Speculatively, Machiavelli doesn't deny Christianity (it has shown us
    the way) - nor does his blame the church for the plight of the
    Italian people, rather the empire.
He does complain that the Church is unable to inculcate this savagery.

From one of his letters: " I have not for many years believed what I
    said or said what I believed, or if I have told the truth I have
    concealed it among so many lies that it can't be found."
    -what can one make of a man like that?

Bad Prince: acts the role of fox and lion for his own good.
Good Prince: acts the role of fox and lion for the good of the people,
        for their betterment and education, etc.

Professor Allen, Political Thought of the 16th Century = at bottom the
    Prince is consistent with the ideals that Machiavelli expressed
    fragmentarily in the Discourses.
    Allens conclusion: the Prince was only necessary to obtain the
        good that was desired in the Discourses.

Mc Coy asks: Is the Prince consistent at the bottom with the views
    expressed fragmentarily in the Discourses or is the Prince at
    the bottom of the Discourses? They both have the same premises.

Hitler said: human rights are above state rights. His words seem to have
    as much value as Machiavelli's.

    Mussolini is thought to have been influenced very much by the
        Prince. Always had a copy in his desk drawer.

The exam will be based mainly on t e considerations made of Aristotle's
    Politics.

            ...curtain.

philosophy of nature

    ...principle of individuation

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Philosophy of Nature: The principle of individuation

Charles De Koninck

June 26, 1950

Aristotle's notion of intelligibile matter-mere words.
   Historians wonder why he used this word "intelligibile matter"
   we are not interested in the historic Aristotle but we are in
   need of something that answers the notion: intelligibile matter.
   This notion is essential to a study of mathematics and the
   study of the individuation of natural things.

We will first take up the principle of individuation in mathematical
   entities.

This notion (Arist. and S. Thom.) always used in respect to quantity.

   Bk. V,chap. 13 Meta. "Quantum est quid est divisibile in partes
   es quae insunt etc.

Quantity means that divisible into 2 or more constituent parts, each
   of which is by nature a one and a this.
   ea quae insunt=to exist within; does it mean constituent?

The parts are necessarily homogeneous parts

Homogeneous (e.g. line; it is a quantum divisible into homogeneous
             (parts. Each part is a line.

Hetero-   (e.g. house. These differ by form. Each part is formally
 geneous   (different. Each brick has orderliness of figure (form)
           (and are homogeneous parts of the whole. Wood, door,
           (bricks etc. would be heterogeneous parts, i.e. differ by
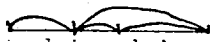   form.

The parts of quantity are homogeneous; other than another not be; by
   formal difference. e.g. how designate the otherness of two spheres
   billiard balls? this one is not that one. There is no otherness
   of form.

We designate their otherness only by signation; what is the proper
   cause? If not the form (as the sphere and dube by very definition)
   then in homogeneity, it is-one is here and one is there.

If they do not differ "quid est" where is the cause of multiplicity
   within identity of form; they are other by a subject. This is some
   thing other than the form being within; it is common to all things
   quantitative.

E.G.
   Continuous quantity:
   -a line (quantitative)   partes
   one part does not differ by form but each is a "this"   quae
                                                             insunt
   -equilateral triangle △
   homogeneity of angles; they do not differ but by a "this".

   Discrete quantity:
   No, 6. Where is homogeneity in number 6? Is it strictly quant-

-2-

Discrete Quantity:
   e.g. take no. 6; where is strict homogeneity in no. 6? Where are
   the homogeneous parts of no. 6.

   (1-atom, 1-horse, 1-sneeze, 1-angel, 1-beer bottle, 1-relation)

   What gives us 6? One cannot retain the differences, must leave
   them aside. Unless one can find something simply common, one
   cannot consider them as homogeneous. But there is nothing univoc-
   ally common with the angel in the group.

   You can have six ones but not one six; one can have the "numerus
   numerans", but if it is to be quantum in the strict sense one must
   have homogeneity.

Wherever we can have-one is this and the other is that and no other diff-
   erence, orthe notion is univocally the same, then one can have
   quantitative number.

This raises the problem of individuation: there is no quantity without
   individuation. This is not necessarily individuation of the natural
   doctrine, i.e. materia separata et quantitata.

Individual - quod est indivisum in se et divisum ab aliis.
               -that which is merely this and nothing else presents
                an individual difference and not a difference of form.

What is the principle of this homogeneity essential to quantity?

What is quantity? Quantity est ordo partium in toto.
               -quantitative parts are synonomous with homogeneous parts.

E.g. Socrates is not quantity, he is a quantum. He has quantity.
         -how can we say that he has quantity?
         We know quantity by measure.

         (Mensura est id quoy primo quantitas cognoscitur.
           Quantitas est id quod mensura cognoscitur.)

      Socrates is extended in a manner that can be designated -
      he has extensive quantity. One part is exterior to another;
      one is here, another is there. There is exterior homogeneity.
      Even if there were no formal difference between his head and
      his feet, par example, he would still have quantity.

If quantity is the order of parts in a whole, then the parts are not
            quantity but the order is quantity, the order of these
            homogeneous parts.

      These parts are the parts of what? They are the parts of
      substance. The reason for the homogeneity of the parts is
      the substance...that in which is intelligible matter.

      These parts, homogeneous because of the substance, are other
      than form (by which)-they are that (in which) and is called
      intelligible matter.

The Principle of Individuation

Charles de Koninck
le 27 Juin
Second class.

de Koninck
5/27/50
page 2

Quantity is the order of parts in a whole: that is, of homogeneous
parts in substance.
Quantity cannot be conceived of without substance.

The object of the order is intelligible matter. Homogeneous exteriority
proper to quantity.

Homogeneous parts are called matter because they are parts and they are
the more matter because they are homogeneous.

But why call it intelligible matter?
-the parts are parts of substance which is attainable only by
intellect. The parts are of quantity which also is attainable
only by intellect.

Sensible Matter (De Anima, bk. II)

per se sensible:
a) proper to one sense and not to another.
e.g. as sound to the ear.
b) common to more than one sense.
(the sense of sight best attains the common sensibles)
-these are always either quantity or quantitative modes.

The quantity in mathmatics prescinds from sensible matter, but it is
attained thru the proper sensible matter.

Mode of attainment: we go from the proper sensible to the common
sensibles.

Intelligible matter is prior to sensible matter.
-surface, as such, can be known only by intellect prescinding
from all matter.

cf. De Trinitate q. 5, art. 3, corpus.
-in this text forma = quantity as abstracted from sensible matter.
If matter is not essential to the form, form can be abstracted.

