ROUND TABLE DISCUSSIONS

LOGIC AND METHODOLOGY DIVISION: *Panel:* FRANCIS X. FITZGIBBONS, FRANCIS X. MEEHAN, MUREL VOGEL.

Problem (*a*): An Evaluation of Symbolic Logic

At the meeting last year Father Gerard Minoque of Brooklyn, New York, concluded his paper with the following statement:

> It seems to me that a primary desideratum of the present day is the finding of a metaphysical foundation for mathematical logic, since the keenness and excellence of this new organon are far too precious to lose. I suspect that this can be found, since mathematical logic when applied to reality 'works'.[1]

In this paper I shall try to explain that metaphysical foundation by an analysis of the instrumentality of mathematical or symbolic logic. Since Scholastic analyses in this field are relatively recent, or at any rate not readily available for reference, it will be necessary to touch on several tangential topics in order that the resolution of the central point may be made clearer. Furthermore, the final conclusion as to the nature of symbolic logic is, for me, still tentative. Again, the limitations imposed by a paper of this kind will necessarily leave the greatest portion of the field unexplored. It is hoped, however, that others will be stimulated to give increasing attention to this most pertinent question.

The following statement by one of the modern symbolic logicians will serve to show the extent of our present analysis:

> Symbolic logic has so far been a domain of mathematicians. It grew from the soil of mathematics, it found its first successful applications in mathematics, and it remained accessible only to those who were trained in the mathematical technique.
>
> The present textbook of symbolic logic is written on the assumption that the new logic has a wider meaning, that it is on the march to replace the traditional Aristotelian logic in all fields, and that it can be taught to students who have no special mathematical training. I came to this opinion when I saw that an analysis of science and a general theory of knowledge demand the use of the methods developed in symbolic logic as much as does the analysis of mathematics; and I found overwhelming evidence for it in more than twenty years of academic teaching, which showed me both that symbolic logic is the best initiation to a scientific philosophy and that it can be taught to all who seriously desire to learn it.[2]

Among others, the writer makes the following points:

1. Symbolic logic has been identified with mathematical logic.
2. The 'new' logic is to replace Aristotelian logic.
3. An analysis of science demands the use of such a logic.
4. Symbolic logic is important as an initiation to philosophy.

[1] *The Three Fundamental Laws of Thought in Their Metaphysical and Logical Aspects,* Am. Cath. Phil. Ass., Proceedings, 1946.

[2] *Elements of Symbolic Logic,* H. Reichenbach, Macmillan, N. Y., 1947, p. v.

Because of the limits of this paper we pass over the second statement, the 'new' logic is to replace Aristotelian logic, and the fourth, symbolic logic is important as an initiation to philosophy, with only a categorical denial. The first statement, namely that symbolic logic has been identified with mathematical logic is important only to the extent that it show the dependence of this new logic on mathematics. It is in the third statement, however, that we shall find the clue to the surprising fertility of this method: An analysis of science demands the use of such a logic.

If we contrast the scientific method of Dirac and Milne with that which was in vogue even fifty or sixty years ago, we see more clearly the truth of such statements as the following: 'Physics . . . (is) a correlating of measurements, instead of a deciding of the ultimate nature of anything.'[3] 'That science is concerned with the rational correlation of experience rather than with the discovery of fragments of absolute truth about an external world is a view which is now widely accepted.'[4] Dirac and Milne, possibly more than any of the other modern physicists, have made extensive use of matrices and group theory in the development of their physical science. The employment of such 'mathematics' in physical science makes the distinction between classical mathematics and modern mathematics more obvious. In the discussion to follow, if we bear in mind the notion that modern mathematics developed largely from the attempt to furnish a rigorous proof for propositions in mathematics, the present tendency to make it a 'formal logic' applicable to all knowledge will be understandable. The only additional notion required is that of certain of the modern epistemologists who claim that the 'exact' physical sciences are the criteria for all true knowledge.

Both Aristotle and St. Thomas saw the necessity of 'impressing an intelligibility' on things by the use of mathematics. Their development of the notion of a *scientia media* whose object was both natural and mathematical was the first step in the analysis of the eternal dialectic involved in the attempt to know natural things *in concreto*. However, we should notice that the mathematical sciences which they spoke of were those developed by the early Greeks and summarized by Euclid in the *Elements*.

