…eration of the world of calculation

1. Relation of identity:

$$[\,S \equiv S\,]_{r_1}\,[\,(\equiv) \equiv (\equiv)\,]_{r_2}\,[\;\equiv\;]_{r_3}$$ | Ex inde, no predication: just A(A).

Necessity of symbolization: no relation of id. has a name, here.

We can take the symb. as a wholly transcendent form: e.g. a letter of alphabet which can be made to stand for anything.

Symbol: a mark 'out there'. Not same as words: there no mechanical import.

Process carried on ad infinitum; and this, in turn, ad infinitum.

…rchum animae; may be
…s' produced by act of reason. (5)
…e 'outthereness'. This, too,
…e, intuitive, substitution, the
…the basic laws of calculation:
…itutive. In following sample,

…equal. & ineq.

…s between classes. They do not
…d by the unit'; no question of
…numbers in sense of …. Such
… explic.)
…;' (rel. of id.) and 'symbol'
…'. Former defined by a
…ter by a convention, but not
…sible about them.
. the notion of class and
some things are called
Av., is the notion of class
on a man?' Is the notion
a class of classes'?
VM. III 1674-5)
…ned 'as an object' not the
…me, and as good). Similarly,
…n of definition', includes itself,
…s from any other definition by
…13 b. [Log. Iª, q. IV, a. 2])

But suppose I say … 'definitions,' is not the definition of definition contained in this class?

What does 'the class of all thinkable concepts' mean? What do we mean by 'all' and 'whole'?

The 'notion of notion' is a notion. If a I add 'every,' the notion of notion is referred to; and this is a particular kind of notion revealing the reflexive nature of intellect. But, is 'every notion' the same as the 'totality of notions'? Every integer a proper notion…

(10) The relations in question 'consequuntur actum animae' may be carried on ad infinitum. They are 'opus' produced by act of reason.
These may provide the stuff for symbolic 'outthereness'. This, too, an opus. Put 'out there' by a primitive, intuitive, substitution, the material or stuff thus produced reveals the basic laws of calculation: the commutative, associative, and distributive. In following sample, three types of symbols:

(a) $\boxed{\cdot\ \cdot} + \boxed{\cdot\ \cdot\ \cdot} = \boxed{\cdot\ \cdot\ \cdot\ \cdot\ \cdot}$ equal. & ineq.

(b) $2 \quad 3 \quad 5$

(c) $a+b=b+a$.

The symbols represent classes and relations between classes. They do not stand, formally, for 'pluralities measured by the unit'; no question of measure, here; nor for relations between numbers in sense of.... Such classes are symbolic structures (ut supra explic.)

Note difference between 'ens rationis' (rel. of id.) and 'symbol' as 'out there'. Former 'in ratione tantum'. Former defined by a particular kind of operation of reason. Latter by a convention, but not as in case of name; then, by operations possible about them.

Note, further, the difference between the notion of class and a given class; between the reason why some things are called classes, and the classes themselves. Ar., is the notion of class a class? Same as 'is the notion of man a man?' Is the notion of 'a class of classes' the same as a 'a class of classes'?

Consider Russell's antinomy. (W.M. III 1674-5)

Remember: 'intelligere as an act' and 'as an object' not the same; diff. ratione. (As in being as true, and as good). Similarly, the 'notion of notion' or the 'definition of definition', includes itself, gomo. But the def. of def. differs from any other definition by definition. (cf. J. of S.T., C. Ph. I, 133 b. [Log. IIa, q. IV, a. 2])

Item, 'genus' is definable; but what is defined is a species. ∴ genus a species.

But suppose I say 'the class of definitions' is not the definition of definition contained in this class?

What does 'the class of all thinkable concepts' mean? What do we mean by 'all' and 'whole'?

The 'notion of notion' is a notion. If a I add 'every' the notion of notion is referred to; and this is a particular kind of notion revealing the reflexive nature of intellect. But, is 'every notion' the same as the 'totality of notions'? Every integer a proper notion...

(12) Classes { not cont. – normal
contain. – non-normal

Remember examples: { mathematician
thinkable concepts.

<u>Normal</u>: We distinguish between { 'mathematician': not a class.
'Every mathematician' (in Engl. 'all'): n a
'Totality of mathem.': symb. construction.

If 'math.' meant 'every,' 'all,' or 'totality of': not predicable:
E.g. '**Euclid**' is 'every math.', 'all', or 'totality'.
'Every (omnis)' refers to many, but indefinitely. E.g. 'every kind of triangle,' and 'every equil. tr.' 'How many' there are or can be, is extrins. to 'what it is to be...'.
The multiplicity may be finite or infinite. If infinite, in pot. or in act. If finite, 'whole.' If 'inf. in act,' also whole.
But notice, when from 'every' or 'all' to 'whole,' no longer in order of predication. Thus the word 'triangle' stands for a [illegible] genus; and genus not a class. We can state explicitly the kinds of figure that the genus triangle can be said of. But these kinds do not have a common term, name, expressing their specific differen[ce]. 'All the kinds of triangle' both definite and indefinite, neither part. nor univ. But we can put down a symbol, and make it stand for 'equil.', 'isosceles,' 'scalene'. Only then is 'equil.' a 'member' of this class. And they are a finite totality. Symb. constr. essential to expression of collections.
'All' may refer to an ~~[illegible]~~ infinite multitude: e.g. 'all equil. tr. of equal basis.' But, may we then go on to say that there <u>is</u> a class of 'all eq. tr.'? If not actually infinite, not a class. If actually inf., then a whole, and a class. (Note: this a normal class.)

(13) [Note, too, that we have put aside the 'notion' of 'triangle', or of 'equil.'?]

But we can put down a symbol to stand for 'every and all' as a whole.' E.g. for 'the totality of points on a line'; 'the totality of whole numbers' } 'greater than 'the totality of even...'; or 'the totality of fractions betw. 0 & 1.

The dist. betw. inf. in pot. & inf. in act not exclusive. Unless we could prove, operationally, that contr. Proof would have to be extra-Math. The point is that the probl. of such a distinction cannot arise from p.o.v. of calculation. Whether whole numbers are already there, given, or not.... Whether 3 three ones or one three; divisible or divided, indiff. The operation upon numbers ~~is~~ ~~not~~ does not affect the numbers because they are not considered as natures, but as classes.

Non-normal. 'The class of all thinkable objects.'

To us, 'every thinkable object' ~~may~~ refer to the 'notion of object,' ~~which~~ as a thinkable ~~of~~ object. But, does every thinkable object, or 'all thinkable objects', constitute a class? We may have a symbol stand for ~~it~~ an infinite class whose characteristic 'thinkable object.'

(Note: taken univocally!)

But, as soon as we do so, we no longer apply to ~~object~~ thinkable object as such, but to instances of it. Now, the 'notion of thinkable object' is itself a thinkable object. Does this mean that it is part of itself? Nullo modo. Only reveals reflexive nature of intellect. This not possible if intellect like sense. Then either regressus ad inf., or... This has nothing to do with classes.

The 'class of all thinkable objects' is ~~either~~ considered either as a class or as an object. The def. of 'class' and of 'object' not the same, although all classes may be objects, not all objects are classes.

(14) But there is a diff. between the case of 'mathematician' and that of 'thinkable object'. The latter transcendental. The notion of math. is not a mathem. But 'thinkable object' is an object. A diffi. would arise, in our context, if 'every thinkable object' ~~were a class~~ meant the same thing as 'the class of all thinkable objects,' in the sense of a whole. ~~Hig~~

The 'class that is a member of itself' unduly compresses the respect in which something is one with the respect in which it is many: Not part and whole in same respect. ~~(leg, part of a man, but whole leg not part of a leg.)~~ Compresses 'absolute nature' with 'nature qua subject of intention of universality' and 'intention of universality' with the particular instances.

On the other hand, if we have to do with classes, the problem of classes members of themselves should not arise, unless 'every' is 'whole'.