No accidental form can be abstracted, for accidental means to
be in; there can be no in which without the subject.

The first accident of substance is quantity: quality is secondary.
e.g. If Socrates were a point he could have no qualities. Not as
Socrates, that is: he would have no quantity.

Quantity can be considered in substance prior to sensible qualities
and hence sans them.

Substance and Quantity as considered apart from sensible qualities
are still matter: THEY ARE INTELLIGIBLE MATTER. ?? (intelligible?)

This quantity, which is prior to sensible qualities is abstract. It
owes its separation to the mind.
Math, is separated from sensible qualities in the mind, but not
in reality.

We can know quantity without sensible qualities precisely because it
is prior to them !!
Mathmatical entities can be considered in separation, but they cannot
be in separation.
Man cannot be considered in separation, forsensible matter is of his
very nature.
The above cited text proves that the notion of intelligible matter is
a valid one.

cf. De Trinitate q.5, art. 2, ad. 3.

The various parts must have a form that is of the same nature.
-based on Bk.VII of Aristotle.

Summa, q.85, art. 1, ad. 2

Common matter
-e.g. the bones and flesh of man.

Proper matter
-e.g. the bones and flesh of Socrates.

The subject of the sensible qualities is the body.

Intelligible matter = substance as the subject of quantity?. (?)

Universal and Particular Intelligible Matter
(Metaphys. bk. VII, cap. 10 - # 1492 - 1497)
-in this passage we have Aristotle on individuation.

Form =
(forma partis: the form as in $\frac{f}{m}$
(
(forma totius: form as species.
-the definition.
(as, par exemple, the definition of man in-
cludes matter, common matter. The matter
added to it is the proper matter of Socrates.)

ratio = nature
ratio = the expression of the nature, too.

To be a circle is the same thing as circle. (A thing is its defini-
tion.) But in the case of humanity, Socrates is not his
humanity.
This identity is of universals! Man is not a soul: he also
is matter.

What parts are included in the definition?

Partes:
a) partes speciei : the parts of Man the universal.
b) partes individui: the parts of Socrates.

The partes speciei are included in the definition. This shows
that the scholastics were not existentialists.

Mathmatics alone prescinds from esse - its form is strictly abstracted
by the mind and cannot exist in the real order.

Objecti mathematicorum possunt definiri sine materia, sed non
esse sine materia.

This abstractness is proper to Mathematics. It is in a way even
more abstract than logic, for in logic you cannot consider a
second intention without some reference, if only a very con-
fused one, to a first intention.

Only the universal is definable.
-can't define circle A or circle B. We can only define the
universal circle.

This is important because it shows why symbolism is absolutely
necessary to mathematics. (He promised to give more on
this later.)

There are somethings common to the individuation in Mathematical things
and Physical things.
Before considering what is proper to the individuation of these
two realms or degrees of abstraction we must first consider
what is common to both. (sensible)?

Intelligible singulars are like the (mathematical) singulars.
-there are individuals in mathematics because we have many
circles, for instance.

...to be continued.

Commentary on quelques textes sur la Matiere Intelleligible

1494 -

singularia = mathematical singulars. A circle (the
individual) differs from Circle (the universal).
There is some sort of individuation.
intellectus = imagination.
-because of the imagination, the matter is called
intelligible. That is, "intelligible" is an
adjective derived from intellectus understood as
imagination.

There are two kinds of individuation:
a) one that calls for imagination.
b) another that does not.

1495 -

The passive intellect = imagination. Yet it is called
imagination intellect.
Singulars of any kind are not known by the definition.

(A sense attains its object only when that object
is physically present.
But we can read about an object, know it, attain it
even in its absense.)

When circle A is not considered it doesn't exist. Circle
the universal, whether or no I am now considering it, has
the existence proper to it.   an object
If the sense doesn't at the moment know, we can only say
that it continues in existence by opinion. Of course, if
we see other things that could not exist unless that
original object still exists, then we have certitude of the
continued existence of that original object. This is a
consideration apropos of certitude.

Insofar as I know the universal, I know the singulars con-
tained under that universal as having the nature of the
universal, but I do not know them as singulars.

The universals are attained only by intellect.
When we represent a singular circle to ourselves we use the
imagination. Intellect alone is not sufficient in the
knowledge of singulars.

Thus we have now three circles:
a) The circle. (the universal)
b) Circle A. (the mathematical circle)
c) This bronze circle here and now.

Aside: Homogeneity is essential to the operation of mathematics.

Even using Mathematical Logic as the be-all and the
end-all of philosophy would leave us with a very crass
materialism.

Mathematics is, in a way, less abstract than Physics.

But, of course, in another way Natural Doctrine is less
abstract than Mathematics.

1496 -

Intelligible matter is in sensible (natural) things, but
not insofar as they are sensible. It is prior to sensible
qualities, but although it is prior to the sensibilia, it
could not exist without them. It simply could not exist
in the manner in which we consider it.

(Why is it that mathematical objects, or why is it
that intelligible matter, can only exist as conceived
by the mind in the mind, and not in reality?

No substance can be without a quality, and that a
quality proper to it. Qualitas est dispositio
substantiae.)

- Mr. de Koninck went on with his explanation quoting from
1497 which we do not have. Cf. # 1497. -

Is there intelligible matter in the parts of the forma
speciei of Mathematics?
Must intelligible matter be contained in the definition
of circle?

1520 -

A natural thing, as such, cannot be defined without
matter.
But Mathematical things can be defined without matter.

1521 -

Parts in matematics, e.g. a semicircle, are not parts of
the forma speciei, but partes materiae. Circle, as such,
cannot be divided into parts. Parts are outside the nature
of circle - if they were not they would be included in the
definition.

When a thing's nature is not identical with Its quod quid
erat esse, it can only exist with material parts. Angels,
however, have a res (nature) that is identical with their
quod quid erat esse. This does not mean that Gabriel's
existence is identical with his nature.

1522

Sortes cannot be his humanity. If he were, there could
be no individual Sortes, but only Sortes in general.

1530

Composites: two types
        a) Man
                body + soul; this is definable.
        b) Socrates
                body + soul; indefinable.
The principle of individuation is not a definite nature, it is
indeterminate, and that which results from this individuation can't
be definable. Therefore the individual is indefinable.

#1760
" Intelligibilis autem materia dicitur ...
... et ab hoc materia non
abstrahunt mathematica."

#1761

As soon as a thing can be a _res_ because of something out-
side, there can be many of the same species.