These *scientiae mediae* in the strict sense soon reach their limit. The world is soon perceived, in its further concretion, as non-mathematical. For this reason, while this method in its first steps is scientific, further explanation according to this mode tends to be more and more dialectical.[5] The light of mathematics becomes too dim to illuminate the natural subject in itself. It is here that the human intellect is able to step in with a super-dialectic already devised by the earlier students of 'pure' mathematics. The development of this organon, which can be traced back to

[3] *Time, Knowledge and the Nebulae,* M. Johnson, Dover, N. Y., 1947, p. 148.

[4] Quoted by Eddington in *The Philosophy of Physical Science,* Macmillan, N. Y., 1939, p. 184.

[5] Cf. St. Thomas, *De Caelo et Mundo,* Bk. I, lect. 3, n. 6; also I *Posteriora,* lect. 15, nn. 5-6, and lect. 17.

the first applications of mathematics by Archimedes, has given modern mathematics a character entirely different from that of Euclid.[5a]

This transition from the domain of classical mathematics to that of the modern dialectical construction is easy, once started. There are many similarities between them from the noetical point of view. In fact, these similarities have confused most writers to such an extent that they insist upon identifying them. The first similarity is found in the notion of 'constructability' used by modern mathematicians. As Cassirer says:

> The logical nature of the pure functional concept finds its clearest expression and most perfect example in the system of mathematics. Here a field of free and universal activity is disclosed, in which thought transcends all limits of the 'given'. The objects, which we consider and into whose objective nature we seek to penetrate, have only an ideal being; all the properties, which we can predicate of them, flow exclusively from the law of their original construction.[6]

This notion of the constructability of mathematical concepts has been mistakenly identified with that found in classical mathematics. Such interpreters should be warned by Cassirer's statement that in this modern construction we transcend "all limits of the 'given'". This is not true in Euclidean mathematics. In this latter science we are always held to the exigencies of *quantitas interminata.*

The second similarity between classical and modern mathematics is found in the abstraction by which each attains its object. The objects in classical mathematics are abstracted from sensible matter. This is true also of the entities of modern mathematics. Again, however, the identification on this ground is too facile. The modern mathematician is dealing with *entia rationis,* while the classical mathematician has a quite different object. This distinction has been clearly set forth by John of St. Thomas:

> Thus mathematics considers quantity precisely as to its having non-terminated extension and according to what it has from matter, but not according to the termination and mode which it has from form by reason of which it is made sensible. Therefore mathematical quantity has a positive concept of non-terminated quantity in the way in which quantity can be found whether imaginarily or sensibly in the nature of true being. Thus it is permissively related to the nature of real and true being: neither positively including and considering adequately nor positively excluding by repugnance the reality of quantity itself. In this respect it differs from purely imaginary quantity which is *ens rationis:* the latter is repugnantly related to real quantity because it is *ens rationis.*[7]

A third similarity between the classical and modern notions is found in the adaptability of both to the field of experimental science. For example,

[5a] This is important in any evaluation of modern comparisons between 'Euclidean' and 'non-Euclidean' mathematics. Too often writers assume that Euclidean mathematics is merely an earlier attempt to establish a 'rigorous' foundation for its proofs. By so doing, of course, they miss the point completely.

[6] *Substance and Function,* Open Court, Chicago, 1923, p. 112.

[7] *Cursus Theologicus* (Solesme ed. 1931), T. I, Disp. VI, a. 2, p. 534a.

we speak of motion in a straight line and we speak of an ideal gas which is a fixed relation of pressure to density and temperature.[7a] Here again we must examine the two more closely. A sensible gas approaches the perfect realization of this fixed relation as a limit, but whether there *is* such a limit is unknown. On the other hand, a sensible line approaches the mathematical straight line as something known as existing. The case of the ideal gas is comparable to the principle of inertia in this respect. When we call a gas 'ideal' we mean that it is a mental construct suggested by experience but in no way directly based upon experience.

If we pause for a moment and ask why it is that this new and very effective instrument, developed as it was from algebra, analytical geometry, the calculus and number theory, happens to be available, we would find something like the following: The mathematicians had been developing this method in order to overcome the classical dichotomy between arithmetic and geometry. They were trying to 'rationalize' the continuum by reducing it to number.[8] The mathematical physicist, on the other hand, had been uncritically using the mathematical notions developed by the mathematicians. The recent developments in physics were made by Heisenberg, Eddington, Dirac and Milne when they realized that this mathematics is fundamentally a dialectic.