(15). What is the origin of Russell's antinomy?

Unwarranted assumptions concerning classes and concepts.

Confusion of symbolic constr. with universals.

When it is said "if we consider a class as a concept, then the class of all concepts in the world is itself a concept, and thus is a class which is a member of itself", we should like to know what is meant by a class, by all, and by concept (not to mention the 'world'), before we can begin discussing the expression 'a class which is a member of itself.'

In our way of thinking: a class is a 'bundled plurality' (coacervatione unum). A concept, either 'by which' or 'what' the intelligibilem intelligentia attains. First, the knowledge itself; second, the known, res. All, taken either as id. with every, as in 'all water is'; or in the plural, as "all the waters..."; or "the whole of the water".

The thing known, and my knowing of it.

Concept: Now, 'concept' and ~~the class of all conc~~ 'a class of concepts' may both be concepts, viz. 'concept of concept', and the 'concept of a class of concepts.' This latter requires symbolic. Thus, 'a finite number of concepts' may express a notion; as such, it is not a finite number of concepts, but the notion of a finite number of concepts, e.g. the kinds of triangle. But if I want to put down such a class, I cannot do it in one word: 'triangle' does not mean that? —

The word 'concept' analog., referring to diff. notions one in proportion; like being div. into categories.

Concept of number not a ~~concept~~ number, but one kind of concept. And the concept of class is not a class, but of class. The concept of class may be ~~a member~~ ~~The class~~ considered as one kind amongst many. The class will be a class of concepts, not 'the concept of class'.

(17) Origin of R's antinomy: hidden assumptions concern. 'classes' & 'concepts'

Kasner & Newman's statement: "...if we consider a class as a concept, then the class of all concepts in the world is itself a concept, and thus is a class which is a member of itself." (p. 217). This a 'non-normal' or 'extraordinary' class. Then example of 'all mathematicians'...

Hence { A: class not member of itself.
        B: " member of itself.

Now Γ: class of all A's
    Δ: class of all B's

Is Γ an A or a B?
- If Γ is B, should not be B: it should contain only classes A.
- If Γ is A, it ought to be B.

Difficulty arises from slurring over 'class, member of itself.'
Have called attention to what we mean by 'class,' 'concept' (etc), 'every', 'all', and 'whole'. (Note: 'every', 'all,' syncategorematic terms.) [Syncat.: "significatio ejus non repraesentatur ut res per se, sed ut modus rei, i.e. exercendo modificationem alterius rei."]
- Also seen diff. between 'concept', 'class of concepts', and 'concept of a class of concepts'; item, diffic. of 'class of all concepts.' Are terms of this class given? Then actual infinity. But in fact pot. infin. The possibility of more is given, but no corresponding actuality. Then not a class, unless 'open class'.
- Quid 'all concepts (in the world),' if not merely 'every'? If 'all' whole, then problem like 'squared circle'. For 'whole' is said of one, not of many. Does 'all concepts' signify a totality,
+ 'Concept' is a universal or logical whole, predicated of each part. Integral whole not predicated of parts. "Door not a house." Thus, 'the class of all concepts is a member of itself' implies that there is a universal whole which is an integral whole (Remember Met. V, 26; l. 21, 1098-99)
Actually, 'concept' may be considered 3ter. Cf. de Ente, c. 4.

{ Definable: totum integr.: natura absoluta A
  Predicable: totum logicum seu univ. B
  In particulars: C

These must be distinguished!

(19) To assume the notion (?) of "a class member of itself" implies that there is a universal whole which is an integral whole. Memo:

Totum: "cui nulla suarum partium deest, ex quibus." { "continens est contenta", i.e. contenta quid unum in toto

Totum universale: "unumquodque contentorum a toto continente est ipsum totum continens." Each part is the whole. Thus animal 'contains' man and horse.

Totum integrale: "ex partibus unum, ita quod non quaelibet partium sit unum illud"; de nullo suarum partium praedicatur.

We considered the case of definition: 3fold:

A. Qua definable: natura absoluta: totum integrale: oratio.
B. Qua predicable: as in 'definition of number'; here ut genus: totum univ. seu logicum; number differ.
C. In particulars: the "rationes explic...", e.g. "a plurality measurable by the unit." Prime: measurable only by the unit. Composite: measurable by the same number"

'Definition' may be taken in all three ways, but they are distinct: A, B, C. But "the defin. of defin." is not an instance of "definition of definition," but of an instance of "definition"; just as "rational animal" is not an instance of rational animal; yet it would be if we could telescope A, B, C.

Now case of name: the name 'name' may stand for:

A. Totum integrale: the definable: "oratio significans...."

Dist. to define 'name', and to define it as logical whole. → B. Totum logicum seu universale: as in 'man' is a name, i.e., a "vox signif. ad plac."

C. Pars subjectiva: this name 'name'? ~~"Name" is the vocal (or written) term signifying~~

An instance of a part identical with the whole. But it does not follow that 'what it is to be a name' is the same as "the name 'name'."

The definable nature, such as 'animal', as in the particular, is not the definable nature. Thus 'Man is an animal' does not mean that the definition of man and animal are the same.
Item, definition is said of "a plurality...."
It does not follow that this definition is the definition of def.; ~~of number~~ or the definition of definition the definition of number.
Thus, the definition of definition is both that and an instance of definition, but not in the same respect. For that would mean that the integral whole is identical with its parts in the way the the logical whole is.

Whence the "class of all concepts in the world" is itself a concept, and thus is a class which is a member of itself"? The concept of the class of all concepts.... ambiguous, as we saw. Hence, first, "the concept of class" as such, or "as a concept" not the same. (Note: words are used here.) They differ by definition.

A class member of itself means a multitude of subjective parts ~~exp of definition~~ each of which is identical with the integral whole that ~~defines~~ is the definition of what they are. The result: 'to be a man' ≡ to be Socrates.
And: man (rat. an.) is an instance of man.
But this not so clear in case of definition, concept, name. This manifests reflexive nature of intellect. Yet here too the respects remain different.

(21) Russell's antinomy, qua expressed in words, arises from distinction of classes into: A, normal or ordinary; B, non-normal or extraordinary. Not concerned with his theory of types, but with notion of B.

Circa hoc (a) reflexive nature of intellect: in case of rel. of identity; in case of 'object,' of 'concept,' of 'notion,' 'definition,' and 'name'; (b) dist. of states of universality.

(a) Implies no new notion. The relations of identity are 'new' as individuals only; and difference between the named and the defined, that of confused integral whole and distinct. But the 'named' and the 'defined' are identical, while name & definition are not.

(b) Different states of the same. E.g. 'definition,' or 'name.'

E.g. definition (α) ratio....
(β) predicable: rel. of univ.
(γ) a definition, comparable to 'man' is an 'animal' (i.e. a subjective part)

A.v.: 'definition' may signify a definable nature, a logical whole, or a subjective part.

Then we noted that each subjective part is identical with the logical whole; Now, R's view is that the three (α, β, γ) are identical. Dist. ① that which is defined, predicable, and subject, idem, C.; ② definition is an 'ratio'...; the relat. of univ. is the definition; rel. of univ. a subj. part, N.

Yet ② is implied when we say that 'man' means 'all men.' (But are they a coll.
Thus 'man' would mean its subjective parts.

Note, however, circa hoc, that the 'expressio' 'normal ~~cl.~~ class' shows no awareness of this distinction: the 'class of all men's' not a distinction considered irrelevant. Why not? No answer except by a mathematician. Yet this is a problem. This distinction does not appear in B.

The necessity of this distinction in transcendentals: abstract term and concrete term signify same: ens et entitas; verum veritas.
Because in transcendentals: abstract term and concrete
Besides, we abstract term predicated of another: humanitas est animalitas. Distinction only in mode of signification.

but not 'all' the parts together id. with whole. Multipl. of parts not essential to universal whole. Only 'aptitude'... But not then a class with one member.

'Being' signif. not only 'quod' but also quo. de Ver. 21, a. 4, ad 4.