Angels - only one angel in any given species. Matter is
the principle of individuation - hence, no matter, no
plurality.

Feuerbach - in effect, he identified man with separated
substance.
Marx - took his notion of the generic being of man from
Feuerbach who, in turn, derived his notion from a
misunderstanding of St. Thomas.
As these men have a great influence on modern thought,
a treatment of the Thomistic doctrine is relevant
today.

\# 707 -
Can abstract from sensible matter, leave it behind, and still
retain intelligible matter.

Circle and the nature of circle aren't the same, just as man
and his nature are not the same.

It is thanks to common sensible matter that individuals exist.
It is thanks to intelligible matter that mathematical objects
exist.

\# 711 -
Why imagination is necessary apropos of mathematical entities.
intellectus passivus = imagination.

\# 712 -
This flesh - known by sensation.
(
Flesh - known by intellect.

- the comparison of the universal and the singular
must be done by the same faculty.
-the intellect must know sensible singulars and
the universals. But perhaps there are many ways of
knowing the singulars.

\# 713 -
To know Flesh and this flesh requires the same intellect.
One (Flesh) is knwon directly, and the other indirectly -
we know singulars indirectly by a reflection on the
phantasm.

\# 714 -
The snub is with nose. The curve is snub only in the sub-
ject. Just so the straight can only be straight in some
continuum.
Just as to know the nature of a sensible thing and the sen-
sible thing we must have recourse to that sensible thing,
so in Mathematics we must have recourse to the imagination.

Imagination is necessary even in the most abstract matematics!!
This is contrary to the opinion of Bertrand Russell.

e.g. a = a
-whatever _a_ is, as soon as I have the homogeneity of a
and a, we have an intelligible matter which will re-
quire the imagination and this in a very particular
way.

Wherever quantity is, there also is homogeneity!

---

x x x x x x
x = an empty symbol, it can equal anything. I can take x
over and over again. It is the same and yet each time
it is different; it is this particular x. There seems
to be a kind of homogeneity. Yet there is no quantity.
All these x's do not co-exist. At any given moment, I
am considering one x; there is no co-existent homo-
geneity.

Today, this sort of thing is calle pure mathematics, and
the abstract mathematics of the ancients is called
applied mathematics.

\# 716 -
In Physics we retain common sensible matter. It is a ques-
tion of the division of sciences, of degrees of abstraction.

Physics or Natural Doctrine:
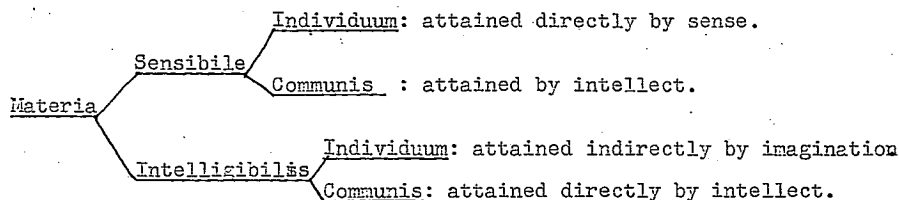Common sensible matter.
Mathematics:
Intelligible matter.
Metaphysics:
No matter.

o

Tomorrow we will consider the division of Sensible matter and
Intelligible matter, each of which can be divided into
individual and common...

Intelligible matter = continuity. Is there, then, intelligible
matter only in geometry and not in arithmetic? No. In Ana.
Post.II, 1.9, n.5 we find that it is said of number as well.
Why? Because number, too, involves homogeneity. Continuity is
presupposed in number, it is because of continuity that we can
have number, which is more abstract than continuum. De Trin.
q.4, art. 2, ad. 6.

Most Complete Division of Matter   (natural)
I, q.85, art.1, ad 2 (p.10). Mathematical abstraction relinquishes
the individual. Math, prescinds from common sensible matter - goes
farther.

Materia
- Sensibile
  - Individuum: attained directly by sense.
  - Communis : attained by intellect.
- Intelligibilis
  - Individuum: attained indirectly by imagination
  - Communis: attained directly by intellect.

o

To know that Sortes is a man, must know Man and that Sortes is a man.
Man = universal; Sortes = individual. Man is attained by intellect;
Sortes by sense (or in his absense by imagination).

To know a circle that we wish to divide, must know: Circle, by intel-
lect; and The circle by imagination and only it - there is no
external sense involved. Can't verify this circle - not by the
imagination derived from the external senses. It is the imagina-
tion required for individuation and no more.
A billion circles - all of them are individuals. They are imagined
by this individuating imagination. These circles are not derived
from the per se sensibilia - our senses have never perceived such a
number. Not sensible quantity, as that had thru external senses.
The senses are limited. This imagination is different and it will
be defined in terms of individuation. (It is important for an
understanding of non-Euclidian geometry.)
A curved space - cannot imagine this in the sensible imagination.
Curved space is dictated by this mathematical imagination. It is
the same as when we try to imagine a polygon of 53 million sides.
Only in this sense can we imagine such a thing. We go from in-
tellect to imagination - the other imagination goes in the other
direction.
The notion of rectangle is not of per se sensibilia.
We toured all those texts in order to find that there is this mathe-
matical type of imagination.

Scientia
principle	term
apprehended by sense.	judgment.

In the judgement, there is a reference to different things in the
different degrees of abstraction: Physics refers to the thing;
mathematics refers to the imagination; metaphysics refers to the
metaphysical being.

We define this special type of imagination in terms of individuation.
This individuation is necessary because of the homogeneity of the
parts - the imagination is necessary because of the singulars.
+
What about pure mathematics?
Quantity implies homogeneity. But it is claimed today that Math.
is no longer about quantity. Bertrand Russel proceeds thus in this
claim: quantity = number. But all mathematics isn't about number.
(This is explained thoroly in the Laval course on Non-Euclidean
Geometry). We reply that no matter how much we generalize, taking
for example a as anything. Even a must be referred to something,
if only to itself by identity. There you have a relation: two
terms: a = a. (In reality identity doesn't require two terms.
When I say Sortes = Sortes, I am employing a relation merely of the
mind - it is only a relation of reason: and this identity is the
clearest examply of a relation of reason. Obviously in reality
Sortes is not two concrete things. The predication of Sortes of
Sortes is all in the mind.) We have a = a. We proceed: (a=a) =
(a=a). And we can go producing multiplicity and there is also
homogeneity. And we can go on to produce relations of inequality,
similarity and dissimilarity and any relation. e.g. of inequality:
a ≠ (a=a). Because we have individuation we have quantity. And
we have homogeneity. Even if 2 doesn't equal 3, they have some-
thing in common. In order to have any relation, they must be in
the same genus. However, this homogeneity we have attained is
negatively abstract. If it were positive, it would be quantity
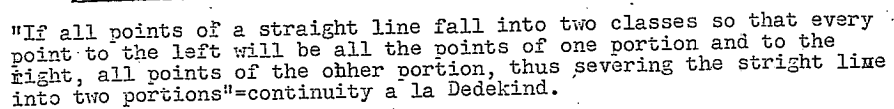proper. Thus it both is and isn't mathematics.
By this production of or multiplicity from a, we do produce relations
of every kind, but they are all in the mind.
We could also by the same method of starting from a produce extension
and various kinds of extension at that. One a is outside another.
a is outside of a_1. This exteriority is homogeneous. I can com-
pare this homogeneous exteriority to that of points on a line.
And I can consider a new kind of exteriority as a new dimension.
Russle in his expose used points - by using a we have more perfectly
satisfied his desire for abstraction.
Continuity is easy to produce. a is outside of a. Continuity =
something that is divisible ad infinitum.