Historically, the mathematical development begins with algebra which no longer is restricted by distinction between arithmetic and geometry, but considers these as homogeneous.[9] Following this, the development of analytical geometry and the calculus led Descartes and Leibniz to envisage a universal mathematics which would be an organon for all knowledge. With the 'algebra' of Boole we see "for the first time that a complete and workable calculus is achieved, and that operations of the mathematical type are systematically and successfully applied to logic".[10] Finally the *Principia Mathematica* of Russell and Whitehead defines all arithmetical ideas in terms of 'logical notions'.

As indicated earlier, this organon is exceptionally useful in any further development of experimental science. However, to confuse it with the organon of Aristotle overlooks the fact that, for the latter, problems in logic are problems concerned with proofs and demonstrations. By this we mean,

[7a] The chemist would rather say that the ideal gas is 'one which would follow a fixed relation of pressure to density and temperature such that:

$$\frac{P}{dT} = \text{a constant.'}$$

[8] I hope to be able to go into this notion in more detail at another time. Those interested may refer to an article by A. Fraenkel, *Continu et discontinu,* published in the Travaux du IXe Congres International de Philosophie (Hermann et cie, Editeurs, Paris, 1937) for a fairly complete statement of the problem.

[9] Cf. *Procedures and Metaphysics,* E. W. Strong, U. of Cal. Press, Berkeley, 1936, pp. 73 ff.

[10] Cf. *Symbolic Logic,* Lewis and Langford, Appleton-Century, N. Y., 1932, p. 9.

as John of St. Thomas points out,[11] that the principal object of logic is the syllogism and, among syllogisms, demonstration. Aristotelian logic, since its end is knowledge, and not just any knowledge, but certain knowledge through causes, places its greatest emphasis on demonstration. This means that the 'Posterior' analysis, concerned with the matter, is an essential and even more principal part of logic than the 'Prior' resolution, concerned with the form. Symbolic logic, on the other hand, as can be seen by its historical development, is primarily concerned with the form of reasoning. This will become more evident when we examine more closely the intrinsic nature of this dialectic.

The metaphysical basis for such a 'formal logic' has already been adequately analyzed in an article by Dr. Charles De Koninck.[12] It consists in the construction of a 'rational' system out of the intellect's capacity to predicate a thing of itself with identity. For example, if we take anything whatsoever, we can predicate it of itself. Again, we can predicate the proposition of itself. Then, again, we can predicate the predication of the proposition of itself of *itself*, etc., etc., in infinitum. Such a structure will contain all the so-called 'primitives' used in the construction of modern mathematical systems. In the continued iteration of identity we develop a 'system' from which we can derive all the 'logical' notions used by the symbolic mathematicians.

A brief summary of such a construction can be seen in the following exposition. If we start with any object and symbolize it with the letter 'A', we can, with iterated, identical predication, derive the following system:[13]

[11] *Cursus Philosophicus* (Reiser ed., Marietti, 1930), Logic, p. 265. Respondetur, quod syllogismus, qui fit per tertiam operationem, est principale obiectum Logicae, et inter syllogismos ratione materiae principalior est demonstratio. Ratio est manifesta, quia, ut docet S. Thomas 1. Periherm. lect. 1. et 1. Poster. lect. 1., omnes tres operationes intellectus ita se habent, quod prima et secunda ordinantur ad tertiam ut ad principalem, in qua perficitur ratiocinatio et discursus, qui recte disponitur per syllogismum et certo ac firmiter resolvitur per demonstrationem. Cum ergo scientia Logicae proprie et essentialiter sit directiva rationis, illud est principale obiectum, ubi principalius invenitur ratiocinatio et discursus cum certitudine et sine errore. Contingit autem error vel ex defectu formae vel ex contingentia materiae. Ergo principale obiectum ex parte formae est syllogismus recte dispositus, principale autem ex parte materiae est demonstratio, in qua invenitur processus rationis sine errore tam ex parte materiae quam ex parte formae. De his tamen, quae pertinent ad alias duas operationes, etiam per se agit Logica, licet minus principaliter.

[12] *La dialectique des limites comme critique de la raison,* Laval Theologique et Philosophique, Vol. I, no. 1, pp. 177 ff. 1945.

