34-1

In the same Old Testament, to which ~~Mam~~ A.M.Turing's instance of a theological argument refers, there is also question of one who declared: Non serviam! Whereupon ~~this person got himself involved in~~ a multitudo negotiationis (Eze.xxviii.16). St.Thomas, ~~in a mood of baroque scholasticism,~~ explains that "the ~~[illegible]~~ activity of the Bearer-of-light, averted from the First One, was bent upon the many of inferior things, and it was their primacy that he coveted" (QQ. Quodlibetales, qdl.5, a.7) Thought and action were ~~turned towards the [illegible] mere multitudinous and the scattered for their own sake.~~ Now the sophist, too, ~~in his elaborate ratiocinations,~~ unrestrained by truth, draws at will upon the infinite store of ens per accidens (including an infinity of logics), thus contriving to make the worst appear the better reason and ~~make to seem the least~~ that which is most; ~~so that~~ whatever there is most of appears what more truly is, and nothing is but what is not. For instance, of Mr.Smith, not to mention the events composing the bundle, there are far more particles than men on earth. The choice will therefore be simple, seeing that the many conveyed by 'a mere bundle' of fleeting 'occurrences' have far more the nature of sheer many than have the integral parts of a totum per se. Hence, to meet the new standard of being, what could be more suitable than to father the rational animal as a mere bundle of occurrences rather than as a substance, ~~for substance, as Lord Russell once decreed, rousing [illegible] loud cheers from the gallery, is~~ a "hopelessly muddle-headed notion" ~~[illegible]~~ what he has in mind ~~while using~~ that word, we ought to agree with him).
Accordingly, ~~MrxS~~ 'Mr.Smith' becomes a collective term, as in 'My name is Legion.' On the other hand, man, in the order of operation, is lowered to the level of a mere tool, and thus can do no more than serve; a tool, like any instrument, being of its nature movens motum. (A rather strange tool, at that, an instrument without a principal agent, ~~like~~ like a sign ~~that~~ does not signify, or a relation adrift, without terms; for the maker and agent is buried when one tool gives birth to another, seeing that Russell has shown ~~that~~ something to the effect that we may have tools of tools and nothing but tools without end, so that everything may well be in the service of nothing.)—Whether or not we believe in Sacred Doctrine is not at present to the point. The plain fact remains that the literature to which Turing refers sets forth a strophe for which we have provided an antistrophe: we have echoed the non serviam, ~~and~~ the attending desire of primacy for the sheerest many; not in the romantic way of Karl Marx quoting Aeschyles's Prometheus, nor by merely enslaving "some of these infernal machines" (as E.T.Bell calls them) to do the repulsive drudgery; but in identifying science with the mechanical process itself, making what goes on in the computer to be one and the same with the highest form of life, namely, thought. And it is worthy of note that the keenest joy is expressed when such reductions are made. Similarly, the scattering thought finds an entitative counter-part in the mere bundles that are Mr.Smith, Earl Russell, etc.; a dispersio that should be carried on and extended to the universe as a whole—the supreme heap of bundles that outbundles them all. Russell has said that we may one day blow up the universe, and he is appalled at the horrible ~~[illegible]~~ prospect. Yet, it being by nature already so much out of joint, one can hardly see what there is of the universe to explode, or that it could make much difference; nor why any one should really care, seeing that, to whatever there is in the universe it will be as if it had never been. Besides, it would all happen most legally.

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QUÉBEC, P. Q.
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PREFATORY REMARKS

The purpose of this Introduction is to show what are the subject and principles of what Aristotle calls the science of nature or natural philosophy. We shall attempt this by way of an exposition of the first two books of his Physics. The reader ~~But~~ should ~~must~~ know from this very beginning that we are not wholly unaware of the extent to which the meaning of the words just employed has changed — viz. 'science,' 'subject of a science,' 'principles of a science,' and 'science of nature.' It would be difficult to find a single instance in which the same words still mean the same thing. Plainly, we cannot afford to neglect this fact.

Many a teacher called upon to give an elementary course in the Philosophy of nature — which sometimes goes under the title of what is actually only one of its parts, viz. Cosmology — will feel impatient when we show some measure of sollicitude for the scientific climate that is proper to our day. Why should we bother about it, he might say. We have the mandate to teach the subject, so let us get down to business. Precisely, can we reasonably get down to it? In fact, is there such a subject? For some reason or other we may be already convinced that there is. But that is not the point, when in teaching one must begin from what is known to the listener. Now his preliminary ~~the~~ information — if only that which was gathered from the headlines ~~— with which he~~ #75 ~~is already equipped by the time he turns up for~~ a an elementary course in some special subject of philosophy, is very different from ~~what has had by~~ those of the beginner of some half century ago. One would compound the confusion by ignoring the difference. No philosopher we hold in esteem thought he could neglect the opinions of his times. It would be most unfair to let the student believe that what is meant by science in the philosophy of nature must be roughly the same as what is meant by science today — only to learn eventually that they really have no more in common than a dog and the constellation that goes by the same name. Yet the fruits of modern science grow with cosmic violence. What then?

The first thing to be noted is that all that will be said in this Introduction will be expressed by means of words.

1. - Science of nature and the use of words.

It has latterly become rather obvious that the giant strides of the mathematical study of nature are concomitant with a gradual emancipation from the use of words. The mathematical physicist does not know what he is talking about until he can have recourse to symbols that are not names. At the same time this very statement uses nothing but words, and it is difficult to see how one could make such statements in any other way. One might suggest that this statement could be symbolized by the sign 'S.' But the interpretation of the symbol would have to refer to the statement made in words.

When Sir Arthur Eddington shows so convincingly that the exact science of nature can get nowhere until it has reduced its definitions to measure-numbers, and that these are expressed in terms of mathematical symbols, not words, he uses words to show ~~this~~ so. Even the terms 'exact,' 'science,' 'symbol' and 'nature,' he employs as words intended to mean something in the way that words do. Indeed he does so while showing just how the physicist obtains his measure-numbers and is concerned only with them. By length, for instance, which is otherwise defined as 'what is extended in one dimension,' he, as a mathematical physicist, means 'when we take a reasonably fair copy of a certain platinum-iridium bar kept in Paris... and apply it successively or by division to know the distance between A and B, the result of the operation may be expressed by Lx.' Thus defined, the standard of length can of course have no length, when there is no other standard, so that length only is once the measurement is had. In turn, weight is 'when using a weighing-machine...', and so on for all the basic definitions. The importance of 'when' in these definitions can hardly be

give reference

exaggerated. If the physicist said 'length is...' instead of 'length is when...' he would revert to a mode of definition which aims to state 'what' a thing is absolutely, and not merely what the name or symbol is intended to mean. Having thus defined length he may tell us "this is length," but this is only another way of saying that that is all he can be concerned with. In mathematical physics definitions should be no more than interpretations of the symbols ~~that were~~ chosen, by describing how the measure-numbers were obtained. It is interesting to note that if only this type of definition were valid in any field, then the definition of 'man' would have to be like 'when I tread on something and it produces a series of sounds such as "Where do you think you're going?" And that is man.'