cf. Dedakind, Theory of Numbers. If all points of a line can be
divided into two classes and those of one class will be to the
left and those of the other to the right, they can be divided by
one definite point.

He ended with something on how we construct a line.

By positing under that line points corresponding to those of the line
we can increase their density until we have a second line. But
can we? Has a point any quantity? If not, even an in-
finite number of things not having quantity could seem to give us
quantity.
...to be continued.

The definition of continuity according to Dedekind is not wholly inept.

 p=point

"If all points of a straight line fall into two classes so that every
point to the left will be all the points of one portion and to the
right, all points of the ohher portion, thus severing the stright line
into two portions"=continuity a la Dedekind.

Our definition: continuua-those things whose parts are conjoined.
Continuity: those things whose parts are joined with one common term
(divisible ad infinitum). If two lines have a term identical with each.

Contiguity: (as opposed) each line has a term distinct from the rest.

Dedekind departs from our concept of continuity (divisible ad infin-
itum). But his definition is reminiscent of Scholasticism's definition;
We accept that the point p divides the line into two sections and all
the points of one lie to theleft and all of the other lie to the right.

The limit of the straight line is continuity. Our continuum=construt
cted from a term; we can construct the whole general form of all mathe-
matical entities, matter. And all this from the simple representation
of an individual term.

e.g. _____   The straight line will be the limit
....   of positing these points. One cannot
                        reach the limit.

Note: all this is purely mental representation of the same. This in-
dividuation is merely abstract individuation.

Abstraction: for these reasons-
a. negative abstraction, as our term can stand for anything; takes an
   empty form.
b. the individuation is due to the mind alone and is for the mind alon
   alone.

example: (again) Sortesé Sortes   Relation of identity.
                                   -the multiplicity is established for
                                   the mind alone. Two Sortes.

When the individuation is abstract in this way it must not be con-
fused with quantitative individuation, i.e. divisible in partes (homo-
geneous parts) which are a "one" and a "this". Unum per se.

The unity of S≠S is one according to reason only and not quantitat-
ive unity, yet they are homogeneous. Why not quantity proper? Unity
only according to the mind alone and the whole is not actually a given
whole). Here the simultaneity is also only of the mind.

ens rationis=instar instar ens; in the likeness of being.

What is the principle of this individuation? None other than the
act of reason terminating in the imagination. Imagination is necessary
i.e. the imagination which can retain images. S=S ; this is not that
even where formally the same. They are different only by reason of

individuals and not form.

Although more logical than mathematical, this individuation is
still in the imagination. We give quite a scope to imagination, just
as we do to Cogitative power. (not the intellect; it is only in man-
almost infinite in capacity)

This is repetition, and the knowledge which the mind has of its
repetition is almost infinite. able to repeat the same act (per se)
over indefinitely.

"Des Sciences de Poincare". Poincare points this out. He reduces all
Mathematics to this power.

Why is there irresistible evidence of 2+2=4. It is none other than
the repetition of the same act once the mind knows this as possible.

But, does repetition generate number? No, not this abstract individ-
uation.
        e.g. call a, a, a, etc.
                   1   2   3
It is only possible to generate a 1:1 ratio. This generates only
a logical correspondent to number. Numbers are strictly one, the pred-
icamental number. What does this mean?

Cf. Meta. V, 16, n.992.   3+3 are not the same as 6. Homogeneous
                           terms are necessary. We can call these six
                           but we are using the symbol, not the pred-
                           icamental number. We have six ones but not
                           one six.

Cf. Meta. VII, 1.13, 1588-1589.

A line is one in act, many in potency. If number should be per se
one, the many in it must be potential with respect to some principle

Democritus was right when he said that from 2 one could not be made
or from one could come two. But he did not distinguish act and potency
and was left with multiplicity.
Magnitude must be composed of indivisibles; if no distinction of act
and potency, then magnitude would be impossible. Kant's (?) difficulty.
When we speak of infinity in potency (as in the line) Kant said if
there exists infinity in potency then there must be infinity in act
(potency always in relation to act) which is impossible. Complete mis-
understanding!

If number is merely a composition of units then it cannot be one.
Therefore number in this strict sense cannot be possible.
To consider 3 in its constituent parts, you are not considering
number formally.

Cf. Meta, VIII, les. 3, no. 1725.

Three differs from two by ultimate unity; number has it specific
kind because of the ultimate unity. Must be homogeneous. Any term
can serve as the ultimate term.

De Koninck

Cf. Thome I, Cursus Theologicus Part II, q.6, art.2

By destroying this ultimate unity, there is left only groups and classes, according to Russel. Those who say they are absolute entities cannot say by which number they are "one".

Number is defined by the function of the class; this is merely our numerus numerans. This of reason; aggregatio=class, bundle. The es species of number does not equal class.

Cf. Bk. 7, q.7, #1673 If one takes unity only from parts as from a class then one has only an accidental number. One is merely considering the aggregate. Species secundum quid, not per se. Its unity is from something extrinsic to the part.

The multitude which the mind creates is not strictly mathematical, quantitative number.

Number is a division of predicamental being. But the abstract multiplicity does involve homogeneity, but predicamental number implies homogeneity in simultaneity and whose existence would only be realized materially.

_ _ _ _ _ _ _ _ _ _ _ (end of sixth day) _ _ _ _ _ _ _ _ _ _ _ _ _

7th Day

Rapid Review:

Intelligible Matter: its notion has no basis in reality or in the Aristotelian system itself. Confusion in Aristotle's mind.