[13] For those who may prefer a geometrical image of this process I append an illustration of the same basic construction. This can be seen in *The Open World,* Hermann Weyl, Yale U. Press, New Haven, 1932, p. 65. If

$$\begin{gathered} A \\ A = A \\ (A = A) = (A = A) \\ [(A = A) = (A = A)] = [(A = A) = (A = A)] \\ \text{etc.} \end{gathered}$$

Here we have created a purely rational world of relations. As De Koninck says:

> The relation of identity of A to A can be turned back on itself to infinity. By so doing, it (the intellect) produces a purely rational order of time and position. It can compare these relations with each other and thus generate all the species of relations: equality, similarity, priority, etc., as well as their contraries, all purely rational. Notice that throughout this proliferation it is always the same object in itself that is being carried over; it is the identity of this which assures the rational purity of the system in question. In itself, it is the same A that we encounter at every point in the system. Between any two terms, however close they may be, we can interpolate any number of relation-terms and make the system as dense as we wish. In all this proliferation, however varied it might be, the object in itself is never separated from itself. Any term of the system is capable of a prolifera-tion identical to that of any other. Each of the terms of the ensemble can be elaborated into an ensemble identical at every point to that of which it is a part. What is said of the whole can then be said, in its turn, of any part. In each member of the ensemble we can encounter the same ensemble, etc.[14]

It should be noted that he has said that the intellect " produces a purely rational order of *time and position* ". In fact, the very nature of a created intellect imposes this necessity: " In going from one thought to another, every created intellect manufactures time." [15] Obviously the time that we

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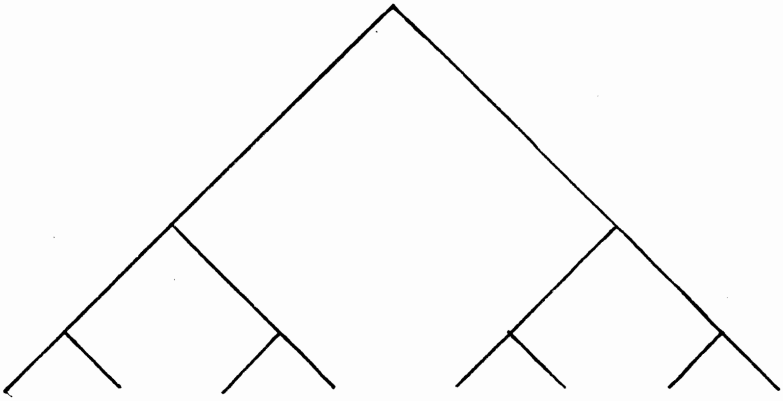
we use equal angles and maintain a fixed proportion, less than 1 to 1, of the later arms to the earlier, we shall derive the following figure of infinitely increasing infinities:



[14] *Op. cit.*, p. 183.

[15] *Ibid.*, p. 179.

have generated is not *numerus motus secundum prius et posterius;* at least we cannot take motion in the strict sense of *actus entis in potentia in quantum huiusmodi.* Rather is it a time which John of St. Thomas defines as *illa mensura quae constat ex pluribus indivisibilibus operationibus, quarum una succedit alteri sine motu fluente intermedio.*[16] This is called discrete time because it lacks the *numerus motus* and retains only the *prius et posterius.* The relevance of this for modern physical theory has been pointed out by Dr. Martin Johnson:

> The point common to the relativities of Einstein and of Milne is the fixed number 'c'. With Einstein the logical status of such fixing is seen in the empirical fact that the velocity of light as measured is constant for all observers. This includes, for instance, the fact that astronomical data on double stars would never have been capable of coordination if the constancy were not true to high accuracy. On the other hand, in Milne's treatment c is an agreement between observers to use the same numerical factor in building coordinates out of each man's immediate temporal experience. Milne's treatment enables (a) the basis of physics in temporal consciousness, and (b) the ground for communicability of any scientific information, to become interrelated.[17]

The facility with which this rational system can be applied to physical science is startling only if we fail to realize the dialectical nature of experimental science. If we understand more clearly that the experimental scientist is never able to abstract the nature of the object which he studies, that his reasoning is always hypothetical, we shall be in a better position to see why this organon developed by modern mathematics 'works'.