It is also plain that when interpreting the time-symbol t the mathematical physicist does not intend even a nominal definition of the word 'time', as this term was and is still used, without specific reference to the way in which the measure-number is obtained. The same holds for the very expression 'mathematical physics,' meaning a certain type of knowledge about 'nature.' He would not try to define in terms of measure-numbers what the word 'nature' stands for, although we might point out that even his kind of definition has something to do with what we call nature. Take, for instance, the following statement made by Einstein: "It is my conviction that pure mathematical construction enables us to discover ~~the~~ the concepts and the laws connecting them which give us the key to the understanding of the phenomena of Nature. Experience can of course guide us in our choice of serviceable mathematical concepts; it cannot possibly be the source from which they are derived; experience of course remains the sole criterion of the serviceability of mathematical construction for physics, but the truly creative principle resides in mathematics." He makes clear what he means by physics when he adds that by itself such construction "can give us no knowledge whatsoever of the world of experience; all knowledge about reality begins with experience and terminates in it. Conclusions obtained by purely rational processus are, so far as Reality is concerned, entirely empty."(1) We do not know how he would have interpreted the names 'Nature' and 'Reality' though he might have suggested that to the physicist they are what the measure-numbers somehow refer to, and the test of the relevance of rational construction to his

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(1)- On the Method of theoretical Physics, Herbert Spencer Lecture, Oxford, 1933, pp. 7, 12.

purpose. We are confident that he would not have confined himself to 'Nature is when using such or such a standard of measure...etc.,' — although in doing so there would be reference to nature, and to what he already knew 'reality' to mean.

2. - <u>The symbolic world of mathematical physics, and the 'symbolically constructed fictions' of mathematical logic.</u>

Once Eddington has made it clear that from the mathematical physicist's standpoint the world is a symbolic one — in the sense that what he knows of it can be conveyed only by symbols and involves a generous share of fiction, starting from and referring to metrical structure — but that whatever the symbols convey is not all that the world is, ~~and~~ he goes on using words to bring home his thoughts on the subject. Hence, to employ either words or symbols is not a matter of choice : now one, then the other, is imposed upon us according to what we wish to express. We are sometimes led to believe that the use of symbols is a way of economizing words. They This is not the whole truth. Their use certainly economizes thought. But it is far more important to realize that the mathematical physicist, as well as the mathematician, does not use symbols instead of names for the sake of abbreviating his equations, but because he could not resolve them by operating on names.

As we shall see further on, the art of calculation simply cannot be concerned with objects in the sense of what names refer us to, like 'man,' 'horse,' or 'nature.' Even what the ancients had named 'number' or 'figure' is of no formal interest to it. "Mathematicians do not study objects, H. Poincaré said, but the relations between the objects; it is therefore indifferent to them when the objects are replaced by other objects, so long as the relations do not change. Not the matter, but the form is their concern."(1) AND Now the objects that are of no concern to

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(1) - Science et hypothèse, p. 32. — To illustrate Lord Bertrand Russell's saying that "Mathematics may be defined as the subject in which we never know what we are talking about, nor whether what we are saying is true," the French mathematician M. Jean Hadamard chooses the following example: "Having bought 6 metres of cloth at 12 franks a meter, how much does one have to pay? In raising this problem, are we really talking about cloth? Not at all. Instead of asking the price of 6 metres of cloth at 12 franks a meter, we could just as well have asked the price of 6 pounds of meat at 12 franks a pound. We might have replaced the meat by copra, and the pupil could have provided the answer without even asking the teacher what copra is. Hence, in raising this problem, one does not know what one is talking about; or, to put it otherwise, there is no need to know it. Here, then, in a first, simple instance, we have the notion of mathematical abstraction." In Encyclopédie française, section "Mathématique," 1,52-3. — I do not think that M. Hadamard wishes to state here all that is intended by what Lord Rusell said, inasmuch as the numbers in question are themselves only material with regard to more abstract forms which ~~do not~~ say ~~anything~~ nothing about numbers.

him are not merely those like horse or apple, but numbers and figures as well.

Now what about the objects that neither mathematician nor mathematical physicist is concerned with? What has happened to the number, e.g. 'three,' which we had named before putting it into an equation, or to the 'time' we named before we manufactured the measure-number by the clock? The operations upon the symbols may have been so proficient that we forgot, or believed we should now forget, what those names meant while we were using them. Could we really replace what the word 'man' meant by referring henceforth to no more than the mathematical physicist's view of him as a swarm of electric charges? This no doubt man is, but is it 'what it is to be a man'? It must be true that if the physicist could produce that particular kind of swarm he would have indeed produced a man. But why would we call it a man unless it were like what we already identified as a man?

3. - <u>Where words remain in use.</u>

If neither the mathematician nor the mathematical physicist can be no more than hampered by the use of names, apparently they must use them when they want to convey what their knowledge is about and especially what it is not concerned with. In saying that they cannot be concerned with things as they are named, they are using names to say it, though ~~while~~ THEY ARE NOT SPEAKING HERE in doing so they are admittedly not saying it qua mathematician or mathematical physicist.

The question we are trying to raise here is this : can there be true knowledge of what the names we use about nature are intended to mean? Can the things they refer to be defined and used to demonstrate something in a way which deserves to be called scientific? Must the term 'science' be restricted to the art of calculation and its application? What did we mean by 'change,' 'movement,' 'infinity,' and 'time,' before we defined them by measure-numbers; has their meaning now become mere fancy? It has been suggested that the only reason why we shall continue to use words is that they are necessary to communication in the order of behaviour — that language is essentially practical. No court of law would excuse manslaughter as being no more than a disturbance produced in a particular swarm of electric charges by another swarm reasonably like the former. So we continue to believe that Mr. Smith is there in some fashion or other perhaps not too clear, and that he after all still has rights and obligations, even as we do. But it seems that so soon as we forget about the practical order — about how we should behave and treat our neighbour, and all such things expressed by names -- and apply ourselves to scientific investigation, things like man and his doings are irretrievably left behind. If the thing (while even 'thing' may be distressingly unscientific) we call 'man' does persist, it is only as what turns up for breakfast or is summoned to pay taxes, or allowed to sleep, and in some event even to study physics.

It is no doubt significant that words are used to tell us these things, and that these things would not be told unless in using words our thoughts were turned to something recognized as their meaning. Nor is it less significant that the practical life should force their use upon us. And there is no denying that many of the words which for centuries remained basic in philosophy, like 'matter,' 'form,' 'action,' originally referred to the order of making and doing, and not to the things of nature; and 'time' may well have meant something we do not have enough of. Surely these facts are worth looking into, however little scientific a curiosity about such things may seem.

4. - <u>If all definitions were to be of names or of symbols only.</u>

If it must be assumed that there can be no true knowledge of things as we name them, but only of what can be expressed by the symbols of logic and in calculation, then what we say about this or any other kind of knowledge in using words could hardly be true. Let us put it still another way. If, as Stuart Mill said, "All definitions are of names, and of names only," such that the things named cannot be defined in themselves, however tentatively, meaning that we cannot know what they are but only what (in that) ~~the~~ name ~~is that~~ we use to signify them, and since there cannot be a science of the names themselves inasmuch as they signify no more than by convention, it is clear that there can be no science of anything to the extent that it is named.

reference ?

On the other hand, what Mill says applies literally to the symbols of the art of calculation, whether applied in mathematics or in physics : to define is simply to interpret the symbol by explaining how it is to be taken, not by stating what the thing named is. For instance, when asked to define the 'number two', the art of calculation will never try to tell us what two is, because what two is never enters into the operation of calculation. In that activity, 'two' is only a term with a function similar to that which it fills in an equation like 2 plus x = 5. Whether two here is actually 'one two' or 'two ones' can make no difference. The only unity two possesses in such an equation is the unity of a symbol; and whatever sort of unity 2 may enjoy apart from that assigned to it as an operational symbol is quite irrelevant to a definition derived from its operational use alone. Lord Russell puts it this way:

"We naturally think that the class of couples (for example) is something different from the number 2. But there is no doubt about the class of couples: it is indubitable and not difficult to define, whereas the number 2, in any other sense, is a metaphysical entity about which we can never feel sure that it exists or that we have tracked it down. It is therefore more prudent to content ourselves with the class of couples, which we are sure of, than to hunt for a problematical number 2 which must always remain elusive. Accordingly we set up the following definition: —

The number of a class is the class of all those classes that are similar to it.

