.*, q. V, art. 1, ad 2.

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