(22)[a] Yet must be made: Thus, the definition as α, does not signify it as β and γ. True the def'd is a subjective part β: but 'to be γ' is not same as to be α or β.

Plainly, 'class' refers to γ; not to α or β. If it did, then we would have B and the antinomy.

Seems, then, that the real expression B has no verification.

Question: why would we need B classes?

→ The problem of the existence of infinite classes very different.

To define is to 'set apart'.

Existence: Many meanings.

Ens naturae { phys. / math. * / metaph. } indiv.; universal. { absol. / log. whole / subject. part }

math. indiv. → indiv.

Ens rat. { relation of negation (~~[illegible]~~) → real. / logical: second intentions; also 'problems'? / fiction { normal: Homer is a poet / imagin.: centaurs. / scientif. { physical: models; weight is... / mathem.: number is.... } } }

Place 'physical truth' of machines.

Way a cube ~~of~~ or a ball go down-hill.

(22)b Oswald Veblen & John Wesley Young, in *Projective Geometry*, vol. I:

"The notion of a *class* of objects is fundamental in logic and therefore in any mathematical science. The objects which make up the class are called the elements of the class. The notion of a class, moreover, and the relation of *belonging to a class* (being included in a class, being an element of a class, etc.) are primitive notions of logic, the meaning of which is not here called in question. [Fn.: Synonyms for classes are *set*, *aggregate*, *assemblage*, *totality*; in German *menge*; in French *ensemble*.]"

Nagel & Newman, in *Goedel's Proof*: "This fatal contradiction [Russell's antinomy] results from an uncritical use of the apparently pellucid notion of class."

H. Weyl, in *The Mathem. Way of Thinking*, p. 1849:

"However, it is surprising that a construct created by mind itself, the sequence of integers, the simplest and most diaphanous thing for the constructive mind, assumes a similar aspect of obscurity and deficiency when viewed from the axiomatic angle. But such is the fact; which casts an uncertain light upon the relationship of evidence and ~~of~~ mathematics. In spite, or because, of our deepened critical insight we are today less sure than at any previous time of the ultimate foundations on which mathematics rests."

These difficulties are all for the better, inasmuch as they oblige us to make a fresh start, to go back to the simplest things, about which we had made tacit assumptions. The notion of class is one of them. Thus, if 'class' is taken as an 'assemblage' (coacervatio) *and* a 'totality', and if our definition and division of whole were accepted, a class is either finite or *actually* infinite. The pot. inf. not a class. But the parts of *a* whole may be in it potentia.

"Cui nulla earum partium deest, ex quibus."

Euclid: the sum of .... equal to two right angles. (Parabolic geom.)
Gauss & Bolyai: .... less than two r.a. (Elliptic) positiv. curved
Riemann & others: .... greater than two r.a. (Hyperbolic) negat. curved.

"Less, equal and greater: three contradictory statements."
H.W. Turnbull,
WM. 159.

Actually, this seems not more contrad.
than legless and twolegged. They
differ by definition.

On saddle-shaped surface, i.e. negat. curved, angles < 2, or less than 180°
On sphere or positively curved surface, 3 angles > 2, more than 180°.

See "The Univ."
A Sc. Amer. Book, Gamow: on the
Universe, pp. 59 et sq.

(23) Actually infinite classes do not contradict the notion of class, but pot. inf. classes do, if the definition of 'whole' or 'totality' is granted. [Note concrete 'whole' & abstract 'totality', convertible, directly predicable: Cf. T. of S. Th., C. Phil. I, II P., Q. V.

But suppose it were said that definitions are free. Then we distinguish between, on the one hand, interpretations of names or of symbols, and, on the other, definitions of what the named is: quid rei. These are not free, but good or bad; virtually true or virtually false.

Now, if we take interpretations of names, then either designation, or an ~~description, or def~~ expression sufficient to set apart...; e.g. 'featherless biped', or 'desire for vengeance'. Many nominal definitions of same thing possible. But not free in sense of 'as we please'? This true of names as 'conventional'. But definitions not 'conv.' in that sense. Yet this in common: once chosen, must be consistent.

Hence, if we have agreed on the use of 'class' ~~[illegible]~~ to signify an assemblage that is a whole, and if we have defined 'what a whole is,' as we did, then we cannot use the word class to signify the potentially inf. ~~[illegible]~~ except by changing the imposition of the name 'class.' Then we could speak of "classes that are not wholes." The point is that we must realize that we have changed the meaning. This is what happens when we say 'the class of all classes,' unless it is actually infinite. Whether or not there is such a thing? Problem not clear to me. Requires some kind of demonstration.

Like 'square' with only three sides;

~~We have already defined an infinite class as an infinite multitude having the nature of a whole. Thus, a line would be an infinite class if 'all the points on a line' formed a totality. The line is a totality, but all the possible points are not. Yet, as we saw, 'symbolically,' 'in imagination,' 'unimportant,' this distinction is not necessary, because the no act of potency. problem cannot be raised in our way. Hence, when speak of infinite classes overlooking our definition the words ought not to stand as name of class,~~

(24) But we can certainly define what an infinite class would have to be, viz. an infinite multitude having the nature of a whole. Thus, if the natural numbers are both infinite in multitude and a whole, they are an infinite class. But are they a class? Similarly, any finite line would be an infinite class if 'all the points on the line formed a totality,' apart from the line being a whole.

~~Philosophic~~ Metaphysically, these two problems have been solved: both are potentially infinite, not actually.

Yet this does not exclude infinite classes. But apart from defining what they ought to be, how do we show that they are in the manner of whole numbers, triangles, etc.

How do we go about it? We define

1° 'same number' or equivalent classes.

2° 'number': cardinal number.

3° The cardinal numbers of finite sets are called natural numbers.

But what about 'infinite sets or classes'? Here is how Hans Hahn goes about it:

"But are there really any infinite sets? ..... of all odd numbers is denumerably infinite." WM 1594-5.

Also, Courant & Robbins, What is..., pp. 78-88.

I cannot follow this method of reasoning, if I have to understand 'totality' as defined.+ If we change its meaning, as we do in this context, just what is it that we are faced with?

The only infinite I know first (and this is what the word means first, as in 'there is no greatest whole number') is potential. Then I can define what an actually inf. mult. would have to be. I can then establish certain properties, e.g. that the multitude of its members cannot be expressed in terms of ~~a~~ whole numbers.

(25) Compare a pos. inf. series A with B ad. inf.

They are not equal. A tends towards equality, as $1 + \frac{1}{2} + \frac{1}{2^2} + 1 + \frac{1}{2^3}$.
Thus B defined as the limit of A.

Kinds of infinity. [Series of integers > series of even, for each even two integers: particular series. [Unless:

(a) $2_1\ 3_2\ 4_3\ 5_4\ 6_5 \ldots$

(b) $2_1\ 4_2\ 6_3\ 8_4\ 10_5 \ldots$]

Is the limit of the series of integ. > of even? Both tend to infinity.]

Take B, and its members each composed of members y. Then, the multitude of B < multitude of y.

Let B stand for an infinite class whose members are sub-classes each which is an infinite class a. Then the multitude of B < the total multitude of a. The totality of the total multitude of B ~~an and~~ and that of all a's ≧ respectively. Like in case of a group of men and their hairs.

Post.: in some instances ~~[illegible]~~ whole infin. = whole

(25) "Are there really any infinite sets?"

A. There are infinites in potency { in nature. / mathem. (in abstr.). / reason. } No postulate.

B. "Are" means: can we state what they would be? We can, and in this sense the infinite set is. But of reason only, until

But we do not do this by passing from A to B. Not necessary, { unless 'existence' of B held same as that of A. / or, unless A merely defined as B.

Creative definition. Symbolic construction. Status: fiction. } The postulate in B, "Infinite multitude + totality" Symb. constr.

Is the definition fruitful? By comparison of B with A. If B.

B > any set of A. A for 'all', not a totality.

B lim. of A.