Thus the number of a couple will be the class of all couples. In fact, the class of all couples will be the number 2, according to our definition." (1)

It is admittedly difficult to see how any other way of being two could be relevant to the equation 2 plus x = 5. In this context, therefore, Aristotle's definition of number as 'a plurality measured by one' must appear awkward, and is certainly useless. But Aristotle was trying to convey what number is, not what an operational symbol may stand for.

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(1) - Introduction to Mathematical Philosophy, p. 18.

Definitions of the same type appear in connection with geometry. Hermann Weyl had this to say in illustration of what he meant by 'creative definitions' :

"Thus, in geometry, the concept of circle is introduced with the help of the ternary point relation of congruence, OA = OB, which appears in the axioms, as follows, "A point O and a different point A determine a circle, the 'circle about O through A'; that a point lies on this circle means that OA = OP." For the mathematician it is irrelevant what circles are. It is of importance only to know in what manner a circle may be given (namely by O and A) and what is meant by saying that a point P lies on the circle thus given. Only in statements of this latter form or in statements explicitly defined on their basis does the concept of a circle appear." (1)

Especially deserving of attention is the neat statement that "For the mathematician it is irrelevant what circles are."

Further on, Hermann Weyl lays puts down his understanding — most mathematicians do share his view — of what is now meant by the 'concept' of number :

"If one wants to speak, all the same, of numbers as concepts or ideal objects, one must at any rate refrain from giving them independent existence; their being exhausts itself in the functional role which they play and their relations of more or less. (They certainly are not concepts in the sense of Aristotle's theory of abstraction.)" (2)

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(1) - Hermann Weyl, Philosophy of Mathematics and Natural Science, p. 8

(2) Ibidem, p. 36.

Turning now to the mode of definition in mathematical physics, we have Eddington's incontrovertible statement about what a definable weight is : "Never mind what two tons refers to; what is it? How has it actually entered in so definite a way into our experience? Two tons is the reading of the pointer when the elephant was placed on a weighing-machine."(1) It was never intended to reveal what weight is apart from this particular mode of defining, viz. by describing how the physicist obtains this kind of measure-number.

5. - Just what is implied when we are told that science is no longer concerned with 'objects.'

We have been told that the mathematician is not concerned with objects, he cannot get very far with the number, like two, of which Lord Russell says that it "is a metaphysical entity about which we can never feel sure that it exists or that we have tracked it down."(2) And to the geometer in particular, it is also irrelevant 'what circles are.' We must be aware of the implication of this fact with regard to what was previously called mathematical science, and which had to do with quantity, this being either number, the subject of arithmetic, or continuous quantity, the subject of geometry. According to Aristotle,

1) - The Nature of the Physical World, p. 251.

2) - Introduction to Mathematical Philosophy, p. 18.

these subjects are to be defined in metaphysics, while the mathematician assumes them, but replaces them by symbolic construction or creative definition, as we were told. Now it is important to note that these constructions are not intended to replace those subjects absolutely. The latter are simply left out, because, it is said, we can never feel sure that they exist or that we have tracked them down. Hence, they who would continue to apply their mind to those subjects would be seen as moving about on slippery ground.

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Beach says: "develop"

We may perhaps make clear what has happened by comparing what Poincaré said to be the concern of the mathematician — viz. the form, and not the object that he also calls the matter — with what the Greeks (Plato and Aristotle in particular) called the matter and the form of a number. Aristotle distinguishes a matter and a form that constitute a number intrinsically, they being related as potency to act. The matter of a number is the units that compose it in the order of material cause, like the pieces of wood that make up a table, or, better still, like the limbs that make up the body of a man. By the form of the number, he meant the particular kind of unity and order which is exhibited by adding a unit to a unit, a unit to the number, obtained, and so on for all the integers. The addition does not fabricate the number but merely brings to mind new kinds of number which, though they are not conceived as existing in reality the way Socrates does, nevertheless are considered as having certain properties which are true even when we do not actually consider them. Number, thus understood, is defined as 'a plurality measured by one' — one being the principle of number. Now, any proper measure must be one in kind with the measured, meaning, here, that to be in number the units must be of the same nature. The particular kind of unity that is proper to any given number depends upon the homogeneity of its components. Otherwise we have no more than "a sort of heap" (<u>Metaph.</u> 1044a). The number two, then, is not the same as two mere units of any kind.

explain here more perhaps

Still, even when objects are not of the same kind, we can count them nevertheless, like the disparate objects in this room — persons, desks, chairs, coughs, absences, the relations of reason that we have in mind, and even those which we ought to have but do not. Thus we have a number that applies to the heterogeneous elements of a heap or mere aggregate, a number which we use to express how many objects are in the room there. This type of number arises in the act of sheer counting. It is the number of the art of calculation which was called logismos or logistikè. Whatever unity such a number may have, it is provided in the operations of addition, multiplication, subtraction and division. Its unity is independent of what the things are that we refer to as being such or such in number; this is the number which has been defined by Russell, as the class of all those classes which are similar to it. Thus the number 2 is the class of all couples, no matter what their kind or the kind of their elements. Nor do the couples or their units have to be coupkes or units in any positive sense, for if number is defined by the operation, whatever the operation may be applied to will by that very fact be such a number, like zero, or a fraction, or an irrational number. And number, thus understood, is admittedly not an object in the sense in which the number two that is one two is an object. It is a convenient fiction which our mind has produced. Though a fiction, it is nonetheless proficient, as can be seen from the fact that we can count things regardless of what they are. Thus, what the things are is of no account to the calculator. The indifference of this number with regard to the nature of the numbered is equalled only by the indifference of the heap as a heap.

Explanation needs [illegible]

The [illegible] the heap.

The science of arithmetic, as Aristotle and Euclid understood it, is about the numbers that are per se one;

([illegible])

unlike logismos, it does not abstract from what the things are that it applies to : ~~they must, as~~ to be the subject of any science as they understood this term, they must be one per se. What Whitehead says about arithmetic would apply only to the art of calculation which the science employs :

"Now, the first noticeable fact about arithmetic is that it applies to everything, to tastes and to sounds, to apples and to angels, to the ideas of the mind and to the bones of the body. The nature of the things is perfectly indifferent, of all things it is true that two and two make four. Thus we write down as the leading characteristic of mathematics that it deals with properties and ideas which are applicable to things just because they are things, and apart from any particular feelings, or emotions, or sensations, in any way connected with them. This is what is meant by calling mathematics an abstract science." (1)

Perhaps we ought to make explicit that the nature of things is indifferent to the point where all that Whitehead mentions can be gathered under a single number.

6. - The expression 'mathematical science' now has a new meaning.

Arithmetic, as it is understood in this context, has nothing to do whatsoever with the subject of what the ancients called by that same name. In fact, most moderns would say that what the ancients had in mind ~~it~~ was not a science at all. This is what Lord Russell implies when he says its subject would be something "about which we can never feel sure that it exists or that we have tracked it down."

(1) - An Introduction to Mathematics, p. 9.

On the other hand, there is no doubt about the class of couples: anything, thing or no, will belong to it, if it is a couple no matter what of. Thus mathematics as it is understood today has put aside everything that might be called into question in any way. To possess what is left we do not even have to discuss whether anything corresponds to the fictions, not whether these are only in the mind. To save their value, even 'logical' in 'logical fictions,' does not have to be tied down to what is in or of the mind.(1) It is enough that 'logical' should refer to logismos; ~~and~~ whereas it should certainly not refer to logic in the Aristotelian sense of this term.