Is there anything in Mathematics or in quantity that calls for intelligible matter? This is the study of the subject itself. As soon as things are distinct only numerically (this and that) we must have a subject to make this homogeneity possible.

Next, we want to show that intelligible matter is in St. Thomas and Aristotle.

Quantum=comprises homogeneous parts, they are parts of substance and so are intelligible matter. Intelligible matter is not quantity but that of which quantity is the order.
Matter=nature of Subject.
Intelligible=substance attained only by the intellect.

There is sensible individual matter and mathematical individual matter. What a thing is and the thing are not the same.

Bk. 7 and 8 of Meta. It depends on the extrinsic principle of individuation, "materia signata et quantitata". Mathematical=intelligible matter. The intellect cannot attain singulars directly. There is a dependence on external sense and imagination.   Intellect depends:
1. on sense=sensible singulars
Bk. III De Anima.   2. on imagination=math. sing.
The intellectual imagination is comparable to "vis cogitativa" i.e. ratio particularis, the internal sense,

De Koninck -2-

This imagination is defined by the term of knowledge (not the principle); knowledge here must terminate in imagination.

sense knowledge   imagination (--- intellect

Radi Radical difference between:
1. sensible quantity (per se sensible, common sensible)
2. intelligible quantity (not per se sensible etc.)

Cf. De Trinitate and Ia q. 85
Intelligibilis-in the defintion equals
   a. known only by intellect; materia intelligibilis communis.
   b. known because of intellectus=phantasia, imagination; materia intelligibilis individualis.

So this intellect (imagination) can be called Intellect as "ratio particularis" can be called Ratio.
. . . .

There is materia intelligibilis in number as well as the continuum. The reason: homogeneity in both, hence both intelligible matter. Continuum is given usually since 1.) it is easier to see. 2.) it is prior to number (though absolutely number is prior) and 3.) the continuum has definite materiality.

      Then we had   sensible quantity
   the observation of: imaginable quantity   -to bring out the proper
                         geometry   character of mathematical
                                                 imagination.

Here, the difficulty of the comparison of the difference of mathematical imagination from sensible imagination.
. . . .

Then we applied the notion of intelligible matter to Pure Mathematics. Is there intelligible matter in pure mathematics? The occasion of this was the statement of Russel-"most of Mathematics is not about quantity at all."
We took the most abstract form of quantity conceivable; a. Even extension and space could be deduced without real extension or real space.

In all these cases, individuation was necessary and therefore there was some kind of intelligible matter. (homogeneity).

Is pure mathematics strictly mathematics? No! it is not about quantity proper, so non-mathematical. It is about a mental construct with similarity to mathematical entities.
This abstract mathematics is not about predicamental quantity; not about number or continuous quantity. Number here can be no more than a class; not number unum per se, predicamental. Russel was aware of this.
. . . .

Reading of Emmanuel Kant requires a knowledge of Intelligible Matter. e.g. his notions of space, extension etc.   (to be considered later)

Cf. Bergson "The Immediate State of Conscience" cap.2, p.15ff.

Number="collection of units", "synthesis of one and many." One must add that these units are identical among themselves. Number is the multiplicity of parts absolutely similar to one another.

Predicamental number can be given a name; numerus numerans-not name but symbols, only in the mode of name..a class.

Every clear idea of number will require space. Space=homogeneous exteriority-individuation. 1+1+1 will only equal 3 if they are homogeneous.

— —

Quote: We must first study the basic things. The great minds are interested in these "little things." The platitudinous minds roam among the great things with no knowledge of the little. I have a great respect for Leibnitz for trying to prove 2+2=4."

— —

Cf. "Critiques etc." of Kant. Very interesting to study in the light of intelligible matter. His "a priori space". It comes from the mind and terminates in the imagination. When we apply Mathematics to Physics we are applying this "quasi-a priori space" to the sensible. That is not to be criticized.

Cf. Phy. II, les. 3, no.8. Space in Mathematical-physics is not such a simple thing. That space is two things, the sensible and the intelligible. This is the difference between Math. and Physics and the Scientiae Mediae.

e.g. linea abstracta (math)
 |
 linea visualis (phys)

Cf. also Post. Anal. les. 41, no.7.
Three kinds of space can be had:

Space

Abstract=a over again; a is a is a. (even if only in mind)

Mathematical= i.e. math. exteriority.
 1. geometry
 2. arithmetic (exteriority matter only)

Natural= actually is place. Ultimate innersurface of surrounding bodies.

abstract space=the hyperspace, transcendental space of Modern Philosophy;

Mathematicism=a philosophy according to which all reality can be known only in or through Mathematics. It will never get beyond transcendental space.

Why is Mathematicism a type of Materialism? They say that all reality is amenable to Mathematics. There is the difference of abstraction of natural doctrine and abstraction of Math. In Math. we can consider separated matter, but it cannot be in reality as separated. If so, one's intelligible matter would become physical matter. If reality were the concretion of mathematical thought, then we would have nothing but matter.

Logically, most modern philosophy can be reduced to this materialism.

The question of Spiritual Homogeneity:
 -this is a question of homogeneity in separated substances. In angels, there are not many individuals of one species. Nor can the mode of thinking of Gabriel and Michael differ except specifically.
Let us consider one angel: in his thinking process, he can consider this object, forget it, then consider it again. These two operations are distinct in number but not in kind (the mode of thought is the same). There is a certain homogeneity - can we say, then, that there must be some sort of matter even in spiritual substances? But there can be no intelligible matter, even, in separated substances. The answer is to be found in the fact that the two thoughts do no coexist. They aren't parts of the same simultaneously given whole. They are not partes ex partes quoad se. They can co-exist only thanks to the imagination. There is here no predicamental number - any number must be of an actual whole.

Spiritual substances have no imagination in the sense of an internal sense. (If by imagination is meant the representation of things absent, angels have much more imagination than we, of course.)
But if imagination is essential to mathematics, can angels know math? They don't need imagination - their means of knowing is prior to the thing. They know imperfect, inferior things without becoming imperfect or inferior, because: quidquid recipitur ad modum recipientis recipitur. The good is in the thing. To love the bad is to be bad. But we can know the bad without becoming it.