[A cannot be a member of B, except individual of indiv. B.

Distinction between universal & particular.

Whatever follows from 'If B' necessarily is true 'If B.' But because something follows necessarily does not entail that B is true.

"Infinite set" { integr. whole (a) / logical whole (b) / subjective parts → ∞ (c) }

Each subjective part an infinite set

Each an infinite set of subject. parts of B?

Is there an infinite set of subject. parts of B?

Note: cannot compare c and a in this respect

But we can ask: Can the members of an infinite set be infinite sets in their turn?

Classical analogy: infinite set of men, each having n hairs. Multitude of men < hairs.

Now Inf. set B of infinite sets a, b, c .., we cannot compare B with a as (a) or (b) with (c)

Equiv. of part and whole. But we can compare them as quantitative wholes.

Thus the infinite set a = B. But is B = a + b

The members of all the subsets > B. No contradiction: more legs than men.

But this implies diff. aspects, brought together symbolically. Actually, parts of a, b, etc., not proper parts of B.

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Let A stand for an infinite set, with members $a_1, a_2, a_3 \ldots a_n$
Let the members be, in their turn, infinite sets.

Analogy: { a set of men, A
each man, a set of atoms, a.

cf. III^a, 10. Two infinite lines, a & b. They are equal.

When compare A to a, not comp. whole with subj. part. It is like comparing a set of boxes, each box containing as many marbles as there are boxes. Same would hold of infinite sets.

If not $\rightarrow \infty$, then intellect destroyed as reflexive power.

Possibility of regressus ad ∞ shows man not a machine.

Why can fictions be fruitful?
Fictions, here, not mere entia rationis. Remember importance of symbolization illustrated in case of number as mere collection. Introduces need of number as mere collection. Yet reason is not sensible: external or internal. Yet reason is not, as obvious in calculation: resort to symbols. Still, there are artifacts.
General case: Art, as dist. from nature, can be fruitful: medicine; wheelbarrows; houses, machines ...
Nature cannot do these things: determined to one.
Mind can bring together things that cannot be together as unum per se.
Why could not mind produce artificial structures more efficient than nature? The mater. artifact, out there, unum per accidens; yet, in mind, unum per se.
Hence, to know nature, we dispose of means that are not natural, and more efficient than what nature provides. Eg. telescope. Ars cooperativa naturae

Math. requires calculation, ~~art~~ art. Math. phys. starts from artifacts (stand. of length, weighing machine, clock, thermom.). These → equat., again artifacts; → Better knowledge of nature.

Basic isomorphism between nature (intrinsic princ.) and art (extrinsic). Nature is an artifact. (Phys. II) Only by use of art can we join the divine artifact. But never the same, i.e. ident. Likewise, on other plane, art of calculation never substit. for sc. proper. (Explain creative definitions.) Early example was, in physics, the mechanical model: definition of the atom was creative: small planetary system where this was conceived mechanically. (Physical truth of machines.) Symbolic construction also creative. But the ultimate term not an artifact. If so, then frustration, as is the case of present foundations of mathematics.

The arts we need for the purpose of mere knowing are remedial. Our knowledge must come from the things. Indetermination of our mind. Hence, logic; even in simple apprehension. Hence calculation. But they do not have the nature of a term, a final cause, a good in itself.

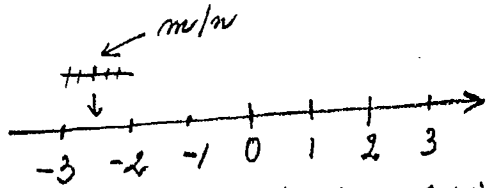
Otherwise 'pour l'ange'. Or, pure humanism: we rest in our own works. Yet, even God rests only in Himself.

Example of equivalence: "any finite set of circles of radius 1 is equivalent to the set of their centers."

A sub-unit obtained by dividing the original unit 1 into n equal parts is denoted by the symbol $1/n$; and if a given quantity contains exactly m of these sub-units, its measure is denoted by the symbol m/n. This symbol is called a *fraction* or *ratio* (sometimes written $m:n$). Where m/n is divested of concrete reference, and considered as a pure number, an entity in itself, on the same footing with the natural numbers. When m and n are natural numbers, the symbol m/n is called a rational number.

Natural numbers, i.e. the positive integers.

"The system of all rational numbers – integers and fractions, positive and negative."



Thus 'rational number' and 'rational point' interchangeable.

Take relation $A<B$ for nat. numbers. Then, on this number axis: if $A<B$, then A to left of B. Then $A<B$ and $B>A$ if $B-A$ is positive. Hence, if $A<B$, the ~~the~~ points (numbers) between A and B are those which are both $>A$ and $<B$. Any such pair of distinct points, together with the points between them, is called a segment, or interval, $[A, B]$.

The distance of a point, A, from the origin, considered as positive, is called the absolute value of A and is indicated by the symbol $|A|$.

It seems that calculus is applicable to nature because nature is not exact: i.e., there is a point at which the tendency towards infinity will compass nature because of its exactness. E.g. physical sphere torsion.

- Existence
  - real
    - phys.
    - math.
    - metaph.
  - rationis
    - rel. & neg.
    - logic
      - log. — 'what meaning' in interrogatio
      - dial. (interrogatio)
    - logistical or symbol.
- fiction
  - normal: Homer is a poet
  - literary:
    - Oedipus
    - Centaurs
  - physical: inertia
  - mathematical

Exist. of math. indiv. in the imag.

demon. proced.

~~23~~ Saw in what sense we cannot speak of ∞ classes. If ~~you~~ we want to speak of classes in some other way, e.g., by <u>choosing</u> your definitions, we are free to do so, in the sense that we are free to define man as a solid line, or a cubic fish, or a point endowed with an agent intellect. This possible so long as you do not have to define the parts of your definition, except nominally. There will be no problem so long as man is not a name but a symbol. Thus

a is (b + x).

If Math. is confined to choice of rules for game with symbols, then, what is strictly math. science: one or the other or both? The game is mechanical; the rules are chosen.

How do arbitrarily chosen rules exist?

Definitions are arbitrary, ut supra.

We don't agree about what { definition is.
a word is.
the word 'is'.
etc. ad ∞

And we don't have to agree about anything. We are machines.
Like in case of a machine. You don't have to agree with the machine: you don't wait till the machine agrees with you.
The point is: we want to show that we are merely machines.

What is characteristic of man? Counting? Binary also, machines do. Syllogism? Reduced to same.

This unpoetic universe

If found. of math. less certain today than ever, could it not be because of assumptions concerning simplest notions? It was, historically, the case of the notion of class. (Cf. E. Nagel, Gödel's Proof)

---

(18) Take case of 'notion of notion'. One expressed by a name, other by definition. Idem: def. of def. which is one definition. The name 'definition' signif. in one way, i.e. as a name; the def. of def. (oratio explicans...) signifies the same, but in terms of what is more known.

Name signif. ~~the~~ indistinct whole; def., distinctly.

Item, 'definitum' qua named & qua defined.

Now 'definition' is a logical whole, i.e. qua predicable. Its parts inf.

Also an integral whole, e.g. 'oratio naturam rei aut termini exponens'.

To consider definition as a 'definible' and as 'predicable' very diff.: first, no intention of universality; second, yes. Otherwise, 'man' is an intention of univ., relation of reason. Case of 'name': as a 'name'; as standing for a definable nature: vox significativa ad placitum": this a:

[Praeterea: Notion of notion equiv. if both refer to same, same way.

But if 'notion', and 'definition of notion'?

Otherwise, like relation of identity and its identity: no new object, except numerical repetition of same act.]

Memo, a propos transc.: act of intell. an 'object'; act of will a good. But act of will and good not convertible: But 'act of will as an act' and 'act of will as a good' are con

Take the concepts of 'class' and of 'individual' and of 'individuals of one kind'. Three concepts. They are an instance of the class 'three'. This class is an instance of class. But 'class' can never be an instance of 'class'.