A further point is worth noting here. The art of calculation does not take into account whether a number is a group of actually divided elements, or whether it is a one

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(1) - "When we have decided that classes cannot be things of the same sort as their members, that they cannot be just heaps or aggregates, and also that they cannot be identified with propositional functions, it becomes very difficult to see what they can be, if they are to be more than symbolic fictions. And if we can find any way of dealing with them as symbolic fictions, we increase the logical security of our position, since we avoid the need of assuming that there are classes without being compelled to make the opposite assumption that there are no classes. We merely abstain from both assumptions." (B. Russell, Introduction to Mathematical Philosophy, p. 185.)

that is divisible yet not divided. Whatever is to the right of the symbol of equality is essentially the same as what is to the left of it. Thus $1 + 1 = 2$ is <u>exactly</u> the same as $1 + 1 = 1 + 1$. Hence, whether 2 stands for one two, or for two ones of any kind, is completely indifferent.(1) Whether the number ~~that~~ it applies to is actually one or actually many is of no account here. Such is the case of all the basic laws of the art. We may neglect, then, whether a number is "an aggregate of units, as is said by some [e.g. Thales, who is said to have defined number as a bundle of units]; for two is either not one, or the unit is not present in it in complete actuality."(2) Likewise with regard

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(1) - This has been clearly exhibited by Courant and Robbins, <u>What is Mathematics?</u> chapt. I.

(2) - <u>Metaph</u>. VII, xiii. Here is the context: "A substance cannot consist of substances present in it in complete actuality; for things that are thus two in act [i.e. each having a complete and distinct actuality of its own], are never one in act, whereas if they are two only in potency, they can be one [in act], like things that are double, as two halves potentially; for the complete actualization of the halves divides them one from the other; therefore if the substance is one, it will not consist of substances present in it and present in this way, which Democritus describes rightly; he says one thing cannot be made out of two nor two out of one; for he identifies substance with his indivisible magnitudes. It is clear therefore that the same will hold good of number, if number is no more than an aggregate of units, as is said by some; for two is either not one, or the unit is not present in it in act." (1039a1-15)

to magnitude, whether the line is actually divided or only potentially so, is irrelevant to the art of calculation when applied to it. Moreover, whether a ~~plurality is finite or infinite in any way, is indifferent : All that is required is that~~ line contains an infinity of points in potency or in act, is indifferent. Of the infinite no more is required than that we should be able to define it operationally. The distinction between act and potency is simply beside the question. Infinite classes can indeed be easily defined in this way. But whether there is an infinite class in the way there is a number that is per se one is a matter irrelevant to what the art defines and applies to. To it, such questions can be no more than obtrusive.

7. - <u>The 'mathematics' that abstracts from the distinction between 'per se' and 'per accidens.'</u>

All this implies that <u>logismos</u> side-steps the distinction between what is <u>per se</u> and what is <u>per accidens</u>, either as to being or as to unity. That the mind can transcend this division is plain from the fact that we can string together the following incidentally connected 'bald-headed pale barn-building flute-playing thrice-married ill-tempered barber.' We cannot name what it is to be such a particular accidental ensemble — although it may be 'Oscar' — but we can make a symbol stand for it. In terms of the calculus of classes, anything which is all those things together belongs to the class that is the logical product of the classes 'bald-headed,' 'pale,' 'barn-building,' etc., and the product may be represented by the single arbitrary sign ψ.

This kind of abstraction may lead to certain paradoxes which are had only inasmuch as we may still be using names about them. Take for instance the principle that the whole is greater than any of its parts. It is said that this principle does not always apply, and therefore is not universal. Consider, e.g., the series of whole numbers compared to the series of even numbers in the following way :

1, 2, 3, 4, ...
2, 4, 6, 8, ...

"There is one entry in the lower row for every one in the top row; therefore the number of terms in the two rows must be the same, although the lower row consists of only half the terms in the top row."(1) Actually the lower row is not part

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(1) - B. Russell, A History of Western Philosophy, p. 829. The comical paradoxes that are found in his writings have an explanation in the fact that Lord Russell everywhere puts what is per se one and what is only incidentally so on the same footing, in which he may be right as a calculator. To him, e.g., 'Mr. Smith' stands for no more than a bundle of events. (ibid., p. 201.) The example would not be funny, it would lack the proper incongruity, if there were no clash between the per se unity which we cannot help but keep in mind, and the incidental whole that is a mere bundle. It is like the cartoon of the elephant eating the jam and following his trunk so completely that in the last picture he has swallowed himself entire, as you can see that he is no longer there. Now if the cartoonist were not allowed to do such things, how could you expect him to be funny? How could we have Alice in Wonderland?

of the first — it is another row of the same amount. It is a part only because you say that it is. If part were used in its proper meaning, your two rows would have to look like this:

1, 2, 3, 4, ...
2, 4, ...

In the top row you would then have two members for each in the lower. Besides, there is nothing to prevent us from taking as many instances of the single number two as there are whole numbers, so that any single part of the series will be equal to the series of the whole numbers. And why not take two instances of 2 for every instance in the series of whole numbers? We cannot see that 'universal' and 'particular instance' of it could have any meaning in terms of symbolic construction, when they who use the word 'universal' in this context, interpret it as a class or collection. The paradox arises because we have forgotten that ~~in~~ passing from number, first signified as one per se, to a mere aggregate that is one only by symbolic construction, the meaning of the word 'number' has changed, like when 'dog' is used to mean the constellation. If we had not kept in mind the original meaning of 'number,' 'whole,' 'odd,' 'part,' and 'half,' there would be no paradox. On the other hand there would be no statement of paradox if the words did not retain reference to what they meant outside this new context. In the present instance, the paradox may be had because we have kept in mind the "metaphysical entity," viz. the numbers that were considered as per se one, so that we are now using their name out of context. But if, once the abstraction has been performed, this type of interpretation of the word were still legitimate, we could easily have far greater oddities than that of the example given above. For instance, it would follow that a man, having two legs, is like a walking contradiction, seeing that two is more than the one of which they are the part. To have the oddity, all we have to do is forget that in making such a comparison we have ceased to consider the parts as parts, and then still call them parts. Plainly, once we have resorted to symbolic construction or creative definition we should realize that names should never more be used except as linguistic conveniences that will remain confusing inasmuch as they shall continue to evoke what can no longer be intended.

8. - A methodical doubt.

By this time the reader will be aware that we take notice of views that are as striking as they are current. There is, among other, one man whom we cannot nor wish to avoid, and that is Lord Bertrand Russell. We admire him in the way Sir Arthur Eddington did, and probably for the same reasons. But Lord Russell has at times strayed wide from these reasons, especially in the survey he made in A History of Western Philosophy, which brought all of philosophy down, including much of what he himself at one time held. There are many to disagree with his sweeping generalisations. Yet here is a man who made important contributions to what he himself calls 'the scientific outlook' — though it is not always clear where these begin or where they end. If this outlook, as he describes it, must be extended to all fields, which he appears to believe, most of what ~~any~~ philosophers of the past ~~has ever~~ ever said ought to be discarded at once as meaningless jumble. Russell's History conveys that all philosophers, save those of the recent school to which to belongs, have held the most arbitrary positions for reasons not less ARBITRARY so. However, if his scientific outlook cannot reach the domains which were called scientific in quite another sense, this alone would prove nothing against its intrinsic value. There may in fact be some dialectical advantage in attempting, by way of methodical doubt, to see what happens when that scientific outlook is given universal scope.

All this is matter for a note

Now this venture into doubt could land us in places so strange that the reader may feel either that I flirt with irony or that I myself have got beyond my base. Let him entertain no doubt about my own position in the matter. Our aim is to disturb comfort where we deem it to be illusory, by making the reader aware, in a dialectical fashion, of what happens when we choose the Russell alternative, obstructive as it may be of anything we held viable. But if his view proves untenable as a general one, we will find that it does apply in some places — how many or how wide is not the question here.