We can't know the individual directly, not because it is an individual or has matter, even intelligible matter, but because our knowledge is posterior to the thing.
In angels, their intelligible species are prior to the things. Their intelligible species are derived from those of God Who makes the things.
 (thus ends our consideration of intelligible matter)

THE PRINCIPLE OF INDIVIDUATION IN REAL, SUBSTANTIAL, EXISTING INDIVI*
 DUALS OF THE SAME SPECIES.

will folow: Cursus Philosophicus, Tome II, part 3, q. 9, art. 3.4 4

I What do we mean by individuation?
 We can consider it in three ways:

 A. Metaphysically - the ultimate difference which contrasts the species. Man is not the ultimate difference, the individuals are. Not the genus animal, nor the species rational, but this particular individual is the ultimate difference.
 B. Logically - the relation of the 2nd intention subjected to predicates.

The relation of subjectibility to predication to one only.
e.g. Sortes - can only be predicated of himself. Sortes is
Sortes. Although man can be predicated of Sortes, Sortes
cannot be predicated of man, but only of himself.

C. Physically
  - the numerical unity by which a thing is indivisum in
    se et divisum ab aliis.
  e.g. Sortes is not composed of two Sortes'.

1. What is the principle of this individuation?
   Is the individuation of, say, Sortes, due to the
   constitutive parts + the nature, or to something
   extrinsic?

   The principle of individuation must belong to the sub-
   stance of the individual - not something extrinsic.
   Sortes must be an individual, but not be something added
   to him, for that that would be added to him would re-
   quire something added to it to make it individual and
   so on ad infinitum.
   That by which the individual is individual must be
   substantial with the individual. Man individual not
   differing in nature, but thanks to something that can
   be signated, this one is not that one. They differ
   by signation.

   We seek then a principle of multiplicity - a non-essential
   substantial difference, as it were.

2. The term of individuation.
   - the subsistence or suppositality - the substantial
   individual. Not individuated by subsistence itself.
   I subsist - as such I don't differ from some other man
   also subsisting. But the fact that I subsist presup-
   poses my individuation.

   Thus the humanity of Christ doesn't have its proper
   subsistence. If it did, He would be two persons. Not
   He, but They. Christ doesn't have the subsistence of
   man - His human nature is not specifically different
   from ours, merely individually different. This make Him
   closer to us, " and furthermore, that is what was in-
   tended."
   The Divine Person can't be the cause of the constitutive
   singularity of this human nature.

3. Various possible opinions on this subject:

   (a) - because individual implies unity, the individual *Ockham*
       seems to follow simply from being, for being = one.
       If being = one, being = individual.
   (b) - to be an individual is to be divided-from, hence
       individuation follows from (quantity) We speak of
       homogeneous individuality - this comes from quantity.
   (c) - we said that individual = an ultimate difference.
       Why not have that the principle. Sortes differs from *Scotus*

11

man in the same way that man differs from animal?
(d) - difference not essential. Sortes and I are both man.
    Our difference is not essential. We differ only accidentally.
(e) - But the principle of individuation is also the principle
    of incommunicability. But matter is incommunicable (not
    common matter) therefore it seems that (matter) might be the
    principle of individuation.

Based somewhat on the above, the following opinions may be advanced.
  These opinions are natural ones, somewhat like the early philoso-
  phical opinions of the Greeks. Even in their error they follow
  a sort of reasonable order:

Ockham 1. Everything is individuated by its own very being, by its entity.
    Being = one. Individual = being. :. Individual = one.

Scotus 2. The principle of individuation is extrinsic to the nature -
    coming to it individuated - we might call it its thisness,
    or haecceitas. By reason of this haecceitas it becomes indi-
    vidual and indivisible into subjective parts.
    The nature in itself is indifferent to individuation - this
    indifference is expelled by the arrival of haecceitas. As
    esse comes to essentia.

3. Principle of individuation must be that because of which Joe
   differs from Pete. This id quo has the nature of act - hence
   the principle must be a form. And this either the substantial
   form (e.g. Sortes is Sortes thanks to his soul) or, say,
   quantity (the order of the parts of substance).

        (Mathematical abstraction = formal abstraction.
          -quantity taken as form.)

4. Based on the consideration that the individual is incom-
   municable. That which is par excellence incommunicable is the
   ultimate substance. In material beings of the same species,
   matter is the ultimate incommunicable substance.

   However prime matter is communicable, as in a substantial
   change. This prime matter of wood become the prime matter of
   ashes. Here we speak of the substance of Sortes qua substance.
   Substantia = quod est in se et non in alio.

   Now even among those that hold that the ultimate substance is
   the principle of incommunicability we find differences:
   (a) - matter considered as such can't be the principle of in-
       dividuation, for the individual as such presupposes a kind of
       nature. We must have this nature before individuation.
       Therefore it would seem that matter in the pure state of
       potency can't be the principle of individuation.
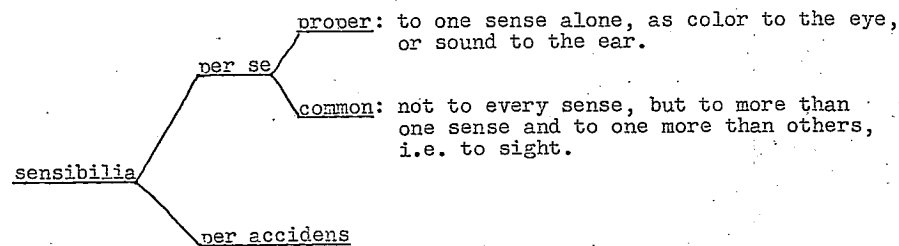   a suivre.

Introduction a la Physique Aristotlecienne par Chanoine Mansion.
- a criticism of Aristotle. His criticism is roughly as follows:.
Mathematical beings belong to the sensible world in the same way
as physical bodies considered in abstraction as universals belong
to that world. Therefore, mathematical beings are just as sensible
as sensible beings because they are quantity and quantity is some-
thing sensible. Therefore there should be no distinction between
Mathematics and physics. Mansions says that precisely because
universals are sensible Aristotle broke with Plato.
Sensible bodies and mathematical bodies are both bodies. Therefore
the Chanoine reasons, that if one has sensible properties and is a
body, the other, being a body, must also have sensible properties.
Why, then, an abstract mathematical world? Mansion says that there
is no such thing as intelligible matter, and that the term is just
another example of Aristotelian verbalism.