But, where pause, not same. Notion of notion is a notion; def. of def. a definition.

Note, however, the 'name' and 'definition' do not signify in the same way. Name signif. indistinct whole; def. distinctly. Definition qua named & qua defined. Thus, the 'definition' qua name falls under namable objects. The definition of definition is one instance of definition. And the definition, in this case, is an instance of 'definition'. Latter does not represent a class member of itself.

To consider something in itself (e.g. man), and to consider it in relation to something else, not the same. 'man' as a definable nature, or man as predicable.

E.g. 'man' as a definable nature, and as predicable. Item definition as a definable nature, and as predicable of any definition - If they were the same, then contradiction. For A is B, C is A, ∴ C is B. Here, man is Socrates; man a member of itself.

To consider 'definition' as a definable, and to cons. it as a univ., two diff. considerations! First, no intention of universality. Second implies latter.

(A) Defin. (oratio...) : this is one

(B) of number of contin. : this many.

Hence, when we say that 'plurality meas. by unit' is a def., we do not mean that it is 'an expression explaining what number is', but 'what something is', or 'what its definition is'.

Now, is the 'class of all concepts' a concept? (Let aside 'all') It is an instance of concept. Concept is not an instance of what is in that class.

Existence in math.

Abstraction according to first operation of the mind, ubi 'what' a thing is. The properties are asserted of it in abstraction. Existence alter { hoc aliquid / compositio

If mathematicalia had to exist outside the state which in mind, ~~universal materialism~~ no science: verification impossible. Hence geometry not natural.

If math. same as calcul., this applying to all, then universal materialism.

But how are mathematicalia in mind? Defined by priority of quantity which makes separate consideration possible. Consider separately and produce, not same.

What sort of things? XI, 1, 1059 b 5. (l. 1, n. 2161).

How different from ens rationis? This either negation or relation. Example: relation of identity. Meta. V, l. 11, n. 912.

(Poincaré: ability of the mind to posit same act over & over again. Symbolization. No reality required, except symbols.)

Symbolic fiction of existence.

Symbol, materially: mark of chalk ....

formally { term in mechanical operation. / sign, to be interpreted. { diversity of opinion

Mathematical abstraction in A's sense.

Different modes of defining. With s. matter, e.g. 'man'. Without according to understanding.... E.g. 'circle': does not refer to copper. Then triangle. Why this possible?

Case of number. Are two men an instance of 'two'. Of a sensible two, yes. But can consider two apart? Then mathematical. Where does this exist? What kind of existence? 'Two is the first even number'. Is, here, does not mean that two exists; but merely that it is the first....

Something of the mind, like a relation of reason? Consider this case. Meta 912 (V, l.11)

The twelve strokes are not taken as twelve strokes. These are an instance of the class 12. ~~What is this class?~~ What is the class 12?

Russell 90. Does the definiendum appear in the definition? Where is it? 94, n.2 ('2 is the n...) Actually 'ostensive'. (cf. p. 88)

Symbolic fiction, symbols defined, not by what they can be made to stand for, but by the operations that can be performed upon them.. Quid generalisation of number? Does not mean that there is a general notion of number of which whole numbers, fractions, etc. are species or instances. Only: the same operation applicable to all. Thus 0 a number.

No question of a predicable universal. The class 2 cannot be said of anything. 'Man' is neither an individual nor many men: stands for neither. Can be said of many. You cannot say These two are the class 2. It is a class of two.

When we say 'two is the class of all twos'

could mean 'to be the class of all couples' is 'to be two'. But 'what is it to be two?' Is it a notion? Or same as one and one? (Here again probl. of 'one per se' and 'per accid.'

Difficulty sidestepped by the operational point of view: 2 is an operative symbol. Neither universal nor particular. Preserved from that. Such questions indiff. to operation. Such questions, it seems, can never lead to agreement. This possible only where the machine can take over.

When two is used as a name, no longer math. When, when A for major term, no longer logical in modern sense.

~~Because~~ It
When a plurality has a proper measure, it can be squared by that measure, actually yielding something one per se (as distinguished from a heap): Subject of properties in strict sense, as 2 the first odd, or 'odd' divisible into two equal parts. This was the subject of the sc. of arithm. But this not same as art of calculation. Cf. de Trin. V, 1, 3m

What do we mean by math. abstract.? Not at all the same as totius, which is common. Clearest exposition, de Trin. V, 3, c.

Div. of sc. based on modes of defining. There are three. But this supposes knowledge of what definition is and what sensible matter means. What is the thing that the name signifies. (Quid sensible matter in other course.)

Russell's solution of antin.: 'a hierarchy of types.'

(a) Lowest type is that of individuals.

(b) Next: aggregates of individuals: higher than the individuals in it. Being of a higher type, it does not contain ~~its members~~ itself.

(c) classes of classes, still higher types.

E.g. 'this couple'; the 'class of couples' (card. n.); ~~a class of~~ number in general, 'a plurality of pluralities of pluralities.'

not plur., sp. of indiv.

We have conceded that a number, as a class of classes, is valid as a symbolic construction. Have seen in what sense the cardinal number 5 ≤ the symbol 5.

But, can we go from there to number "in general"? What does 'in general' mean here? Not universal. What, then?

We have also seen diff. between "All math." & "All thinkable concepts ~~objects~~."

We distinguished betw. 'thinkable concept', a 'class of thinkable concepts' (kinds of triangle), ~~and the 'class of a~~ between 'every thinkable concept,' and 'all thinkable concepts.'

Now: ~~Every~~ 'thinkable concept' not something clear. What does concept mean? Why add thinkable? Grant just concept as enough. Now (a) 'Every concept,' or (b) 'all concepts' distrib.

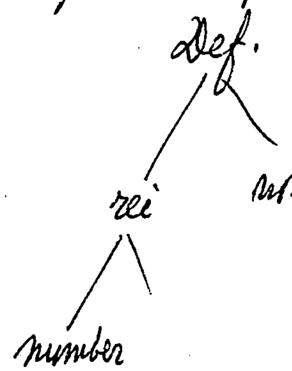
There is 'a' concept, and 'concept.'

The notion of number is a notion, but not a number.
" " " animal " " " , but not an animal.

But the notion of notion is a notion; the definition of definition is a definition. But def. of def. is not the same as just definition, and notion not the same as notion of notion. The notion of notion is one notion, and the notion of number is another. The def. of def. is ~~one~~ def.; the definition of number is another. Item ~~Thus~~, the def. of integer is one; the definit. of odd another. Both are instances of definition. The definition of definition is an instance of definition.

~~Name and definition~~

The 'name' definition, and the 'definition', such as def. of def. do not signify in the same way. To name a definition, and to define it, not the same, although what they signify is the same. The def. of def. ~~does~~ not signify, formally, the definition of number. If we say 'plurality measurable by one' is a definition, we do not mean that 'it is an ~~def~~expression explaining ~~signifying~~ what number is', but that it is an expression ~~signifying~~ explaining what something is.' Thus



If we define numbers by symbolic construction, I don't think that the distinction betw. potent. and act. infinite is relevant; unless we could prove, operationally, that actual infinity implies a contradiction. I don't think that in this case it is possible to disprove the act. inf. The proof would be extra-mathem. I know of no phil. proof.

~~E.g. what~~

Now case of non-normal classes: 'class of all notions'. Is it a notion? Is the 'class of math.' the same as the 'notion' of such a class? It is said no, because the class is not a math.. But, is the 'class of all notions' a notion? There is such a notion: all in sense of every. Then symbolize. But does the symbol stand for the 'class of all notions' and for the 'notion of class of all notions'? It may, but it will need interpretation. Now, the interpretation may lead to contrad. or not. It will not if 'notion' of notion' is an instance of 'notion'. In this respect it does not include 'all notions'. We cannot say that the notion of integer is a notion of notion; ~~nor that~~ this is it is an instance of notion, but not of notion of notion. The notion of notion ~~not bec~~ a notion, not because it is a notion of notion ~~but bec~~ ut sic. Proper reason why man ~~qua animal~~ ~~not~~ is a substance, is not that he is an animal.