As to the modes of defining ~~that~~ were already pointed out, we hold them to be both true, and fail to see any contradiction in doing so. There is nothing irreconcilable in defining man to be a rational animal and interpreting the name by 'when I tread on something...etc.' In fact, there is a domain in which definitions by interpretation of the name or of the symbol are the only ones to open an avenue of rewarding research. The latter type of definition of man can lead to "detailed and precise knowledge of normal and pathological mental processes in a desired direction and thus cure mental ailments."(1) We find little in Aristotle's De Anima to further knowledge along those lines, except that it might help to deem the subject worthy of relief. But the

(1)- Dr. Franz Alexander, Introduction to What Man has Made of Man, by Mortimer Adler, Chicago, 1937, p. xi.

sole fact that the former type of definition may in fact lead to endless discussion, and the latter to little or none, should not compel us to choose the one instead of the other. There is no doubt that if we felt hopeless about 'rational animal' and took the narrower view we would be judged [illegible] more broad in mind. In refusing to follow one course to the exclusion of the other we may be found guilty of wasting time — our own ~~, and or~~ or [illegible] someone else's. It is a chance one may be willing to take. Whatever the outcome, it seems one ought to be allowed that kind of freedom — the freedom to be considerably wrong and to hold no longer tenable positions. Taking advantage of this margin may prove one unworthy of the right-minded; yet in the end it might be worth the risk, if only to protect the freedom of those who use it well.

Say otherwise

This separate

8.

Let us revert, then, to the beginning of this foreword. It is a historical fact that so long as the study of the physical world implied essentially the use of names, little was achieved to further knowledge of the kind now called physics. Where the Greek Philosophers sought to know <u>what</u> the things of nature are, we have deliberately renounced that type of inquiry for the simple reasons that it does not lead to the kind of knowledge about nature which we actually obtained by method of another type, whose possibilities have only begun to reveal themselves. Is there however any clear reason why the former mode of investigation should be abandoned altogether and everywhere? Is it necessarily beside the point to be interested in objects and to ask what a thing is? From the very start the physicist defines movement by the way he measures it and that is what movement <u>is</u> to him. Does this imply that it must be irrelevant to ask what movement is, apart from this operational way of defining? The answer, today, is a fairly general 'no.'

Aristotle's whole treatise that came down to us under the name <u>Physics</u> deals but with a few definitions and is confined to a relatively small number of demonstrations most of which must appear outlandish when we look at them in the light of what is today called physics. Aristotle's intention was fairly clear : he wanted to provide in his work a general introduction to the study of nature, which in treatises that come after this first general approach branches out into particular sciences whose denominations we have in some instances retained. In Book I of this work he investigates which what are the principles of the subject of natural science in its widest acceptation. In Book II, having shown some meanings of the term 'nature,'

That whole treatise of Aristotle's which came down to us under the name Physics deals but with he determines what kind of knowledge we are after in the study of nature, what are the causes or definitions from which demonstration can be had in this field. In doing this he has raised the problem of how the natural scientist and the mathematician differ when talking somehow about the same things of nature; finally he shows the difference between necessity in mathematics and the kind of necessity to be found in nature and in the science of it. Book III starts with the question of what movement is and having defined it as something admittedly obscure, he goes on to the problem raised by movement with regard to infinity. Book IV is about place and time. The discussion is somewhat uneven inasmuch as here and there, though not essential to the points he wishes to make, he makes assumptions drawn from theories expounded in later treatises, which eventually followed the way of the earth at the hub of the universe and of the instantaneous propagation of light — like when he identifies the time he has defined independently, with the movement of the 'outer sphere.' These assumptions, however, are no more than incidental to the definitions he has arrived at. Book V is about the division of movement according to its kinds, viz. in quantity, quality and place. He then presents a few motions such as 'to be in contact,' 'between,' 'next to,' 'contiguous,' 'continuous,' thus leading towards the discussion of movement according to its quantitative parts in Book VI, where he first defines 'continuum,' 'indivisibles' and infinitely divisible;' and there is a first approach to Zeno's paradoxes, which are left unresolved until Book VIII. Both the position and solution of Zeno's problem differ widely from those made in sheer terms of calculation. Books VII and VIII culminate in demonstration of a first unmoved mover. The whole of the discussion makes no sense in terms of mathematical physics, nor was it ever intended to ~~have such a meaning, or to persuade in this mode~~.

Most everyone holds that whatever interest the <u>Physics</u> may still possess can be no more than historical. In this we see not so much a challenge to the particular doctrines it contains but, more important by far, a challenge to the meaning and validity of the kind of questions it assumes that we may raise. They are no doubt of the kind which to this day leads philosophers to the most contradictory positions. Whether this alone provides a sufficient reason for abandoning them even as questions is~~, to my mind,~~ debatable. But it is interesting to note that these opposite opinions are the more easily had when the philosophers themselves have not really raised the questions and assumed that they are solved without proper investigation. This was the case of Descartes who held that movement is one of the clearest things there are, while what he had in mind when using the word was indeed quite manifest.

Whatever the case may be as to relevance we must leave it up to the reader of the present <u>Introduction</u> to judge the extent to which we may still be allowed to ask, in words, just what it is that the study of nature is about; whether it is possible to define movement in the sense of 'what' it is, and not just to interpret what the word is used to mean merely by pointing out some instance of it, like "Mr. Smith moved from street A to street B," and then leave it up to the physicist to define it in his own way; or to define what time is, and so on. Our purpose in the present work is to go into the subject of the first two Books of the <u>Physics</u>. To do so, we must be allowed to establish a few points by way of prolegomena.

Show relevance of the more ancient

Now the first difficulty we run into is the very name 'science' in the expression 'science of nature' or 'natural science.' Aristotle himself appears to be a little help when, in the <u>Posteriora Analytica</u> (I, i-ii), illustrating what he means by 'to possess unqualified scientific knowledge of a thing,' he refers to the demonstrations of mathematics. Accordingly we might follow him and choose an example from geometry, like the first proposition in Euclid : 'On a given finite straight line to construct an equilateral triangle.' Now this is very awkward, in view of what is commonly held today about Euclid's mode of demonstration. Here, for instance, is what Lord Russell has to say on the subject — and, bearing in mind the kind of rigour he demands, I cannot see that one could disagree with him :

"The rigid methods employed by modern geometers have deposed Euclid from his pinnacle of correctness. It was thought, until recent times, that, as Sir Henry Savile remarked in 1621, there were only two blemishes in Euclid, the theory of parallels and the theory of proportion. It is now known that these are almost the only points in which Euclid is free from blemish. Countless errors are involved in his first eight propositions. That is to say, not only is it doubtful whether his axioms are true, which is a comparatively trivial matter, but it is certain that his propositions do not follow from the axioms which he enunciates. A vastly greater number of axioms, which Euclid unconsciously employs, are required for the proof of his propositions. Even in the first proposition of all, where he constructs an equilateral triangle on a given base, he uses two circles which are assumed to intersect. But no explicit axiom assures us that they do so, and in some kinds of spaces they do not always intersect. It is quite doubtful whether our space belongs to one of these kinds or not. Thus Euclid fails entirely to prove his point in the very first proposition. As he is certainly not an easy author, and is terribly longwinded, he has no longer any but an historical interest. Under these circumstances, it is nothing less than a scandal that he should still be taught to boys in England. A book should have either intelligibility or correctness; to combine the two is impossible, but to lack both is to be unworthy of such a place as Euclid has occupied in education.

The most remarkable result of moder methods in mathematics is the importance of symbolic logic and of rigid formalism." (1)

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(1) - Mysticism and Logic, pp. 94-95

Thus, on the one hand we are faced with Aristotle who plainly believed that the proposition about the construction of an equilateral triangle is an obvious instance of true demonstration, while on the other hand this proof appears so pitifully lacking in rigour that we can hardly see what the philosopher really meant by demonstration. This is a sorry beginning when it requires that we know what the aristotelian intends by 'science,' seeing that he defines it as a knowledge obtained by way of demonstration. It is the more puzzling when Aristotle and his followers saw in mathematics the archetype of what science means to us : for in it, they say, that which is most knowable in itself is also most knowable to us -- adding that this is never the case in metaphysics.