The Victorian experimentalist with his notions of substance, matter, cause, etc. was fairly certain that his progress through theory, hypothesis and law brought him to a real knowledge of the world in itself.[18] The modern scientist's more modest claim is summed up in the following statement:

> 'Intelligibility' has thus become divorced from fictitious claims to 'know what thing underlies phenomena' in the sense of devising a causal model of mechanism and expecting it to be a picture reduced in size from perceptual experience. If the differential equations and the appropriate transformations provide predictions quantitatively checked in experiment, we have the degree of understanding which is desired for those aspects of Nature interesting to physics.[19]

In other words, the modern scientist is content to pose questions which Nature can answer affirmatively or negatively. This is the attribute characterizing the dialectical question, as Aristotle points out in the *Topics* (158a 14-20):

> Not every universal question can form a dialectical proposition as ordinarily understood, e.g. 'What is man?' or 'How many meanings has "the good"?' For a dialectical premiss must be of a form to

[16] *Curs. Theol.,* T. II, Disp. 10, a. 3, p. 97a.

[17] *Op. cit.,* p. 98.

[18] Cf. *Science and the Meaning of Truth, M. Johnson* (Faber and Faber, London, 1946), pp. 20 ff.

[19] *Ibid.,* p. 60.

which it is possible to reply 'Yes' or 'No', whereas to the aforesaid it is not possible. For this reason questions of this kind are not dialectical unless the questioner himself draws distinctions or divisions before expressing them, e.g. 'Good means this, or this, does it not?' For questions of this sort are easily answered by a Yes or a No.

This dialectical character of modern science seems to be understood by most of the modern scientists. On the other hand, their conviction on this point seems somewhat doubtful, if we are to believe the following statement by Eddington:

> Keeping to physics, the commonly accepted scientific philosophy is that it is not concerned with the discovery of absolute truth about the external world, and its laws are not fragments of absolute truth about the external world, or, as I have put it, they are not laws of the objective world. What then are they, and how is it that we find them in our correlations of experience? Until we can see, by an examination of the procedure of correlation of our observational experience, how these highly complex laws can have got into it subjectively, it seems premature to accept a philosophy which cuts us off from all other possible explanations of their origin. This is the examination that we have been conducting.
>
> The end of our journey is rather a bathos after so much toil. Instead of strugging up to a lonely peak, we have reached an encampment of believers, who tell us "That is what we have been asserting for years". Presumably they will welcome with open arms the toilworn travellers who have at last found a resting place in the true faith. All the same I am a bit dubious about that welcome. Perhaps the assertion, like many a religious creed, was intended only to be recited and applauded. Anyone who *believes* it is a bit of a heretic.[20]

Thus, while most scientists in their methodological analyses would agree that 'the wave form might better play the part of propositional function', they cannot help feeling that corroboration by experiment should give some 'existence' to electrons, protons, neutrons, neutrinos, etc.[21]

As indicated earlier in the quotation by Hans Reichenbach, most symbolic logicians have been so impressed by the success of their method in the experimental sciences, without understanding just why it works, that they are led hastily to proclaim it as the universal method for all knowledge. The consequences of this are very serious. If this method, lacking a 'Posterior' analysis, is the sole method of arriving at knowledge, those who accept this position should be sophists, or at least skeptics. Fortunately, the better scientists and perhaps even some of the symbolic logicians do not go so far.[22] Unfortunately, the great distance which exists today between perennial philosophy and the physical sciences has caused the epistemological writings of these better men of science to be poorly received by scholastic philosophers.

However, in any epistemological misunderstanding between philosophers and scientists the onus rests on us. Moreover, although there are many scientists who mistakenly talk about the 'existence' of their dialectical

[20] *Op. cit.*, p. 186.

[21] *Ibid.*, p. 112.

[22] Cf. Eddington, *op. cit.*, pp. 189 ff.; and Johnson, *Time, Knowledge and the Nebulae*, pp. 130-131.

constructs, almost all scholastic philosophers, with far less reason, insist that protons, electrons, inertia, energy, etc. be interpreted existentially. But, if the classical physicists erred by imagining that the models drawn from a rationalization of our experience in terms of philosophical notions such as 'substance', 'matter', etc. represented the external world; and if later scientists erred by holding that a description in terms of Euclidean mathematics would adequately illuminate natural things in the concrete; the modern error of seeking the external world in the dialectical constructs derived from the intellect's exigence to create a discrete time is much more serious. Truly we are naturally metaphysicians!