{ Ens et entitas.
Homo et humanitas.

(11) The notion of notion is one notion; the notion of integer another. Different respects. Contr. only when wholly identified. Not. of not. ≡ all notions

The notion of integer is not an integer: the definition of integer is not an integer.

But suppose that "all thinkable objects" were a class, and that it contained the object "all thinkable objects"; is it contained in this class qua "notion of object"? Qua thinkable.

[Reminds one of Averroists: My notion of man is the same as your notion of man: dist. "the notion is identical"; but "my having it not identical with your having it."]

Are "the class of all think. obj." and "the class of all mathematicians" really different types of classes? Is the notion of "class of all classes" the same as "~~the~~ class of all classes"? There seems to be no reason for a distinction of the type suggested. We say: some things reveal reflexive nature of intellect; others do not. As in case of genus: species as definition; but "quod praedicatur de pluribus specie diff...."

Russell's antinomy seems to imply a denial of the reflexive nature of mind. Reminds ~~of~~ us of what St. Thomas says about the order of knowing powers: extern. sense; internal sense..... Unless inorganic faculty, we would never know that we know:

"... visus non potest comprehendere actum suum. Si autem hoc esset necessarium in omnibus quod actus cujuslibet potentiae non cognosceretur a propria potentia, sed a superiori, tunc oporteret quod vel in potentiis animae iretur in infinitum, vel remaneret aliquis actus animae imperceptibilis." I S. d.17, q.1, a.5, 3m.

But in the intell., the infinity which is possible is not necessary. If necessary, we could not understand that we understand except by proceeding ad infinitum...

~~x~~ – Remark on Poincaré. What he says is true but insufficient. Must add symbolic

– Remark on Russell's paradox of classes:

non-normal: contain themselves as members: "the class of all thinkable concepts."

normal: do not: 'the class of mathematicians' not a mathematician.

Let N: class of all normal classes. Is *it* normal or not?

– If normal, it is non-normal, for it must include all normal class.

– If non-normal, it must be a member of itself, hence normal.

Hence N is normal only if it is non-normal.

Reply: (a) 'Intelligere' as an act, and 'intelligere' as an object not the same; though identical in re, different ratione. No contradiction, as this distinction shows.

(b) A class is never a member of itself qua class.

The class of all ~~....~~ thinkable objects is a thinkable object? Dist. Problem of 'all' and 'whole'.

The 'notion of notion' is a notion. If I add 'every,' the this notion is referred to. But this does not mean that it is any kind of notion. 'Notion' is not a class, but *neither one nor many*. There is a notion of class; but *it* is not a class.

Note that the acts described ~~do~~ involve time. But, it, too, is a 'rational time', produced by succession of distinct mental acts. However, this can be symbolized simultaneously. Comparable to case of 'all'.

Also, note the 'productive' character of the objects involved. In S.Th's example we start with the knowledge of stone (aliquo actu intelligit homo lapidem.' Then a new act, bearing, not on the stone absolutely, but upon this act of understanding stone. Here the act of understanding becomes an object; and hence ad infinitum. This infinite has nothing to do with the first object, the stone, but with the act about it, which is in and of the mind, and which remains ~~that~~ in it. But first thing does not have to be a stone: a symbol of anything / These acts, and their succession, are products of the mind; and so is the exteriority, whether spatial or temporal.

All this is in the domain of 'ens rationis'. But we can abstract from this 'act of mind', and 'ens rationis' by applying ourselves to the symbols, i.e. to fictional objects. Only postulate first involved: we can take it over again and again, like 1 (a stroke), or a.

Rational space and time can be associated with rational direction, as $a_1, \overleftarrow{\overrightarrow{a_2}}, a_3, \ldots$. Also a rational 'first'.

Rational 'next'. [(Note two kinds of 'next': the next act, and the 'next act about a'. Those things are next to one another or consecutive which have nothing between them of the same kind (A cow between two horses ...)]

Thus we are provided with a minimal ~~[illegible]~~ starting point for symb. constr.: complete indiff. to whatever the object involved may be. E.g., an error is identical with itself. Sic, sic; non, non.

The relation of identity – unconditional, pure identity: not diversa ratione, eadem subjecto – provides us with a minimal starting point, and with material for symbolic fiction and construction; with infinity.

- Why minimal. Complete indiff. (Ident. of error, of impossible.)
- Why symbolic.
- Why fiction (like 'centaur').
- Why 'symbolic construction'.
- Where infinity.

[No particular relation of identity has a name. The game with symbols is played in silence.]

Neither univ., nor equiv.

Example of ~~[illegible]~~ many 1: symbolic: a, a, a, a, ...

of symbolic constructs:

a, a, a, a, ...

Classes which { do contain themselves as members: 'non-normal.'
do not " " " " : 'normal.'

{ The class of mathematicians is not a mathematician: i.e. is normal.
The class of all thinkable ~~object~~ concepts: 'non-normal,' for the class of all...., is itself a concept.

Let N stand for 'all normal classes':

- if N is normal, it is a member of itself, for it was defined as including all normal classes. But if it does, it is 'non-normal.'
- if N is 'non-normal,' then it must be a member of itself; but if it ~~[illegible]~~ is a member of itself, it is normal.

A.v., N is normal if and only if it is non-normal.

This contrad. results from uncritical use of notion of class.

Numeri compositi: qui communicant in aliquo numero mensurante eos, ut 6 et 9 mensurantur 3, et non solum ad unitatem comparationem habent, sed ad mensuram communem.

Numeri primarii, seu primi, seu incompositi: quos non mensurat alius numerus communis, nisi sola unitas.

The form of the syll. is self-evident, qua an inst. of the principle of compared identity.

If all mammals have horns,
and man is a mammal
Man has horns.

This is not a proof

Just as 'Socrates is Socrates' implies a relation of reason, which can be repeated without end ($r_1, r_2, r_3, r_4, \ldots \infty$), owing to reflexive nature of intellect; so 'a', thus:

$$a_1, a_2, a_3, \ldots$$

symbolizing distinct acts of the mind. We know that this is so. From there on we can produce orderly symbolic structure. The important thing is that they be 'out there' in the way that symbols are.

The same can be taken over and over again by the mind without end. This sheer repetition of whatever it may be appears to provide a sufficient basis for symbolic construction. Ideally, the symbol 'a', taken as identical with itself, can be expressed as follows: $a \equiv a$. No universality involved. Then, the marks $a \equiv a$ can be taken as identical, $(a \equiv a) \equiv (a \equiv a)$. But, once there, as symbols, also $(a \equiv a) = (a \equiv a)$, etc.

$$a \equiv a$$
$$a = a \,;\, (a \equiv a) \not\equiv a$$
$$(a \equiv a) \neq a .$$

Repetition of a : $a_1, a_2, a_3 \ldots\ldots$
$(a_1)_1, (a_2)_2, \ldots\ldots$

All this is well, provided $a + a$

Let A be an infinite collection.

Let each member of A be an infinite collection, eg a,

a sub-class $\overset{A}{\leftarrow a, a_1, a_2 \rightarrow}$

Every subclass will be equal to A in magnitude.

But the totality of subclasses will be infinitely greater than A.

Let A' be an infinity of infinite collections A of infinite collections a. Then,

$A > A'$ and the totality of a's $> A'$ and A.

Thus:

$$\overset{A}{(\leftarrow x, y, z \rightarrow)} : a$$

$$\downarrow$$

$$(\leftarrow \beta, \gamma, \delta \rightarrow) : \alpha$$

Then, the totality of a's $> A$; the totality of α's $> a$ ∴ $\alpha > a$ and A, and $\alpha > (A+a)$. But what does this mean? Distinguo: if of same order: then equality of part and whole. If of different order, as integer and even, then $\alpha > (A+a)$.