We must first bear in mind that what Aristotle intended by 'science' and 'mathematics' is not at all what we usually mean by those words today. Our present Queen of the Sciences, which is called mathematical logic, logical mathematics, logistics, and frequently just logic, has an ideal of rigour which, in the ancient context, would correspond more to the operations of the calculator in arithmetic. The ideal would be had inasmuch as we might reduce the entire operational structure of mathematics or logic to the simple identity of

'A is A,' where A is not an element of a word, but a symbol standing for absolutely anything, thing or nothing; and where 'is' is not to be interpreted as copula, but again as a mere symbol exhibiting obvious and unasserted identity which is no more than a relation that our reason produces in the face of A. If an assertion were involved we could always take it away and just apply to A ≡ A. Once we have the symbol doubled by the relation of identity, we may pass on to exhibit the relation of equality, then to that of similarity, as well as to the converse of all three, which is easily had, seeing that A to the left is not A to the right, etc. From this simple beginning every type of relation involved in the operations of mathematics can be exhibited as well as all the entities defined by means of the operations — not only those of arithmetic, but those of geometry as well.(1)

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(1) - The particular case of geometry has been very well put by R. Courant and H. Robbins in What is Mathematics? The following quotation may be somewhat lengthy here, yet it is of such vital importance as a concrete statement of what is now meant by mathematics as distinguished from earlier use and procedure that we feel quite justified: "Here the reader may be troubled by an entirely legitimate doubt. What is this "point" on the number axis, which we assumed to belong to all the intervals of a nested sequence, in case it is not a rational point? Our answer is: the existence on the number axis (regarded as a line) of a point contained in every nested sequence of intervals with rational end-points is a fundamental postulate of geometry. No logical reduction of this postulate to other mathematical facts is required. We accept it, just as we accept other axioms or postulates

in mathematics, because of its intuitive plausibility and its usefulness in building a consistent system of mathematical thought. From a purely formal point of view, we may start with a line made up only of rational points and then define an irrational point as just a symbol for a certain sequence of nested rational intervals. An irrational point is completely described by a sequence of nested rational intervals with lengths tending to zero. Hence our fundamental postulate really amounts to a definition. To make this definition after having been led to a sequence of nested rational intervals by an intuitive feeling that the irrational point "exists," is to throw away the intuitive crutch with which our reasoning proceeded and to realize that all the mathematical properties of irrational points may be expressed as properties of nested sequences of rational intervals.

We have here a typical instance of the philosophical position described in the introduction to this book; to discard the naive "realistic" approach that regards a mathematical object as a "thing in itself" of which we humbly investigate the properties, and instead to realize that the only relevant existence of mathematical objects lies in their mathematical properties and in the relations by which they are interconnected. These relations and properties exhaust the possible aspects under which an object can enter the realm of mathematical activity. We give up the mathematical "thing in itself" as physics gave up the unobservable ether. This is the meaning of the "intrinsic" definition of an irrational number as a nested sequence of rational intervals.

The mathematically important point here is that for these irrational numbers, defined as nested sequences of rational intervals, the operations of addition, multiplication, etc., and the relations of "less than" and "greater than," are capable of immediate generalization from the field of rational numbers in such a way that all the laws which hold in the rational number field are preserved. For example, the addition of two irrational numbers α and β can be defined in terms of the two sequences of nested intervals defining α and β respectively. We construct a third sequence of nested intervals by adding the initial values and the end values of corresponding intervals of the two sequences. The new sequence of nested intervals defines $\alpha + \beta$. Similarly, we may define the product $\alpha\beta$, the difference $\alpha - \beta$, and the quotient α/β. On the basis of these definitions the arithmetical laws discussed in §1 of this chapter can be shown to hold for irrational numbers also. The details are omitted here." pp. 69-70.

Observe that no predication is required here, neither affirmation nor negation, for non-A is considered as no more than a term, even as the infinite name 'not-man' is not a proposition. Propositions may be replaced by a symbol which is not itself a proposition. The difficulty involved in the use of names, propositions, and syllogisms is thus avoided and the resulting rigour of the completely indeterminate A — in which it is also completely determinate — is achieved. It would seem, then, that all the elaborate theorems of mathematics or logic so defined "are no more than round about ways of saying that A is A."(1)

However odd, it appears of the utmost importance that we realize the extent to which every operation proper to the mind is excluded from the operations of logic thus understood, once the symbolic system has been set up. All of the latter can and sometimes must be performed by machines. In the course of the operation upon symbols, like in passing from x and y to z, the symbols themselves become irrelevant as signs or representations. This is plain from the fact that operational symbols may be fed to a machine which will turn out the correct solution of a computation too prolix or involved for the human brain. Hence, with regard to these operations "man's rationality marks only a difference in degree from other animals, and fundamentally, no difference at all from the machine.

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(1) - H. Poincaré, La Science et l'hypothèse, p. 10.

For modern computers are essentially logical machines : they are designed to confront propositions and to draw from them their logical conclusions."[1]

To those who believe that the computers are therefore endowed with mind as much as man — implying that mind is no more than what is had in the machine while computing — it is sometimes pointed out that thought is still needed to interpret the symbols and the result of the machine's operations upon them. Now there is no question of dispensing with the human mind. The whole point is that here an operation is involved in the course of which no mental activity takes place, so that when such an operation does take place in the human mind, there is an aspect to it that is not mental : one that does not involve apprehension, judging nor reasoning, unless we have identified these with what goes on in a computer. This identification however is implied, at least orally, when mind is defined by the kind of operation so efficiently performed by the machine. Mind, then, in some other sense of the word, may be held indispensable in devising the computer, and we alone to be concerned with the results of its computing. But this should not obscure the simple fact that something is arrived at by the machine without our thinking on it, somewhat in the way thoughts are contained in a book that no one

(1) - A. Kaplan, Sociology learns the language of Mathematics, reproduced in The World of Mathematics, edit. by James R. Newman, p. 1308.

is now reading — except that the machine produces new combinations of symbols without our even looking at them when they are finally there. Something we would have had to do has been done by it. And this is what we call a logical operation in the modern sense of the term — if the quotation from Dr. Kaplan is indeed a true statement of what is at hand. Whether the operation is in a mind or no is quite irrelevant. It is there — whatever 'there' may mean — in living ~~the~~ symbols, which we can interpret if we wish, but the interpretation of them as relevant to this or that material never was the business of the modern logician, and there is no reason to expect it from the machine.

Could probably be said more exactly

This may be what is meant in saying that modern logic has attained to a rigour and detachment hitherto unknown to man. Lord Russell, for instance, as well as any other contemporary logician of repute, has denounced the Aristotelian doctrines of logic as "wholly false, with the exception of the formal theory of the syllogism, which is unimportant."(1) This apparently severe opinion is actually too mild (~~I think~~). For the doctrine contained in the Prior Analytics is, from the viewpoint of modern logic, far too misleading to be merely unimportant. Its symbols, in effect, viz. A, B, and C for major, middle and minor

(1) - A History of Western Philosophy, p. 202.

terms, are restricted to a very particular kind of relations of reason arrived at in a way that is completely unintelligible in the light of what is now called logic. Moreover, the very meaning of syllogism as to form depends, in Aristotle, upon the value of the treatises that precede it in the order of learning. Again, for the purposes of modern logic, whose value cannot be contested, Aristotle's doctrines are as aimless as to go in search of an elephant for the earth to rest upon.