How do we go about a refutation? By distinguishing sensibilia:

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                      proper: to one sense alone, as color to the eye,
                              or sound to the ear.
             per se
                      common: not to every sense, but to more than
                              one sense and to one more than others,
                              i.e. to sight.
  sensibilia
             per accidens
```

The per se common sensibilia are either quantity or quantitative modes.
This is not the subject of mathematics! Now the common per se sensibilia
areobtained only thru the proper per se sensibilia. If mathematics
prescinds from the proper per se sensibilia it must of necessity pre-
scind from the common which are attained only thru the proper.

Quantity, the subject of mathematics, is considered apart from sensible
qualities, but it can't exist apart from them. And in nature we can
not find any quantity sans quality. Nor is the quantity perceived
by the senses ever found in mathematics. Mathematics is concerned
with abstract quantity.

Aristotle's abstraction in mathematics is not abstraction from the
singular in the sense of total abstraction. It is the abstraction
proper to mathematics.

Abstraction:
  I.   abstractio totius
          e.g. man as abstracted from individuals. From man ---
          animal. From animal -- natural living being. And so
          on to being (and non-being: ens verum).
        -the universality involved here is universale in praedicando.
        -the higher you go in this abstraction, the less determina-
         tion.

Greater potentiality, greater extension, but also greater
  confusion.
Total abstraction is proper to all the sciences.
Without this could never demonstrate things that all have
  in common.
As opposed to this universality of predication, the angels
  are capable of universale in repraesentando; they perceive
  things by the divine intelligible species which are in-
  fused into them.

II abstractio formae
      -consider something in separation, but in the mind only,
      cannot exist in separation in reality.
      -as in mathematics, we consider the form outside of the
      subject.
      -quantity (the order of the parts) as form of the subject
      of the order of the parts.
          (cf. De Trinitate, q.5, art. 3)

III separatio
       Had simply, if in considering a thing we prescind from all
       matter, sensible or sensible or intelligible common.
       As spoken of here, in this terminology as used here, is
       the abstraction proper to metaphysics.

                              o

Natural doctrine has only total abstraction (as spoken of here), but
  the others have it, too.   o

cf. IPhysics, lesson 1; Post. Ana. lesson 41; De Trinitate q.5, arts.
     1 and 2.

Diversitas formalis
    -division of the sciences. Scientia est certa cognitio per causas.
    It is demonstration and demonstration is had thru definition.
    (Meta VI, 1) If there are various kinds of definition, there will
    be various sciences.

  I. Definitio sine materia sensibile, sed cum materia commune.
        - as man cannot be defined except with common sensible matter.
          Even the soul cannot be defined without matter: anima est
          actus primus physici corporis.
        - here we also have a total abstraction, e.g. man from Sortes.
        - we have left behind something that impeded intelligibility.

  II. Definitio sine materia materia sensibile et commune, sed cum
       materia intelligibile.
       -here again a degree of intelligibility which is the principle
       of a certain type of conclusions.

  III. Definitio sine materia intelligibile etiam.
                              o
Abstractio formae = the abstraction of mathematics. This differs from
  the degrees of formal abstraction by which sciences are divided.

If total abstraction were the basis of division of the sciences, there
would be as many sciences as there are grades of generality.

Mathematics can be learned earlier by us because it requires less
experience.

Sortes
Man
Animal) all of these are fit subjects for the science that uses the
Mammal   1st degree of abstraction, if we merely leave behind the
etc.   individual sensible matter. They do not present different
problems.

This is all explained by the quidquid recipitur... The intellect is
act. Omnis scientia est in intellectu. Therefore, if any-
thing is to be known, it must become adapted to the intel-
lect, it must become act.

We have no grudge against the individual because it is an
individual, but because it is a __material__ individual, and
insofar as it is material, it is unintelligible.

°

Total abstraction: Figure

                              Plane fig.   Solid

                        Triangle   Circle

                Scalene   Equilateral

—we leave behind differences to find something common, we attain
a more universal predication. The more things embraced, the less
is something particular said of them.

Mension (mentioned previously) didn't distinguish between the mathematical
line and the sensible line.
He confused the universal as received in total abstraction (the
abstraction from something particular, e.g. man from Sortes, Tri-
angle from scalene triangle) as opposed to abstracting a universal
that is more intelligible in act.

                    sensible singular ---------- universal (actual
                                                           intelligibility)

Total abstraction is indifferent ot actual intelligibility.

Sortes (a)----------Man (b) ------------Animal (c)

From __a__ to __b__ I go from particular to universal and this differs radically
from the step from-__b__ to __c__, where I attain only more universality.
If I consider the step from __a__ to __b__ as making something definable -
then I consider it as the formal abstraction of Type I (definition
with common sensible matter) - this, of course, differs formally
from the other two types.

This word "formal" like many others in St. Thomas has many connotations.
It is necessary to consider the word in its context. Philosophy,
after all, is not a dictionary. In fact, history shows us that the
weakest philosophers were the most fecund in manufacturing new words.
Form is used two ways in different types of formal abstraction:
    a) it is used to imply the formal diversity between the three types
       of abstraction.
    b) Type III is as form (act) to II, and II is as form to I. I is as
       form or act to the sensible singulars.
                                            °

To know = habere formam alterius. To know is to be in some way, must be
something. In knowing, we have the form of another. To know an
apple: distinguish it by senses; that by which the apple is in me
and yet still on the tree - I am in some way determined by it.
Form, in this use, means species, not forma partis.

                                    °

Why should there be individuation? What the principle is not:
    One __opinion__: (the giving of opinions ans fighting our way thru
    them makes our knowledge more worthwhile.)
    Being is the principle of individuation. Of course, the principle
    __is__ being in some sense, but not that by reason of which it is
    some nature.

-there can be a being of such a nature that is this particular one and
not that. That is the nature can be 𝑏𝑜𝑡ℎ in both - one is not an in-
dividual by virtue of his nature.
-as it can't be the nature it must be that which receives the nature.
[As in mathematics, two identical spheres - how do they differ, if they
don't differ by that which they are? They must differ by that which
receives what they are, by the subject.[So in physical things, the
subject is the principle of indivudation. Not as that which is matter
in the species - the principle must be outside the reason of the species.

The nature of Sortes is individuated and as such belongs to the nature,
    but not because of the nature.
Cannot be predicated by either of the 1st two modes of perseity:

    1st mode: predicate of the definition or of some part of the def.
        (as we say rational or animal or both of man.) Can't say
        individual of man in this way.
    2nd mode: the subject is placed in the definition of the predicate.
        e.g. straight and circular are placed in the line. The
        subject must be placed in the predicate. Individual is
        not predicated in this way either.
(For the compete treatment of modes of perseity, cf. Post. Analytics I,
lesson 10)