Analogy: an infinite class of men A } a
each has 5000 hairs. a }

$$e^{\pi i} + 1 = 0$$

As <u>number</u>, there is a unique 2, 3, 4, ... So if I want to match

		2, 3, 4, 5, 6, ...
	with	2, 4, 6, 8, 10, ...
and	with	3, 5, 7, 9, 11, ...

you will not allow me to because all have been used in the first line.

Will you allow me to arrange 2, 3, 4, 5, 6, ... and note the trace they leave as I gather them up? (Actually they do not leave traces nor can I gather them up.)

Then let me pick out just the even numbers: 2, 4, 6, ... and arrange them as I had previously arranged the whole set. Also let me take the remaining numbers, 3, 5, 7, ... and arrange them under the 2, 4, 6, ...
Then let me conclude:

Whether I had used all the numbers at once, or only the even ones, or only the odd ones, in each case I would have an unending succession.

x x x

I understood you to say that of course I could make such an arrangement if I used <u>only</u> <u>symbols</u>, because I'd have "as many of these as I pleased". I think that begs the ~~point~~ question. I don't <u>want</u> "as many as I please" in the sense of being supplied with an endless representation of "2", for instance. All I want is positions for each of the symbols 1, 2, 3, 4, 5, ...
Then I wish to use these same <u>positions</u> but fill them with 2, 4, 6, 8, ...

x x x

What was Euclid using when he proved that there is no greatest prime number? Was he using <u>numbers</u> or <u>symbols</u>? Was his proof demonstration, or was he reverting to <u>logistics</u>?

x x x

Why can't we take a different definition of number which will <u>include</u> the integers obtained from the classical definition and give us more besides? Why limit us to the unit "1" as our divisor? After all, in nature there are other natural quotients than those found by dividing by "1". For example: the ratio of the diagonal of a square to its side is always $\sqrt{2}$; the ratio of the circumference of a circle to its diameter is the fascinating transcendental number π.

It seems to me that

$$e^{\pi i} + 1 = 0$$

shows that e, π, i, and 0 are not as artificial as you seem to insist on their being.

x x x

The very fact that the Greek philosophers and, I believe, St. Thomas and St. Albert, included both the theory of numbers and geometry in <u>mathematics</u> seems to me to indicate that they would welcome our extension of the number system.

Didn't the Greek mathematicians use <u>symbols</u> in producing their proofs in theory of number? Otherwise, can there be any <u>meaning</u> in the difference between "three fours" and "four threes"?

I think I don't understand the conditions under which you insist we use numbers for some mathematical proofs and symbols for others.

This is neither well documented nor well organized, but I hope it conveys to you my basic difficulties and problems. It is written in the hospital, where I find myself unexpectedly spending most of my vacation. I had intended to do more reading before I did any of this writing. So if this is not worth considering, please let me know. (T he typing was done by one of the Sisters).

+

a few additional thoughts.

Number without "relation" seems to me barren.

The principle of identity is a rational rather than real relation (if I understand you correctly.)

$$a \equiv a$$

But it gives us unity.

$$C = \pi D \quad (C = \text{circumference of a circle} - D = \text{diameter})$$

is a rational relation which gives us

$$\pi = \frac{C}{D}.$$

Likewise $d = \sqrt{2}\, s$ (d = diagonal of square, s = side)

gives us $\sqrt{2}$.

Why is the "addition relation" admissible to the science of mathematics and the "ratio relation" inadmissible?

In the mystery of the Trinity we find ineffable relations, made known to us only darkly, through revelation.

Jan. 7, 1958.

Dear Dr. De Koninck,

Perhaps this addition does nothing to strengthen the other notes; I am not sure. They are still in a more or less random state and need a great more thought and study. My chief hope is that they will not sound merely trifling to you.

May 1958 be a year blessed mightily for you and yours.

Sincerely,
Sister M. Philip

Idem numerus

(a) Now,

n / 10 / 10_a 10_b... } ineffable

Hence, addition and division mean something diff.: your equation!

numerus } nominable
décem / 10a 10b } ineff.

(b) Note: the classes are held together by the symbol.

The 'nature' is not held together by the name.

(c) Same number: not the same as our 'same number'
(αὐτὸς ἀριθμός. Ours defined through predication with identity. Former stops at 'equal classes':
The classes A & B have the same n if they are equal: stops at the relation of equality. But note that 'equality' itself is one between aggregates which are not quantitative. Valde notandum! Therefore not equality in our sense either. (At best a relation of reason.)

(d) What, then, is the 'cardinal number' of a class? Vd. Kasner & Newman (Math. & the Imag., N.Y. 1949, p. 31): "(Note p. 14)" This symbolic construction. Fiction, like Centaur.
You cannot calculate with a universal. You cannot predicate a symbol any more than you can a name qua name: 'Socrates is a name.' distinguo.

(e) What is the 'generality of ~~a~~ a cardinal number'? Generality of symbol 5 does not differ from 'generality' of any five fingers: these can be put into one-to-one correspondence with classes having

Universal & individual in math.

We distinguish between the number 2 and an instance of 2; between circle and an instance of circle. Cf. Meta. VII, l. 9-11.
The individuals, here, have no names. They are quotational.

Indiv. 2's are not part of two, except as subjective part. Parts materiae, not speciei. Socrates not part of man as rational is part of 'what man is.'

Note, then, comparison of 'man' and 'Socrates,' of 'two' and 'this two,' of 'series of whole numbers' and 'this series'. (Two kinds of infinity!)

Before Russell's comparison of series, note:
No distinction of per se & per accidens. Hence no quantity. No part & whole proper, only fiction.
No act & potency. No addition, no division. Vide Goethe, Sc. of Calc. p. 95, n. 3.

Now Russell's series:

1, 2, 3, 4, 5, 6 .... (A)
2, 4, 6, 8, 10, 12 .... (B)
(Complete) 1, 3, 5, 7, 9, 11 .... (C)

All three have same number. $1, 2_2, 3_3$ / $2_1, 4_2, 3_3$ / $1_1, 3_2, 5_3$

But, are B or C part of A in this comparison?

Consider possibilities: the series stand either

a) for universals: then B and C are parts of A:

(A)	1	2	3	4	5	6
(B)	⊗	2	⊗	4	⊗	6
(C)	1	⊗	3	⊗	5	⊗

b) for individual series: then, either

α) comparison of part to whole;

β) " " whole to whole.

If α) then merely count indiv. terms, and, as many elements in B or C as in A. (Show by subscripts)

c) A for universals, B or C for particulars. Impossible comparison.

d) A for 'class of classes', and B and C as while. Then prescind from univ. and part., from part and whole.

We can go Russell one better:

$1_1, 2_2, 3_3, 4_4 \ldots$

$2_1, 2_2, 2_3, 2_4 \ldots$

There are more twos than two.

Should remain [illegible] construction and

There are no integers here.

Differences are quotional.

Fractions not 'parts'.

All defined by similarity of quotations.

$$\begin{array}{c} A \\ \infty! \leftarrow a, b, c, \rightarrow \infty! \\ \infty!\ \infty! \end{array} \Bigg] \infty!$$

Sc. cannot be solv. by Sc.
But by philos. Not by
"the phil."...
Sc. not the panacea of
good phil. or of good men.

Why not 'abstract quality' as 'abstract quantity'? Why not abstract quality related to sensible qual., as abstr. quant. related to sensible quantity?

We do this of 'figure' when .... but it is 'in quantity' yet non-sensible.

But, is there a quality not in quantity? Neither in sens. matt. nor in intell. matt.

'Science' is a quality. But is it 'not in quantity'?

That there is must be demonstrated.

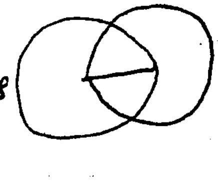
[What would 'non-material' mean? Same as 'immaterial'?
"Is there a 'squarable circle'? Does question contain answer?
Unverifiable expressions have meaning. Like 'diagonal commensurate with side'?]

'Non-corporeal' and 'incorporeal' not necess. the same.