The supermathematical theory of groups is one of the most abstract branches of the game with symbols. Still, it is valid more than in abstraction, seeing that in quantum theory it provides the only means of accounting for what goes on in the atom.(1) One hardly sees how traditional logic, or even mathematics as Euclid conceived of it, could do such a thing. Meanwhile, it is generally agreed that manipulating a group is of interest only when applied in experimental science. It is a game and, as such, it is far surpassed by chess.

What becomes of mathematics when reduced to a game with symbols played according to fixed rules? Mr. E. T. Bell, in a paragraph that goes under the heading of The Queen of Queens' Slaves, says that in using the modern calculating machines

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(1) - Eddington, New Pathways in Science, chapt. XIII.

mathematics "has enslaved a few of these infernal things to do some of her more repulsive drudgery. What I shall say about these marvelous aids to the feeble human intelligence will be little indeed, for two reasons : I have always hated machinery, and the only machine I ever understood was a wheelbarrow, and that but imperfectly."[1] Now where does the 'repulsive drudgery' begin and where does it end? If mathematics, or logic, <u>is</u> the game with symbols played according to fixed rules, and if the machines can play the game, it appears that the only thing left the mathematician id the choice of symbols and the setting up of rules, the choice being so arbitrary that it can hardly be left to the rigorous machines.

Surely, that could not be what is meant by mathematics today. Still,it is difficult to see just what it means besides. No doubt the operations which may be entrusted to the computers have their compensation when they take place in the head of a mathematician,but this could only be inasmuch as they are associated with elements foreign to the formal structure that is somehow contained and whirling about in the machine. Perhaps the typical concern of the human mind in this game is the definitions constructed in terms of the kind of operations that can be wrought by a machine. Admittedly, it is an activity

(1) - Reproduced in <u>The World of</u> Mathematics, vol. I, p. 515.

that is useful when applied to the world of experience, but that could hardly be the aim of pure science when we define this as the pursuit of knowledge for its own sake, or as knowledge had for no other reason than to have it. Yet, what is had here that the machine does not have ? Perhaps the delight that goes with it when it is had in the man who is having it ? Now we remain with the question : Why should a man delight in having it ? What is there about it that is delightful to him ? And how is this expressed by the delighting science ? There ought to be something here that is not to be found in Aristotelian logic, — a logic which is not pursued for its own sake, not delightful, but only weary, if not painful — at least this has been the opinion of Aristotelians to the present day, if there are any left. Nor could modern logic, which is said to be science in its more pure and detached form, provide the joy of knowing the truth when it is defined as "the subject in which we never know what are talking about nor whether ~~what~~ we are saying is true." For there would (might) then be a special delight in not being concerned with truth. Would this really be the whole point ? It is difficult to say. ~~I confess.~~[1]

16. – <u>What happens when the rigour of this logic is exacted from strict science.</u>

If Lord Russell's exacting rigour is to be the canon of strict science, and if strict science in some other meaning is strict nonsense, then of course we have none other than the one which proceeds by way of "symbolically constructed fictions" or "creative definitions," which is the game with symbols played according to fixed rules. This leaves out all of geometry in the ancient sense of science as well as all of the arithmetic which was concerned with what numbers are, apart from the operations upon them. Thus it appears that modern logic, and or mathematics, leaves us with ~~no more than~~ but an elaborate development of what Plato and Aristotle had called <u>logismos</u>

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(1) - The old-fashioned philosophy ought to feel relieved by the exit of so many now delighting themselves in devotion to the activity of mathematical logic. The catharsis would be complete if these could spend all of their time in it instead of wasting breath to tell the philosopher how silly he is when pursing knowledge of the kind that does not define or demonstrate in the new mode. There is a suspicion abroad that if they had to abstain from needling the ancient-minded philosopher (who of course is no longer worthy of being called a philosopher at all except in a slighting way) there would not be much left for them to enjoy.

or <u>logistikè</u>, viz, the art of calculation that was used by the mathematician when he demonstrates. For they distinguished between the operations of calculation and the activity of demonstration even when this involves calculation, which can be shown by the following example from Euclid's <u>Elements</u> (IX, 24):

> <u>If from an even number an even number be subtracted, the remainder will be even.</u>
>
> For from the even number <u>AB</u> let the even number <u>BC</u>, be subtracted :
>
> A C B
>
> I say that the remainder <u>CA</u> is even. For, since <u>AB</u> is even [i.e. 'divisible into two equal parts'] it has a half part.
>
> For the same reason <u>BC</u> also has a half part; so that the remainder [CA also has a half part, and] <u>AC</u> is therefore even. Q. E. D.

Now this demonstration comprises a calculation, namely the subtraction AB — BC = CA. BUT This operation is not the demonstration. The symbols AB and BC stand for terms subject to calculation. But the middle term in this proof, viz. the definition of even number ('divisible into two equal parts'), could hardly be symbolized as such, nor is the definition subject to calculation, although an instance of the definitum may be so; but the instance itself could never be the middle term. Modern mathematics would retain only what can be symbolized, and no more than the operation upon the symbols. It would allow only the calculation which the ancient mathematician then used to demonstrate something that he held to be true. He assumed that there are even numbers (existence here meaning no more than that we can form true propositions about them, like in saying that they are divisible into two equal parts) and that the construction of their series is no more than a means of discovering them. And so it is with the basic assumptions of their geometry. E.g. 'point' and the various kinds of continuity are assumed to 'exist' in the sense we have just noted, and whatever we can construct by their means is no more than an instrument for the discovery of abstract (in a very special sense of abstraction) things and certain of their necessary properties. For instance, in the demonstration of the equilateral triangle, the construction itself is not the demonstration, nor is the demonstration about what is constructed as such. It is actually about what the modern mathematician considers a nuisance to be done away with, something involving the 'naive concepts' (i.e. concepts or notions in the Aristotelian sense), such as 'one,' 'number,' in arithmetic, and 'point,' 'line,' etc., in geometry, and whatever subject can be established by construction with them in demonstration — the basic entities being supplied by some kind of debatable or debated intuition, like seeing the intersection of the circles. Plainly, if these things cannot be granted no other course is left but that of creative definition. Having taken this course, we can move along freely and rigorously and make incontestable achievements none of them to be obtained by demonstration, which cannot, nor was it ever intended to be rigorous in the sense of calculation.

Once we have done away with definition in the sense of stating what a thing is, like 'a plane surface bounded by a single line which at every point is equidistant from the point within called its center,' we have also done away with what is the middle term in demonstration, and therefore with mathematical demonstration in the former sense of this word. Lord Russell is clearly aware of the effect of this emancipation of what we call Logistikè, which was previously considered to be no more than the handmaiden of mathematical science : the result is that mathematics is henceforth concerned with no more than what he calls 'logical fictions.' Now there can be no objection to fictions, logical or otherwise, especially when they can be used to produce results, like in mathematical physics, and in literature. In fact, a geometry of logical fictions (logical in the operational sense of the word) has proved to be far more useful than the one that was developed as a science acquired by syllogistic demonstration. The latter would indeed be false if it had to be real in the sense in which nature is real. This alone should prove that we cannot underrate the value of emancipated logic and/or mathematics. But it is also important to realize that it is held to be concerned with logical fictions and operations upon them, and that not to be concerned with truth is declared essential to it — this being one of the reasons why the physicist can be helped by it to approach the truth about nature, whereas euclidian geometry cannot render him that service.

EXPERIM.