Objection from the role of the efficient cause.
    If the cause is individual, the product must be individual. But
        what is the intrinsic principle of individuality in the product
        or for that matter of the agent. Does something extrinsic make
        the agent individual? Could go back ad infinitum.

What entitas is the principle of incommunicability? Or of what kind of
    incommunicability is it the principle?

Being = one. But there are different kinds of being so there will be
        different kinds of unity.
        If man has this particular kind of unity thanks to his species,
        what constitutes the unity of the species?
But we are interested in the principle of indivudation in this material
    individual. What kind of being individuates a material indivudal?
    If it can't be the matter, it must be a thisness. But what in the
    material individual causes this unity? A being, but what kind of 𝑏𝑒𝑖𝑛𝑔
    being?

If the principle were the form as explained in the contraction of forms
    the schema would go like this:
                    genus
                        . (by contraction)
                        speices
                            . (by contraction)
                            species
                                    .
                                    individual
    (But we can't go from the species to the individual, for this is
    radically different from the contraction from higher species to
    lower species.)

On all this see Cursus Philosophicus, Tome II, part 3, q. 9, arts. 3, 4.

cf. De Trinitate, q.4, art. 2

Both matter (sec. genus) and form (secundum to species) can be prin=
    ciples of diversity.
We want to know how those of the same species can be different in number.
No form can be "this" of itself. The rational soul is individuated but
    not because it is form. This soul is individuated due to matter,
    and even when separated from the body it is individuated because it
    is for this matter.
So long as we consider form only, it can be attributed to several indi-
    viduals. This form can be received in matter. Matter is the reason
    that there can be many individuals. But not matter in se, for it is
    indifferent as to what form it receives - it can receive any form or
    any species. If the subject must in some way be principle of indi-
    viduation, it will be that principle insofar as it is distinguishable.
    When form is received in matter it is not in any matter, but in this
    matter. But matter can be divided only by reason of quantity. Mat-
    ter is this, Sortes' matter is this, only insofar as it is subject
    of dimensions. But these dimensions aren't the principle of indiv-
    iduation, but it is only by reason of these that something can be con-
    sidered an individual.

Dimensions (dupliciter)

    1. Sortes now weighs 180 lbs and was once only 6 pounds. These
        dimensions considered thusly can't be the principle, for every
        type his quantity changed, Sortes dimensions changed too - if
        this were the principle, Sortes would be a new individual after
        every change.
    2. Dimensions can also be considered prescinding from these varia-
        tions. The extension changes, but this extension. This exten-
        sion is Sortes', his dimensions vary, but the varieties of the
        dimensions aren't the dimensions.

            This dimension is never without a certain termination. As
            the sahdes of a color vary, but the color is always the
            same.
                    termination is to dimension
                    as act   is to potency.

        Sortes is divided from every individual in his species, not
        because his dimensions are now such and such, but because they
        are his. That by means of which we can point him out remains
        the same throughout the variations. Matter is the principle
        of individuation insofar as it is quantity considered as
        divisive!
    No other accident could be the cause of the individual, for quantity
        is the only accident that is divisive, and division is what
        we seek the answer to. It is by reason of quantity that the
        accidents can be call 'this".

    MATTER IS THE PRINCIPLE OF INDIVIDUATION WHEN CONSIDERED AS
    QUANTITY UT DIVIDENS. In this respect quantity is prior
    to substance (not with a temporal priority) but prior in the
    order of material causality.

Not by reason of matter as matter, but by reason of matter considered as
   quantity as divisive. The subject of something which divides.
The individual enters into the predicament of substance, so matter taken
   as quantity ut dividens is the principle.
Divisibility (homogeneous division) can be found only in quantity.
   And quantity ut dividens is the cause of quantity ut terminans.

$$a = \quad ° \qquad ° = b$$

   Two identical spheres. a is not b. They have same form, they are
   homogeneous. But quantity is the principle of homogeneous divi-
   sion. But not just quantity - must be something substantial, have
   the nature of subject, that in quo this sphericity is. Can't be
   matter as such, either. Matter is indifferent as to the form it
   receives: must take matter as the subject of quantity as divisive:
   and this is the principle of individuation. But this principle,
   this quantity is not the same as the thing's magnitude. Matter as
   subject of interminate dimensions.

This is the final cause of mere numerical difference in individuals of
   the same species, of repetition. There are individuals either
   simultaneously or in succession. " Tomorrow and tomorrow and
   tomorrow creeps in its petty pace from day to day until the last
   syllable of recorded time."

Why individuation? Takes many bricks to build a house. Many trees to
   perpetuate trees. cf. Aristotle's de generatione et corruptione
   II, cap. 6, b25. Nature strives for the better. Being is better
   than non-being. All things can't have being because they are so
   far from the Being. Have it for a while, but they can't be simply.
   God solved this by making coming-to-be perpetually - the species at
   least is permanent. Corruptible things can be perpetual only by
   being continuously repeated: as opposed to things divine.

We are very far from the Being. To acheive perpetuity must have multi-
   plicity. cf. De anima A25 et seq. BEING, that by which other beings
   are. The most natural act, to produce another like oneself.
   Nothing perishable can be continuous - procreation is an attempt.
   Something specifically one, but not numerically one.
But this repetition is not an end in itself. It must be a means to
   an end, the reason can't be found in the repetition itself.

Let's consider things that are repeated. See how the necessity for in-
   dividuation arises as we move away from the Originative Source.
   We go from God (Pure Act) to Angels (created separated substances
   of which there are many: There are more angels than natural species.
   There is less resistance to multiplicity in them. That individuates
   angels - can't be matter, must be form. Res = quod quid erat esse.
   Thus every angel is a species and the genus angel is only a logical
   one. As angels are more perfect there will be a greater difference
   between them proportionately. Greater difference between angel A
   and B than between B and C and so on down the line. At the limit,
   just before "dropping into the cosmos" there will be two angels who
   will be nearly alike. And yet there are much more very perfect
   angels (because less resistance) and a greater difference between
   them. The lowest angel is more perfect than our whole cosmos, a
   pure and incomparably greater intellect. Each angel is a universe,
   if we call our deal a universe.

As we go down the line from God we find less simplicity) At the lowest
   limit we come to man: composition of matter and form and the quod
   quid erat esse no longer is the same as the res. We no longer have
   simplicity of essence and individualsaim the same species become
   possible. Now we distinguish the species (the res) and the essence.
   Humanity of Sortes isn't his being. The species subsists in itslef.

Quaestio disputata De anima, art. 17, ad 10.
   Many individuals of same species because there is a difference between
   the nature and the one having the nature.
   This is a humble place for man, but it is his place. He owes his
   existence to the most tenuous of causes: fortune.

The most important text: Metaphysics Bk. 7 and some of 8.