Quality first known is sensible quality, and in quantity. Hence this cannot be abstracted from matter. But is there one that is an immaterial quality? To be can be? Is 'science' an immaterial quality? This, with demonstrated, from what is already known: this, with matter. This either sensible or intellig: first could lead to reality; second, no. (Anselm) Hence, third mode of defining depends upon first.

Whence abstractibility of quantity? Priority to sens. quality.
Whence constructibility? Nature of quantity: materially,
repetition: 1, 1+1, 2+1, ... } construction.
Point, line, surface, body.

Hence problem of intelligible matter.



There are math. individuals. E.g. circles
They do not differ. by 'what they are.'
(a) Analogous to case of bowling pins.
Material principle of individuation.
No names. Symbols, A, B.
(b) Other analogy: { When sensation ceases, sensible (Socrates), too.
When math. indiv. not actually considered....

But 'what man is' and 'what circle is' does not depend upon our consid. (Similar triangles one in notion: definable.)

Russell's antinomy. cf. WM. 1674.

A. Mathematician

B. Concept.

1. We distinguish, circa ~~A~~: mathematician

   (a) Totum integrale: definable: not predicated of its parts.

   (b) " logicum: predicable.

   (c) Pars subjectiva: Plato: particulars $\rightarrow \infty$

(margin: ...ti, ...4.)

2. We distinguish, circa B: concept 3ter:

   (a) Totum integrale: definibile

   (b) " univ. seu logicum

   (c) In quo et de quo: particulars $\rightarrow \infty$

Thus: ~~...~~ a class number of itself: a universal whole, containing its parts as an integr. whole, and identified with its subjective parts.

As if say 'notion of number' were a number, and a number ~~the~~ collection of numbers.

Think of our problem: lapis, actus quo intelligitur lapis: ambo habent eandem objecti: quid. No contradiction: merely manifests reflexive nature of mind.

Finally, what about the 'infinity' presupposed by intellect? Calculation does not distinguish between act and potency, ~~between~~ 'all' and 'whole', ~~between~~ actual and posse whole. Hence no problem: We can state what inf. in act 'would be'.

(margin: Sic Repub. ... of silence.)

Goedel's proof. cf. WM 1695. Compare to sense $\rightarrow$ intellect.

If not infinity, intellect destroyed as refl. power.

If not infinity, intellect destroyed as reflexive power

Résumé

What Math. is, schemely exposit. to Arist. & Thomist: To us, the only 'pure' science, scil. from cause to effect. Most certain, model for all others. – Logic about 2d intentions at least remotely based on prior.

If Arist. notion of sc. collapses, so does all of phil. as both Arist. & S.Th. conceived of it.

Today, found. of Math. declared less certain than ever. (Weyl, Russell, von Neumann, Gödel, Nagel, ...) If we take this for granted, then S. Thomas out – except for genuine – includ. theology qua using philosophy.

When we look into problem, we notice that, whatever math. now is actually held to be, it is trying to justify its own foundations. This strange to us: unless math. is most universal of sciences. But if it is, then we still helpless, as seen in Russell's definition.

To save our own skin, must investigate how this change came about.

I. Some recognize, explicitly, and some do not, that the foundations and nature of math. are discussed by means of words, whereas Math. is carried out by means of symbols. E.g. Hilbert, Weyl, Nagel

– Nowhere do we find any clear and thorough analysis of this distinction. [In this regard, semantics avoids basic questions. No one discusses classical literature, e.g. Perih. on names & verbs. This a historical situation. Conseq.: no one concerned with what Arist. meant by 'science.']

– We suggested, timidly, that the distinction may be of some consequence. [Note: our whole discussion of any prob. here, purely dialectical.]

– Both words and symbols are signs. But, we are told, mathematics a game with meaningless symbols, played ..... Rules explained and communicated in words, ..

Hence, what is their diff.? How can signs fail to signify? Has the word 'symbol' been given a new imposition?

Ancient meaning of word and symbol.

We examined diff. in S. Thomas. Noted, specially, that a name 'unum vel nihil significat.' 'Unum' here means 'per se one'. But symbol 'collectionem quamdam importat.'

Coroll.: an unum per accid. can be symbolized. Thus, if a number is a 'coacervatione unum,' i.e. a mere bundle or heap, it cannot be named, but symbolized. Russell: "number is a way of bringing together collections..." His own definition of numbers defines them as collections. — Weyl "numbers certainly not concepts in Aristotle's sense."

Noted, too, that R. calls number a symbolic construction.

Kasner: the 'symbol' 5 is the cardinal number.

Name, symbol, & infinite name.

To make meaning clearer, we discussed name & infin. name; symbol is neither, but is between. When infinite name made to correspond with class, then takes on symbol.

Yet infin. name and symbol quid in commun: "one according to reason" {inf. not in suppos. symbol sic: not in operation}

~~Still, differ: symbol cannot be predicated.~~

Coroll.: Mind can go beyond 'per se'; operate, effecting, using symbols, as in sheer calculating. In fact, must do so in calculation. Diff. between demonst. & calculation: Here no predication.

Diversity of symbols:

(a) Transcendental terms of Prior Analytics: Omnia et nihil [not absolutely general as A is A]

(b) Symb. in Math. &c.
{= as cifers
{= as in AB - BC.

Neither (a) nor (b) stand for universals.

(c) Symbols of algebra:

(α) of rules: i.e. general form of the rule

(β) of a particular equation.

(d) Symbols of math. physics: "when..." as S and t for space & time medius major

minor

$$\begin{array}{c} M < P \\ S < M \\ \hline S < P \end{array}$$

med maj

min.

$$\begin{array}{c} B - A \\ C - B \\ \hline C - A \end{array}$$

This disposition has ti be demonstrated.

In what sense symb. of math. 'abstract'? III

~~What we mean by mathematical abstraction~~

Ex. from Hadamard. Not 'abstractio formae', neither 'totius.'

Russell's definition of number as a class of classes.

Basic to it: 'Same number.'

Compared to our notion of 'same number': not same as R's, because, to us, 'same n' presupposes notion of number, not of a given number. (Arist. Phys. IV 14.)

Must concede: Can know same number of a given class without how much.

But our discuss. irrelevant to moderns: genus, species.

Must even exclude our idea of universal altogether.

Actually, none of these notions involved in calculation.

Cannot calculate universals or with universals.

What we mean by number: "multitudo mensurabilis per unum."

Quid measure.

[What we mean by math. abstraction. De Trin. V, a. 3.]

Then, unity of number.

Calculation transcends this: no dist. { per se, per accid. / act. & potency

Notion of intellig. matter { univ. / sing.: here necess. of symbols. } discr. / contin.

Exam. of Russell's critique of part-whole principle:

paradox due to conf. of univ. & part.

Notions of 'part' and 'whole'

Application to infinite.

Quid "there are infinite sets"?

Problem of existence.

Various meanings.

Real, rational.

What is the existence of mathematicalia: purely rational?

Not verifiable in experience.

Poincaré and Weyl: Integers purely creations of mind; based on ability of mind to repeat.

If so, then mathematicals entia rationis. Neither mathematical nor logical in our sense.

Relation of identity.

Rationis tantum. Repeated ad infinitum circa idem.

Necessity of symbols.

Why can the intell. do this? Reflexive nature.

Rational exteriority; order; time.

Russell's antinomy.

What is its origin?

Assumptions concerning 'class' and 'concept'.

Disc. of 'class member of itself.'

In our terminology: "There is a univ. whole which is an integral whole." Ex. used: definition of definition.

Disc. "class of all classes," and 'member of itself.'

Diff: predication univers. If excluded, no more problem.

The existence of 'infinite classes'. Useful to show 'symbolic existence'. (Then comparison betw. diff. kinds of existence.)

For this is the most important in mod. mathem.

Our definition: inf. multitude + nature of whole.

Noted that transition from inf. in pot. to inf. in act assumed to be self-evident.

Where to stop?