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Aristotle showed great interest in trying to see, in his unhurried way, how people reach misunderstanding of a subject as well as understanding of it. Now there is a passage in the Metaphysics (II, ii) which may help at this juncture to appreciate the difficulty we are faced with even before embarking upon a venture into ancient modes of thought. Here is the chapter we have in mind :

"The effect which lectures produce on a hearer depends on his habits; for we demand the language we are accustomed to, and that which is different from this seems not in keeping but somewhat unintelligible and foreign because of its unwontedness. For it is the customary that is intelligible. The force of habit is shown by the laws, in which the legendary and childish elements prevail over our knowledge about them, owing to habit. Thus some people do not listen to a speaker unless he speaks mathematically, others unless he gives instances, while others expect him to cite a poet as witness. And some want to have everything done accurately, while others are annoyed by accuracy, either because they cannot follow the connexion of thought or because they regard it as pettifoggery. For accuracy has something of this character, so that as in trade so in argument some people think it mean. Hence one must be already trained to know how to take each sort of argument, since it is absurd to seek at the same time knowledge and the way of attaining knowledge; and it is not easy to get even one of the two.

The minute accuracy of mathematics is not to be demanded in all cases, but only in the case of things which have no matter. Hence its method is not that of natural science; for presumably the whole of nature has matter."

Concerning these observations about the influence of temparament and early training upon the mode of exposition we require from the teacher, it is plain from what we have already seen that we must bear in mind the assumptions made here regarding the nature of science in general, and those regarding mathematical science in particular. Science unqualified was not intended to mean just true knowledge of any kind, but knowledge had by demonstration from first, self-evident principles. In view of what we have already seen it ought to be clear that the terms 'science,' 'demonstration,' 'first,' 'self-evident' and 'principles' could not possibly have anything in common with, or even remotely resemble, what they should evoke in the mind when

Lord Russell uses those same words. In fact, we cannot think of a single term in Aristotelian philosophy whose meaning, as intended by it, has anything in common (including this word 'common') with meanings that would be valid once ~~he~~ Russell has applied his canon of verification to them. The reader will feel that we exaggerate. But let him see what happens to 'existence' and 'being' in the following passage : "Since 'is' does not belong to the primary language [for instead of 'A is yellow' a logical language will say 'yellow (A)'], 'existence' and 'being' [as they occur in traditional metaphysics], if they are to mean anything, must be linguistic concepts not directly applicable to objects." (1) Lord Russell may at times begin to use a word which appears to have the sense that we intend. But this appearance is soon dispelled. The word 'universal' is a case in point. To him it reduces to 'similar,' and then 'similar' becomes something that cannot, perhaps, be verified in a satisfactory way. (2) This is not being pointed out to

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(1) - An Inquiry into Meaning and Truth, p. 79. Actually, in this context, Lord Russell's canon seems to be 'If you can convince me.'

(2) - Op.cit., chapt. XXV, This calls to mind the followers of Parmenides, who suppressed the copula 'is' from 'Man is white,' and confined themselves to saying 'White man' instead. Physics I, 185b25.

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question the stature of Lord Russell, but only to warn how naive it would be to think that we might be able to find some common ground with him outside what he calls logic. If such a ground were accessible it would likely turn out to be unimportant.

The tempting advantage of Lord Russell's way of thinking is that the apparently easy questions raised by Aristotle and their difficult answers are not to be considered at all. If anything, this would make for economy of thought. It may even be that in the end a thought worthy of the name would be of the kind that we may impart to the computer. It appears (I think) that Lord Russell would not feel quite certain of this, but only definitely uncertain, which is again, perhaps, not clear. For certitude, in the present context, would have to be defined in an operational way, like in 'I certainly feel too warm,' which is emotional and therefore not scientific. From this point of view it can be shown to the satisfaction of any logical thinker that the reason why in Euclid's construction of an equilateral triangle the two circles intersect is really an emotional one : though middle-headed about his assumptions he wanted to carry on nevertheless. So that Euclid's geometry had strayed into ~~really belongs to~~ the province of ethics, which is unscientific. This I believe to be a fair instance of the way Lord Russell reasons outside the domain where he is indeed at home, which is not very definite if he is to go beyond that of the computers. ~~I think that whatever is valid in his trend of thought may be described as confined to what the machines would think about what they are doing — if they could.~~

Yet it is not a small thing to have shown what 'mathematics' means today — a type of operation that is a tool so strikingly proficient.

Returning now to our passage from the <u>Metaphysics</u>, we can see that Aristotle would have deemed it impossible to teach anything whatsoever on any subject to the one who refuses a statement that does not have the rigour (still dubitable in some regard or other until the computer gets hold of it) of A ≡ A. Must we grant that the non-listener is right ? For that is what our problem amounts to. Aristotle would have said not that we should change the subject, there being no subject left to change, but that we should not even try to talk to him, SINCE seeing that his canon of exactness forbids him to listen at all. BUT So perhaps we may carry on notwithstanding, provided we still have someone ELSE to talk to.

We ought to feel aware of the extent to which all doors have been shut, and realize, too, that this is perhaps not too essentially important for the business at hand. The reader may have gathered the impression that the case of the computers was SO STATED overstated to the point where it seems that the activity once

(1)

In the ~~same~~ Old Testament, from which A.M.Turing quotes ~~to lay~~ bare the hopelessness of theological argument, there is also question of a person who declared: <u>Non serviam</u>, ~~hereupon he~~ got ~~himself~~ involved in a <u>multitudo negotiationis</u>. St.Thomas, ~~in a mood typical of baroque scholasticism,~~ explains that the activity of the Bearer of light, "averted from the First One, was bent upon the many of inferior things, and it was <u>their</u> primacy that he coveted."(<u>QQ.Quodlibetales</u>, qdl.5, a.7, ad 1.) His thought and action became multitudinous, scattered and confusing, like the elaborate ratiocinations of the sophist who, [illegible] by choice ~~drawing by choice,~~ [illegible] ~~unrestrained,~~ upon the infinite store of <u>ens per accidens</u>, contrives to make the worst appear the better reason, and the most, the least of all ~~and that which is the most, to seem the least of all. One plain~~ The assumption is that that of which there is most is ~~xxxxxxxxxxxxxxxxxx~~ also what most truly <u>is</u>, even to the point where nothing is but what is not. For instance, of Mr. Smith ~~is only one, while there~~ there are more particles, ~~of him,~~ and ~~xxxx~~ still more events, than there are men in all the world. Now, the ~~xxxxx~~ 'many' conveyed by 'a mere bundle' of fleeting 'occurrences' has far more the nature of sheer many than the integral parts of a <u>totum per se</u>. Hence, to meet the new standard of being, what could be more suitable than to father the rational animal

(2)

as a mere bundle of events instead of a substance—
this "hopelessly muddle-headed notion," ~~as Lord Russell once decreed.~~ Quite appropriately, 'Mr.Smith' becomes
a collective ~~name~~ term, ~~as in 'My name is~~ Legion.'
On the other hand, mankind is lowered to the level of mere
tools, and so can ~~do no more than~~ serve, a tool being
of its nature movens motum, as any instrument ~~[illegible]~~ is.
(A very strange tool, ~~all the same;~~ of that; an instrument without
a principal agent, like ~~x~~ a symbol without a meaning. For
the principal agency was buried ~~by~~ in the birth ~~fecundity~~ of one
tool ~~making~~ from another, ~~as [illegible] is obvious in automation;~~ while
Lord Russell has shown something to the effect that we
might have tools of tools ad infinitum, ~~and~~ nothing but
tools all along the line; so that everything may well be in
the service of ~~nothing~~ nothing, and 'service' a confusing
linguistic device.) — Whether or not ~~i~~ we believe in Sacred Doctrine
is not at present to the point. The plain fact is that the
literature to which Turing refers sets forth a strophe for
which we have provided an antistrophe: we have echoed the
non serviam, not in the romantic way of Marx, ~~[illegible] identifying~~
not ~~merely~~ merely by enslaving some of these 'infernal machines' (as
E.T.Bell calls them) to do the wearisome drudgery, ~~by~~ but by
identifying science with the drudgery itself, and that which Aristotle
held to be the highest ~~from~~ form of life, ~~[illegible]~~