I can well imagine that when the import of this analysis of the nature of experimental science is realized the objections will be many and vociferous, both those of the scientist and those of the philosopher. For the scientist, it is all right to call physics a 'set of Functional dependences' provided we do not examine too closely what is meant by a 'set of Functional dependences'. From the philosopher we run the risk of collecting the names 'Kantian', 'Idealist', etc. in their epithetical significance if we ascribe to any such notion of experimental science as the following:

> It is the fundamental hope, that the infinite variety of natural facts can be simplified through the physicist's process of abstraction into a manageable finite number of forms, that has given to recent science its striking preoccupation with the mathematical Theory of Groups. Until the last twenty years, few mathematicians and hardly any physicists had taken notice of this simple and beautiful but thought-provoking border-line between pure mathematics and logic. Eddington is the only writer with the audacity to expound it in delightful and readable language. The theory concerns not the grouping of quantities but of 'operations' both logical and quantitative, and of particular importance are the properties of grouping sets of Transformations. I have already explained why the analysis of Transformations is at the root of ascribing any meaning to Nature, because it settles how 'knowing' can become independent of the accident of individual circumstance.[23]

However, these objections overlook the paradox that, in spite of the dialectical nature of this method, or rather because of it, we actually are led further and further into the real external world. In one sense the assertions made on the basis of this dialectic are 'truer' than those given by any previous method. We get a more complete picture of the world in its concretion. On the other hand, we 'attain' this more complete picture even less than we did the earlier ones. We have only a *shadow world of symbols.* It cannot be too strongly emphasized that this kind of knowledge presupposes, much more than did the previous interpretations, a positive knowledge of the *principles* and *terms* of the dialectic. We can never generate 2 from the series 1, $1 + \frac{1}{2}$, $1 + \frac{1}{2} + \frac{1}{4}$, . . . We must know both the 2 and all the numbers composing the series, the limit and the principle of the 'set', beforehand. However, the mathematical example, using the known 2 as the limit, is deceptive. As applied to experimental science our 'sets' are better exemplified by that for the ideal gas. Here we see that the construction of the notion of the ideal gas and of the series leading

[23] *Science and the Meaning of Truth,* M. Johnson, p. 61.

up to it can only be made by an ever increasing knowledge of the nature of the limit and of the terms making up the series. Thus, in practice, this method tends to push the scientist further and further into reality, and also further and further into the field of dialectical construction. It is for this reason that there is a great danger for the symbolic logician who attempts to spin out this web within himself. Cut off from experimental science where this method is applicable, he becomes at best a dilettante, at worst a solipsist.

In the light of this analysis, the problem in modern mathematics between formalism and intuitionism becomes intelligible. Also we can understand the truth contained in the many remarks by Poincare, Hilbert and others classifying mathematics as a game. Mathematics, mathematical logic and symbolic logic, if we consider them in abstraction from the experimental sciences, *are* only a game; a game in which *we do not know what we are talking about nor whether what we say is true.* This profound insight into the nature of modern mathematics which led its innovators to call it a logic and to insist upon its formal character, has been largely nullified by an equally profound ignorance of traditional, or more properly, Aristotelian logic.

In conclusion I wish to repeat that this new organon is good, and not only good but, for the experimental sciences, necessary. Insofar as modern scientists persist in developing this dialectic, they will be drawing further and further away from the perverted notion of science as 'applicable' towards a greater appreciation of its speculative character. By so doing they will escape C. S. Lewis' excoriation of the 'magician-scientist' and be able to give an enthusiastic affirmative to his question:

> Is it, then, possible to imagine a new Natural Philosophy, continually conscious that the 'natural object' produced by analysis and abstraction is not reality but only a view, and always correcting the abstraction? [24]

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Problem (*b*): 'Essential Relevance' in Whitehead

Alfred North Whitehead makes a contribution which must be considered in any account of American philosophy. After he became a philosopher, his general purpose seemed to be a reinstatement of metaphysics. It was perhaps an impossible task, considering his background in mathematical logic, in empiricism, perhaps most of all in his feelings towards what he often called dogmatism, and what this audience might call perennial metaphysics. But the purpose was there, and the genius of the man brought him far along the way of achieving it, far enough to be instructive for us.

For one trained in scholastic philosophy, Whitehead is stimulating, and

[24] *The Abolition of Man,* Macmillan, N. Y., 1947, p. 49.